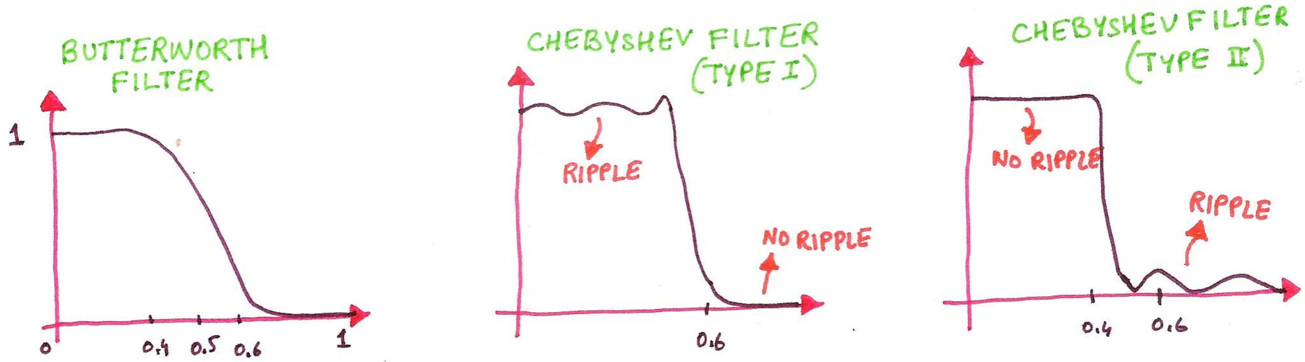
Tentative Problems.

Chebyshev Filters:

Chebyshev filters have steeper roll-off than butterworth filters but at the cost of having more pass band ripple (type I) or more stop band ripple (type II) than the Butterworth filters.



Problem 1:

Obtain the transfer function for an analog chebychev high-pass filter, which satisfies the following specification:

Pass band Edge Frequency = 3000 r/s
Stop band Edge Frequency = 1000 r/s
Pass band Attenuation = 0.5 dB
Stop band Attenuation = 22 dB

Afterwards, build up the circuit realization of the filter ($n = 3$).

Answer 1:

Since specifications for high-pass filter are given but we have the design Equations for designing a low-pass filter. We will convert the specification to match low pass filter.

Pass band Edge Frequency = 1000 r/s
Stop band Edge Frequency = 3000 r/s
Pass band Attenuation = 0.5 dB
Stop band Attenuation = 22 dB

← NEW SPECIFICATIONS

To make our calculations easier, we can reduce the passband and stopband Edge frequencies by some factor and later add these factors in the transfer function.

Ω_p ← Passband Edge Frequency =	1 r/s
Ω_s ← Stopband Edge Frequency =	3 r/s
R_p ← Passband Attenuation =	0.5 dB
A_s ← Stopband Attenuation =	22 dB

→ Parameters of Filter Design:

→ R_p = Passband Ripple [redacted]

→ A_s = Stopband Attenuation [redacted]

→ Ω_p = Passband cut-off Frequency

→ Ω_s = Stopband cut-off Frequency

LET THE DESIGNING BEGIN!

STAGE 1: FINDING PARAMETERS.

STEP (1):

$$\varepsilon = \sqrt{10^{0.1(R_p)} - 1}$$

$$\varepsilon = \sqrt{10^{(0.1 \times 0.5)} - 1} = 0.349$$

$$\boxed{\varepsilon = 0.349}$$

STEP (2):

$$A = 10^{\left(\frac{A_s}{20}\right)}$$

$$A = 10^{\left(\frac{22}{20}\right)} = 12.6$$

$$\boxed{A = 12.6}$$

STEP (3):

$$\Omega_c = \Omega_p = 1 \text{ r/s}$$

$$\Omega_r = \frac{\Omega_s}{\Omega_c} = \frac{3}{1} = 3$$

$$\boxed{\Omega_r = 3}$$

STEP (4):

$$g = \sqrt{\frac{(A^2 - 1)}{\varepsilon^2}}$$

$$g = \sqrt{\frac{(12.6)^2 - 1}{(0.349)^2}} = 35.989$$

$$\boxed{g = 36.0}$$

STEP 5:

$$N = \left\lceil \frac{\log_{10}\left[g + \sqrt{g^2 - 1}\right]}{\log_{10}\left[\Omega_r + \sqrt{\Omega_r^2 - 1}\right]} \right\rceil$$

$\lceil \; \rceil$ = take cieling

$$N = \left\lceil \frac{\log_{10}\left[36 + \sqrt{36^2 - 1}\right]}{\log_{10}\left[3 + \sqrt{3^2 - 1}\right]} \right\rceil = \lceil 2.43 \rceil = 3$$

$$\boxed{N = 3}$$

STAGE 2: FINDING POLES & ZEROS.

