1. (50 points) A baseband digital communication system employs the signals shown below (Figure 1) for transmission of two equiprobable messages. It is assumed the communication problem studied here is a "one shot" communication problem, that is, the above messages are transmitted just once and no transmission takes place afterwards. The channel has no attenuation and the noise is AWGN with power-spectral density $N_0/2$.
   a. Find an appropriate orthonormal basis for the representation of the signals.
   b. In a block diagram, give the precise specifications of the optimal receiver using matched filters. Label the block diagram carefully.
   c. Find the error probability of the optimal receiver.
   d. Show that the optimal receiver can be implemented by using just one filter (see block diagram shown in Figure 2). What are the characteristics of the matched filter and the sampler and decision device?
   e. Now assume the channel is not ideal, but has an impulse response of $c(t) = \delta(t) + 0.5\delta(t - \frac{T}{2})$. Using the same matched filter you used in the previous part, design an optimal receiver.
   f. Assuming that the channel impulse response is $c(t) = \delta(t) + a\delta(t - \frac{T}{2})$, where a is a random variable uniformly distributed on [0, 1], and using the same matched filter, design the optimal receiver.

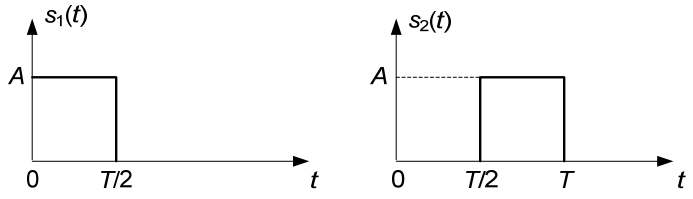

Figure 1

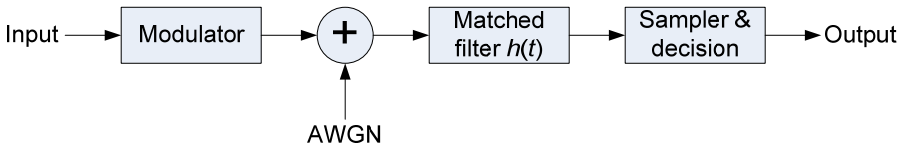

Figure 2

2. (20 points) Consider the transmission of data via PAM over a voice-band telephone channel that has a bandwidth of 3 kHz. Show how the symbol rate varies as a function of the excess bandwidth. In particular, determine the symbol rate for excess bandwidths of 25%, 33%, 50%, 67%, 75%, and 100%.

3. (30 points) Determine the tap weight coefficients of a three-tap zero-forcing equalizer if the ISI spans three symbols and is characterized by the values $x(0) = 1, x(-1) = -0.3, x(1) = 0.2$. Also determine the residual ISI at the output of the equalizer for the optimum tap coefficients.