ABU DHABI UNIVERSITY

Abu Dhabi University

CEN 466 - Advanced Digital System Design

Assignment - 1
Questions 2.6, 2.10 and 2.30

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Section 1

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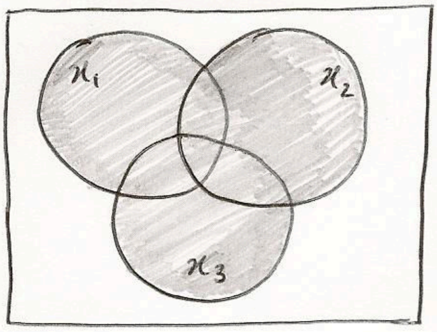
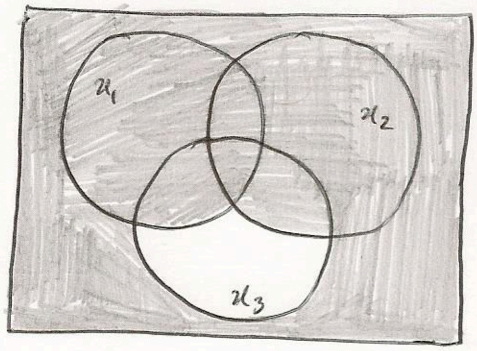
2.6 Use Venn diagram to prove that

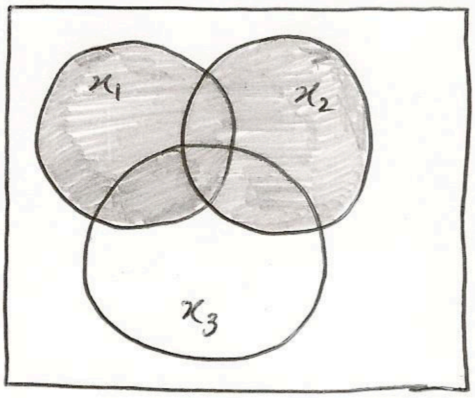
$$(x_1 + x_2 + x_3) \cdot (x_1 + x_2 + \overline{x_3}) = x_1 + x_2$$