STEP 1: $K = 0, 1, \ldots N-1$

$K = 0, 1, 2$

$$\boxed{K = 0, 1, 2}$$

$p_K = \sigma_K + j\Omega_K$

$$\boxed{\begin{aligned} p_0 &= \sigma_0 + j\Omega_0 \\ p_1 &= \sigma_1 + j\Omega_1 \\ p_2 &= \sigma_2 + j\Omega_2 \end{aligned}}$$

Three poles. So Two of them should have imaginary part and should be conjugates of each others. One pole would be real.

STEP 2:

$$\alpha = \frac{1}{\varepsilon} + \sqrt{1 + \frac{1}{\varepsilon^2}}$$

$$\alpha = \frac{1}{0.349} + \sqrt{1 + \frac{1}{(0.349)^2}} = 5.90$$

$$\boxed{\alpha = 5.90}$$

STEP 3:

$$a = \frac{1}{2}\left(\sqrt[N]{\alpha} - \sqrt[N]{\frac{1}{\alpha}}\right) \qquad b = \frac{1}{2}\left(\sqrt[N]{\alpha} + \sqrt[N]{\frac{1}{\alpha}}\right)$$

$$a = \frac{1}{2}\left(\sqrt[3]{5.90} - \sqrt[3]{\frac{1}{5.90}}\right) \qquad b = \frac{1}{2}\left(\sqrt[3]{5.90} + \sqrt[3]{\frac{1}{5.90}}\right)$$

$$a = 0.627 \qquad b = 1.180$$

$$\boxed{\begin{aligned} a &= 0.627 \\ b &= 1.180 \end{aligned}}$$

STEP 4:

For p_0:

$$\sigma_0 = (a\,\Omega_c)\cos\left(\frac{\pi}{2} + \frac{(2K+1)\pi}{2N}\right)$$

$$\sigma_0 = (0.627 \times 1)\cos\left(\frac{\pi}{2} + \frac{\pi}{6}\right) = -0.3135$$

$$\Omega_0 = (b\,\Omega_c)\, \cancel{\cos}\, \sin\left(\frac{\pi}{2} + \frac{(2K+1)\pi}{2N}\right)$$

$$\Omega_0 = (1.180 \times 1)\sin\left(\frac{\pi}{2} + \frac{\pi}{6}\right) = 1.02$$

$$\boxed{p_0 = -0.314 + 1.02j} \rightarrow \text{Conjugate of } p_2$$

For p_1:

$$\sigma_1 = (a\,\Omega_c)\cos\left(\frac{\pi}{2} + \frac{(2K+1)\pi}{2N}\right)$$

$$\sigma_1 = (0.627 \times 1)\cos\left(\frac{\pi}{2} + \frac{3\pi}{6}\right) = -0.627$$

$$\Omega_1 = (b\,\Omega_c)\sin\left(\frac{\pi}{2} + \frac{(2K+1)\pi}{2N}\right)$$

$$\Omega_1 = (1.180 \times 1)\sin\left(\frac{\pi}{2} + \frac{3\pi}{6}\right) = 0$$

$$\boxed{p_1 = -0.627 + 0j} \rightarrow \text{Entirely Real}$$

For p_2:

$$\sigma_2 = (a\,\Omega_c)\cos\left(\frac{\pi}{2} + \frac{(2K+1)\pi}{2N}\right)$$

$$\sigma_2 = (0.627 \times 1)\cos\left(\frac{\pi}{2} + \frac{5\pi}{6}\right) = -0.3135$$

$$\Omega_2 = (b\,\Omega_c)\sin\left(\frac{\pi}{2} + \frac{(2K+1)\pi}{2N}\right)$$

$$\Omega_2 = (1.180 \times 1)\sin\left(\frac{\pi}{2} + \frac{5\pi}{6}\right) = -1.02$$

$$\boxed{p_2 = -0.3135 - 1.02j} \rightarrow \text{Conjugate of } p_0$$

STAGE 3: BUILDING UP THE TRANSFER FUNCTION

STEP 1:

Now the system function is :

$$\boxed{H_a(s) = \frac{K}{\prod_k (s+p_k)}}$$

Assume K=1 unless otherwise told

$$H_{LP}(s) = \frac{1}{(s+0.314-1.02j)(s+0.627)(s+0.314+1.02j)}$$

LOW PASS TRANSFER FUNCTION

SIMPLIFYING:

$$H_{LP}(s) = \frac{1}{(s^2+0.941s-1.02sj-0.64j+0.197)(s+0.314+1.02j)}$$

$$H_{LP}(s) = \frac{1}{s^3+1.255s^2+1.533s+0.714}$$

WOLFRAM CHECKED

STEP 2:

Low pass to high pass transformation.

$$S = j\omega \xrightarrow{\text{REPLACE}} \frac{\omega_0}{j\omega} \qquad \boxed{\omega_0 = 3000 \text{ r/s}}$$

$$H_{HP}(s) = \frac{1}{\left(\frac{3000}{j\omega}\right)^3 + 1.255\left(\frac{3000}{j\omega}\right)^2 + 1.533\left(\frac{3000}{j\omega}\right) + 0.714} \times \frac{(j\omega)^3}{(j\omega)^3}$$

$$H_{HP}(s) = \frac{(j\omega)^3}{2.7\times10^{10} + 1.13\times10^{7}j\omega + 4599(j\omega)^2 + 0.714(j\omega)^3}$$

STAGE 4: PASSIVE CIRCUIT REALIZATION.

FROM THE TABLE ON SLIDE 70.

Order = n = 3

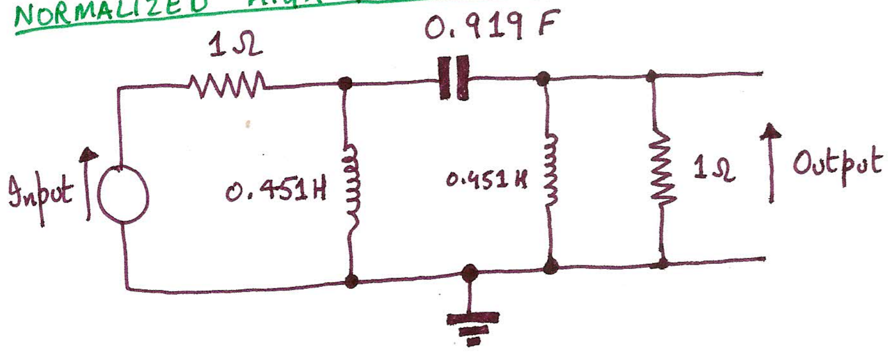
C1 = 2.216

L2 = 1.088

C3 = 2.216

> **WARNING:**
> THE QUESTION ASKS FOR 0.5dB of RIPPLE BUT IN THE TABLE IN SLIDES, THE VALUES ARE GIVEN FOR 1dB RIPPLE. IN EXAM BE CAREFUL.

STEP (1): NORMALIZED HIGH PASS FILTER.



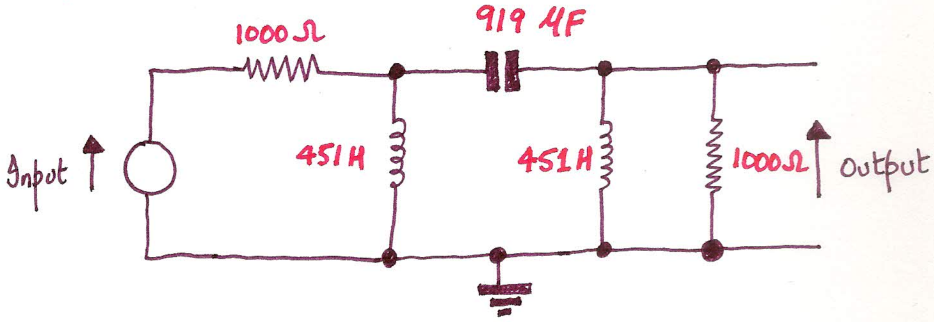
$\boxed{K = 1}$

STEP (2): CONSIDER SOME K. (SCALE IMPEDANCES)

If we choose some high value of K, then the values of capacitors and inductors reach the acceptable level.

