Department of Electrical and Computer Engineering
University of Massachusetts/Amherst
ECE 645: Digital Communications Fall 2011
Homework #1, Due Thursday 09/29/2011 (in class)

4.4 Determine the autocorrelation function of the stochastic process

$$x(t) = A\sin(2\pi f_c t + \theta)$$

where f_c is a constant and θ is a uniformly distributed phase, i.e.,

$$p(\theta) = \frac{1}{2\pi}, \qquad 0 \leqslant \theta \leqslant 2\pi$$

4.11 Consider the four waveforms shown in Figure P4.11.

a) Determine the dimensionality of the waveforms and a set of basis functions.

b) Use the basis functions to represent the four waveforms by vectors $\mathbf{s}_1$, $\mathbf{s}_2$, $\mathbf{s}_3$, and $\mathbf{s}_4$.

c) Determine the minimum distance between any pair of vectors.

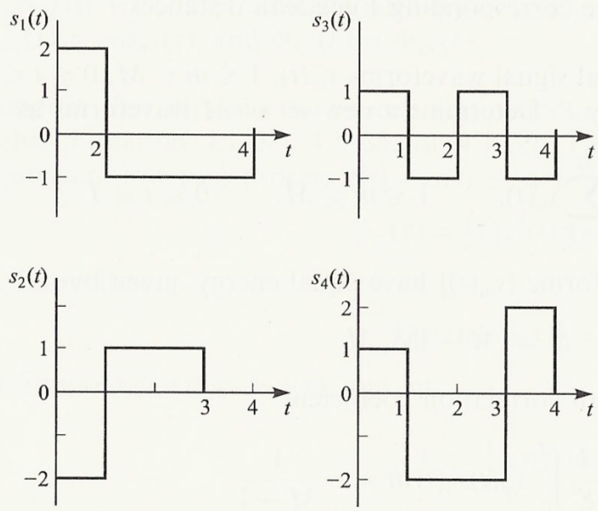

FIGURE P4.11

5.2 Consider the signal

$$s(t) = \begin{cases} (A/T)t\cos 2\pi f_c t. & (0 \leqslant t \leqslant T) \\ 0 & (\text{otherwise}) \end{cases}$$

a) Determine the impulse response of the matched filter for the signal.

b) Determine the output of the matched filter at $t = T$.

c) Suppose the signal $s(t)$ is passed through a correlator that correlates the input $s(t)$ with $s(t)$. Determine the value of the correlator output at $t = T$. Compare your result with that in (b).

5.8 The two equivalent low-pass signals shown in Figure P5.8 are used to transmit a binary sequence over an additive white Gaussian noise channel. The received signal can be expressed as

$$r_l(t) = s_i(t) + z(t), \qquad 0 \leqslant t \leqslant T, \qquad i - 1, 2$$

where $z(t)$ is a zero-mean Gaussian noise process with autocorrelation function

$$\phi_{zz}(\tau) = \frac{1}{2}E[z^*(t)z(t+\tau)] = N_0\delta(\tau)$$

a) Determine the transmitted energy in $s_1(t)$ and $s_2(t)$ and the cross-correlation coefficient ρ_{12}.

b) Suppose the receiver is implemented by means of coherent detection using two matched filters, one matched to $s_1(t)$ and the other to $s_2(t)$. Sketch the equivalent low-pass impulse responses of the matched filters.

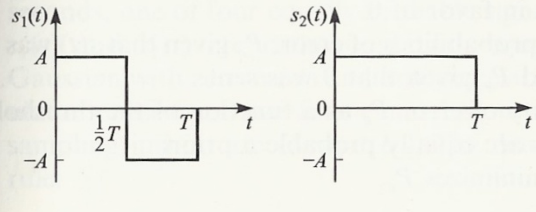

FIGURE P5.8

c) Sketch the noise-free response of the two matched filters when the transmitted signal is $s_2(t)$.

d) Suppose the receiver is implemented by means of two cross correlators (multipliers followed by integrators) in parallel. Sketch the output of each integrator as a function of time for the interval $0 \leqslant t \leqslant T$ when the transmitted signal is $s_2(t)$.

e) Compare the sketches in (c) and (d). Are they the same? Explain briefly.

f) From your knowledge of the signal characteristics, give the probability of error for this binary communication system.

5.19 Consider a signal detector with an input

$$r = \pm A + n$$

where $+A$ and $-A$ occur with equal probability and the noise variable n is characterized by the (Laplacian) PDF shown in Figure P5.19.

a) Determine the probability of error as a function of the parameters A and σ.

b) Determine the SNR required to achieve an error probability of 10^{-5}. How does the SNR compare with the result for a Gaussian PDF?

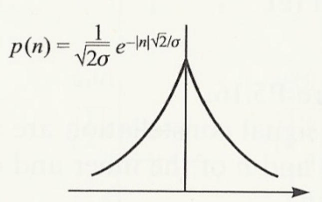

FIGURE P5.19

5.27 Consider the four-phase and eight-phase signal constellations shown in Figure P5.27. Determine the radii r_1 and r_2 of the circles such that the distance between two adjacent points in the two constellations is d. From this result, determine the additional transmitted energy required in the 8-PSK signal to achieve the same error probability as the four-phase signal at high SNR, where the probability of error is determined by errors in selecting adjacent points.

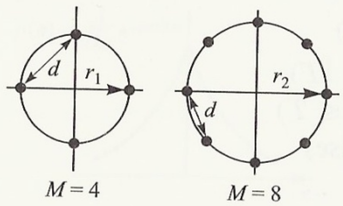

FIGURE P5.27