$x_1 + x_2 + x_3$



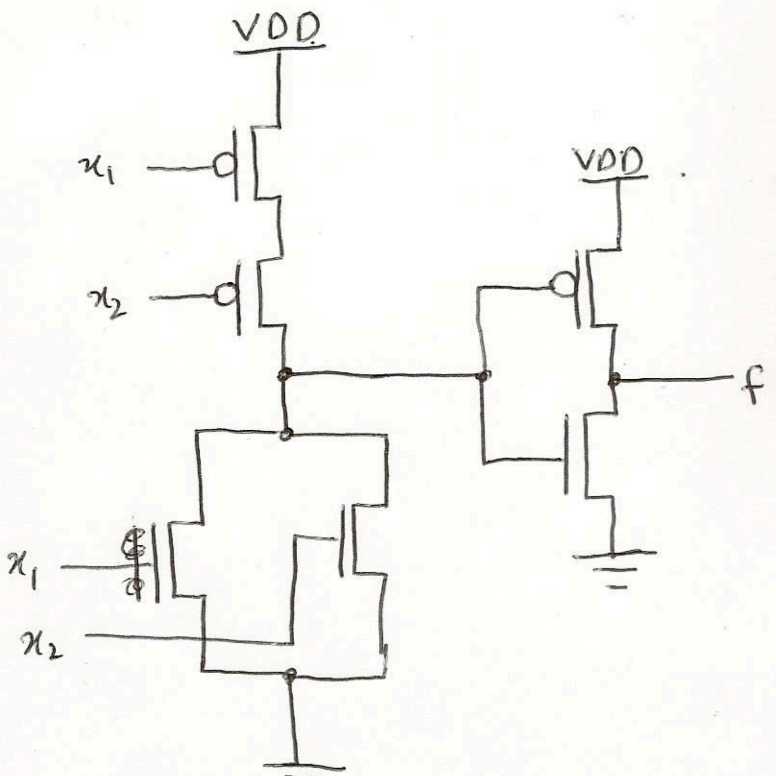
$x_1 + x_2 + \overline{x_3}$



$$x_1 + x_2 = (x_1 + x_2 + x_3) \cdot (x_1 + x_2 + \overline{x_3})$$

CMOS IMPLEMENTATION

$$f(x_1, x_2, x_3) = x_1 + x_2$$



2.10

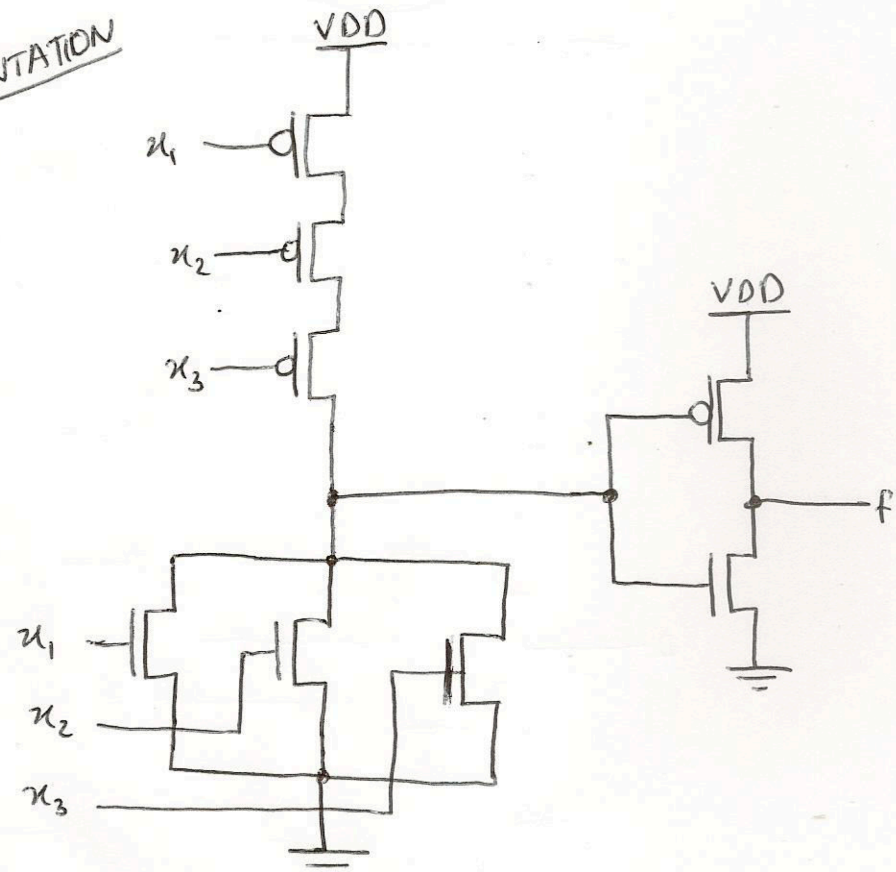
x_1	x_2	x_3	f
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