$\boxed{K = 1000}$ → JUST RANDOMLY CHOOSEN

Now the circuit becomes:-

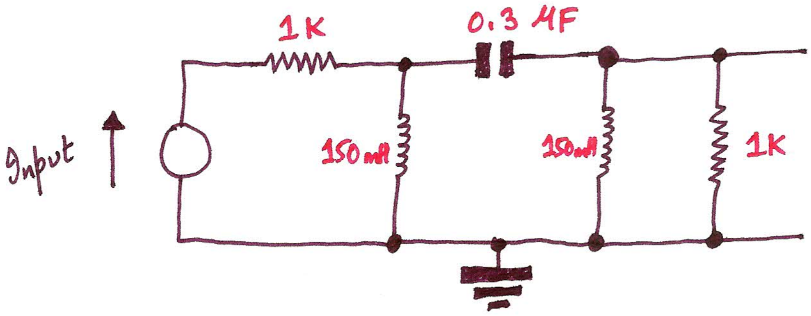


→ MULTIPLY RESISTORS & INDUCTORS VALUES BY K

→ DIVIDE CAPACITOR VALUES BY K

STEP (3): SCALE FREQUENCY.

Now we need to change the frequency at which oor circuit operates to the one required in the question



→ DIVIDE CAPACITOR & INDUCTOR VALUES BY Ω_p

STAGE 5: ACTIVE CIRCUIT REALIZATION.

STEP 1:

Divide the transfer function into first order and second order transfer function.

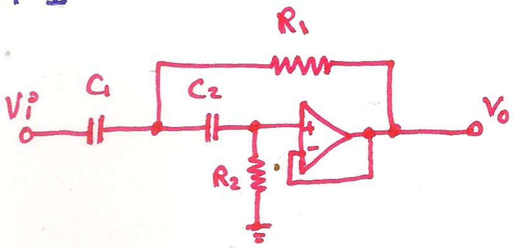
$$H_{HP}(s) = \boxed{\frac{(j\omega)^2}{(s^2 + 1656s + 7.9\times10^6)}} + \boxed{\frac{j\omega}{(0.714s + 3416.7)}}$$

This is a second order Equation which corresponds to Sallen-Key High-Pass Filter.

This is a first order high pass filter.

FOR SALLEN-KEY:

$$H_{hp}(s) = \frac{1.27\times10^7 (j\omega)^2}{1.27\times10^7 s^2 + 2.096\times10^{-4} s + 1}$$



$\therefore T_1 T_2 = 1.27\times10^7$

$2T_1 = 2.096\times10^{-4}$

$\boxed{\therefore T_1 = 1.05\times10^{-4}}$

$$\boxed{\therefore T_2 = \frac{1.27\times10^7}{1.05\times10^{-4}} = 1.21\times10^{11}}$$

$C_1 = C_2$

Let $C_1 = C_2 = 4\,F$

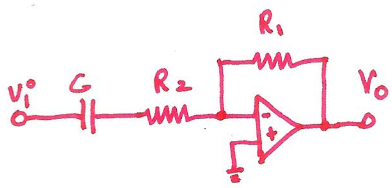
then

$R_2 = 3.025\times10^{10}\,\Omega$

$R_1 = 26\,M\Omega$

FOR FIRST ORDER HP:

$$H_{hp}(s) = \frac{\overset{H_o}{1.4}\; s}{s + \underset{\omega_o}{4785}}$$

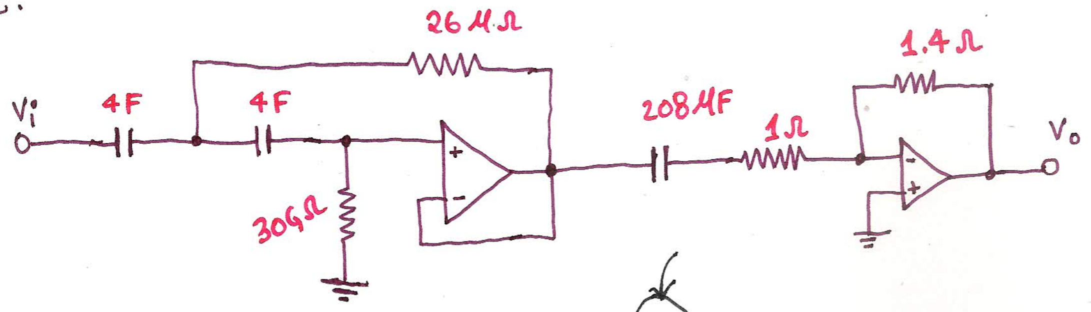


$$\omega_o = \frac{1}{R_2 C} \qquad H_o = -\frac{R_1}{R_2} \qquad |H_o| = \frac{R_1}{R_2}$$

$$1.4 = \frac{R_1}{R_2}$$ If $R_2 = 1\,\Omega$ then $R_1 = 1.4\,\Omega$

$$C = \frac{1}{R_2\,\omega_o} = \frac{1}{1 \times 4785} = 208\ \mu F$$

finally



YALLA ENJOY !