Taking Max. terms

$$\prod M(0) = x_1 + x_2 + x_3$$

$$f(x_1, x_2, x_3) = x_1 + x_2 + x_3$$

CMOS IMPLEMENTATION

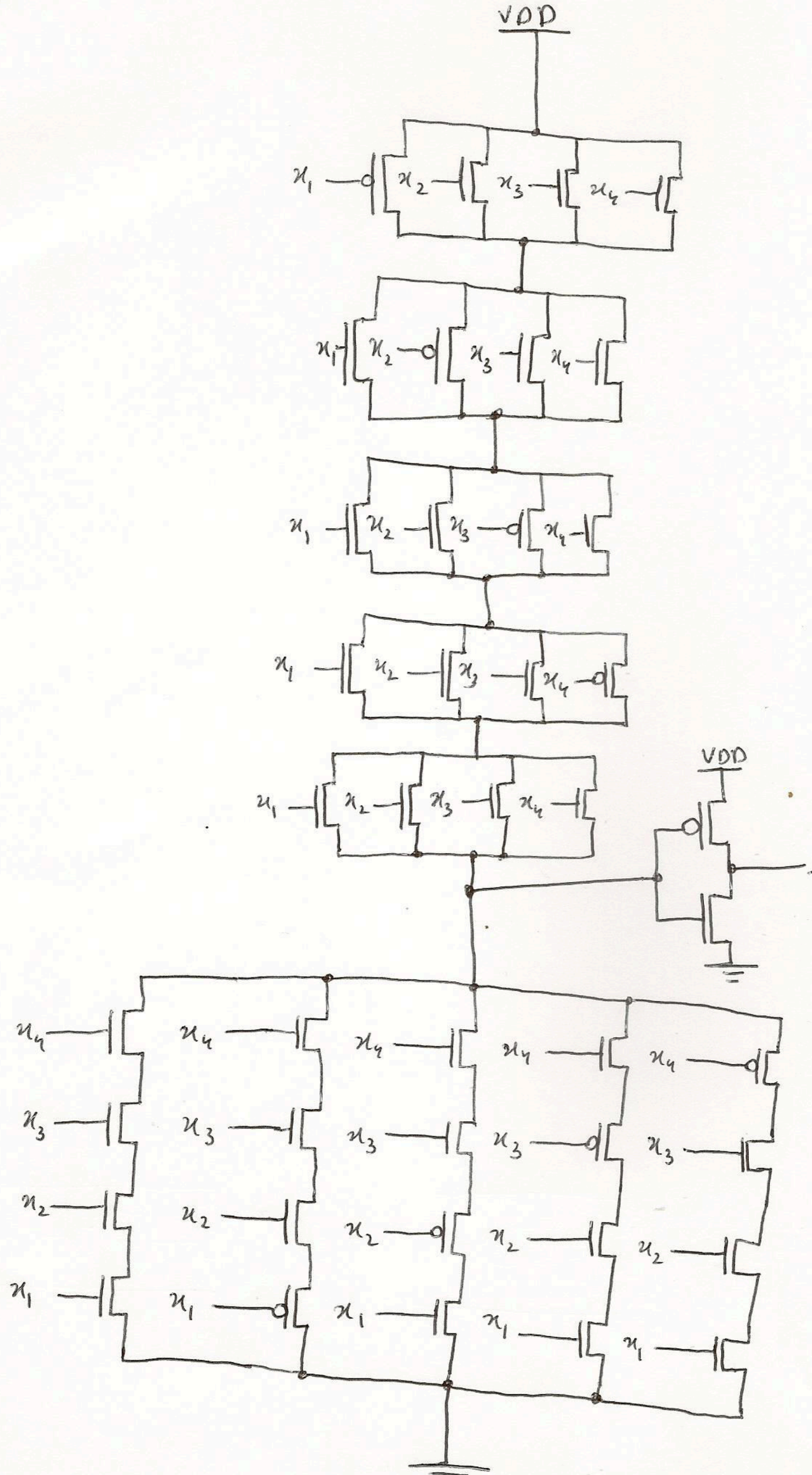


2.30 Design the simplest circuit that has four inputs x_1, x_2, x_3 & x_4 which produces an output value of 1 whenever three or more of the input variables have the value of 1, otherwise output has to be zero. (4-input voting function)

x_1	x_2	x_3	x_4	f
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

$$f(x_1 x_2 x_3 x_4) = \bar{x}_1 x_2 x_3 x_4 + x_1 \bar{x}_2 x_3 x_4 + x_1 x_2 \bar{x}_3 x_4$$
$$+ x_1 x_2 x_3 \bar{x}_4 + x_1 x_2 x_3 x_4$$

CMOS IMPLEMENTATION



Question 11

	x_1	x_2	x_3	f
0	0	0	0	0
1	0	0	1	0
2	0	1	0	0
3	0	1	1	0
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	1

$$f(x_1,x_2,x_3) = x_1 x_2 x_3$$

$$f = (x_1+x_2+x_3)(x_1+x_2+\bar{x}_3)(x_1+\bar{x}_2+x_3)(x_1+\bar{x}_2+\bar{x}_3)$$
$$(\bar{x}_1+x_2+x_3)(\bar{x}_1+x_2+\bar{x}_3)(\bar{x}_1+\bar{x}_2+x_3)$$

$$= \Big((x_1+x_2+x_3)(x_1+x_2+\bar{x}_3)\Big)\Big((x_1+\bar{x}_2+x_3)(x_1+\bar{x}_2+\bar{x}_3)\Big)$$
$$\Big((\bar{x}_1+x_2+x_3)(\bar{x}_1+x_2+\bar{x}_3)\Big)\Big((\bar{x}_1+\bar{x}_2+x_3)(\bar{x}_1+x_2+x_3)\Big)$$

$$= (x_1+x_2+x_3\bar{x}_3)(x_1+\bar{x}_2+x_3\bar{x}_3)(\bar{x}_1+x_2+x_3\bar{x}_3)$$
$$(\bar{x}_1+\bar{x}_2 x_2+x_3)$$

$$= (x_1+x_2)(x_1+\bar{x}_2)(\bar{x}_1+x_2)(\bar{x}_1+x_3)$$

$$= (x_1+x_2\bar{x}_2)(\bar{x}_1+x_2 x_3)$$

$$= x_1(\bar{x}_1+x_2 x_3)$$

$$= x_1\bar{x}_1 + x_2 x_3 x_1$$

Question 35

X		Y		
x_1	x_0	y_1	y_0	f
0	0	0	0	1
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	1
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	1
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	1

$$f = (x_1+x_0+y_1+\bar{y}_0)(x_1+x_0+\bar{y}_1+y_0)(x_1+x_0+\bar{y}_1+\bar{y}_0)(x_1+\bar{x}_0+y_1+y_0)$$
$$(x_1+\bar{x}_0+\bar{y}_1+y_0)(x_1+\bar{x}_0+\bar{y}_1+\bar{y}_0)(\bar{x}_1+x_0+y_1+y_0)(\bar{x}_1+x_0+y_1+\bar{y}_0)$$
$$(\bar{x}_1+x_0+\bar{y}_1+\bar{y}_0)(\bar{x}_1+\bar{x}_0+y_1+y_0)(\bar{x}_1+\bar{x}_0+y_1+\bar{y}_0)(\bar{x}_1+\bar{x}_0+\bar{y}_1+y_0)$$

$$f = (x_1+x_0)\left((y_1+\bar{y}_0)(\bar{y}_1+y_0)(\bar{y}_1+\bar{y}_0)\right)(\bar{x}_1+x_0)\left((y_1+y_0)(y_1+\bar{y}_0)(\bar{y}_1+\bar{y}_0)\right)$$
$$(x_1\bar{x}_0)\left((y_1+y_0)(\bar{y}_1+y_0)(\bar{y}_1+\bar{y}_0)\right)(\bar{x}_1+\bar{x}_0)\left((y_1+y_0)(y_1+\bar{y}_0)(\bar{y}_1+y_0)\right)$$
$$= (x_1+\bar{y}_1)(\bar{x}_1+y_1)(x_0+\bar{y}_0)(\bar{x}_0+y_0)$$

Question 41

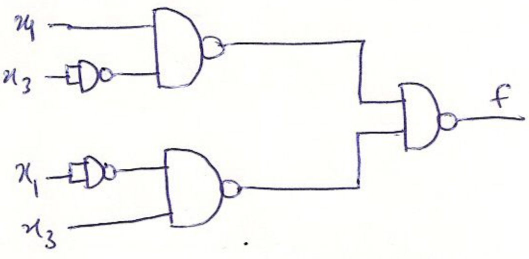
x_1	x_2	x_3	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

$\sum m(1,3,4,6)$

$$f = \bar{x}_1 \bar{x}_2 x_3 + \bar{x}_1 x_2 x_3 + x_1 \bar{x}_2 \bar{x}_3 + x_1 x_2 \bar{x}_3$$

$$f = \bar{x}_1 \left(x_3 (\bar{x}_2 + x_2) \right) + x_1 \left(\bar{x}_3 (\bar{x}_2 + x_2) \right)$$

$$f = \bar{x}_1 x_3 + x_1 \bar{x}_3$$



Question 47

$f_1 = f_2$?

$f_1 \qquad = \qquad f_2$

$$x_1 \bar{x}_3 + x_2 \bar{x}_3 + \bar{x}_3 \bar{x}_4 + x_1 x_2 + x_1 \bar{x}_4 = (x_1 + \bar{x}_3) \cdot (x_1 + x_2 + \bar{x}_4)(x_2 + \bar{x}_3 + \bar{x}_4)$$

$$f_1 = \bar{x}_3 (x_1 + x_2 + \bar{x}_4) + x_1 (x_2 + \bar{x}_4) + x_1 \bar{x}_3$$

$$\cancel{= (\bar{x}_3 + x_1)(x_1 + x_2 + \bar{x}_4)(x_2 + \bar{x}_4)}$$

$$= \bar{x}_3 (x_1 + x_2 + \bar{x}_4) + x_1 (x_2 + \bar{x}_3 + \bar{x}_4)$$

$$= (x_1 + \bar{x}_3)(x_1 + x_2 + \bar{x}_4)(x_2 + \bar{x}_3 + \bar{x}_4)$$