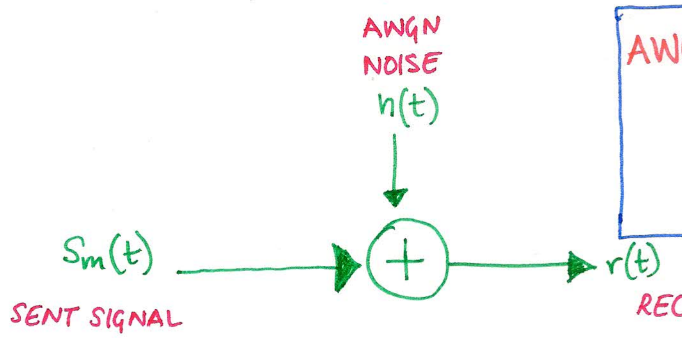
OPTIMUM RECIEVERS FOR SIGNALS CORRUPTED BY AWGN.

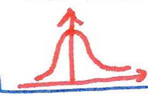
→ What happens to our transmitted signals ?



AWGN = ? = Additive White Gaussian Noise

Additive: It adds to original signal

White Gaussian: Gaussian Distribution



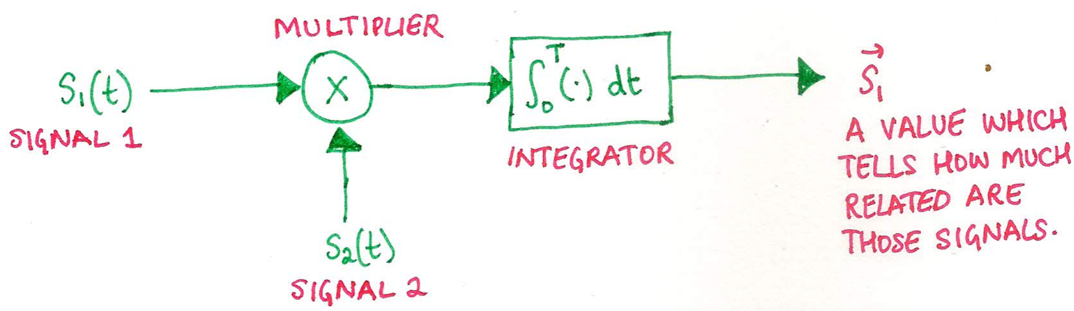
The AWGN, n(t) has power spectral density (power per frequency) $= \frac{N_0}{2}$ (Watts/Hz)

→ So how do we seperate this AWGN Noise ?

We do the prediction of original sent signal by two steps :-

1. Compare the recieved signal to the base signals.

   Comparison of two signals is done by multiplying two signals together and integrating from $-\infty$ to T.

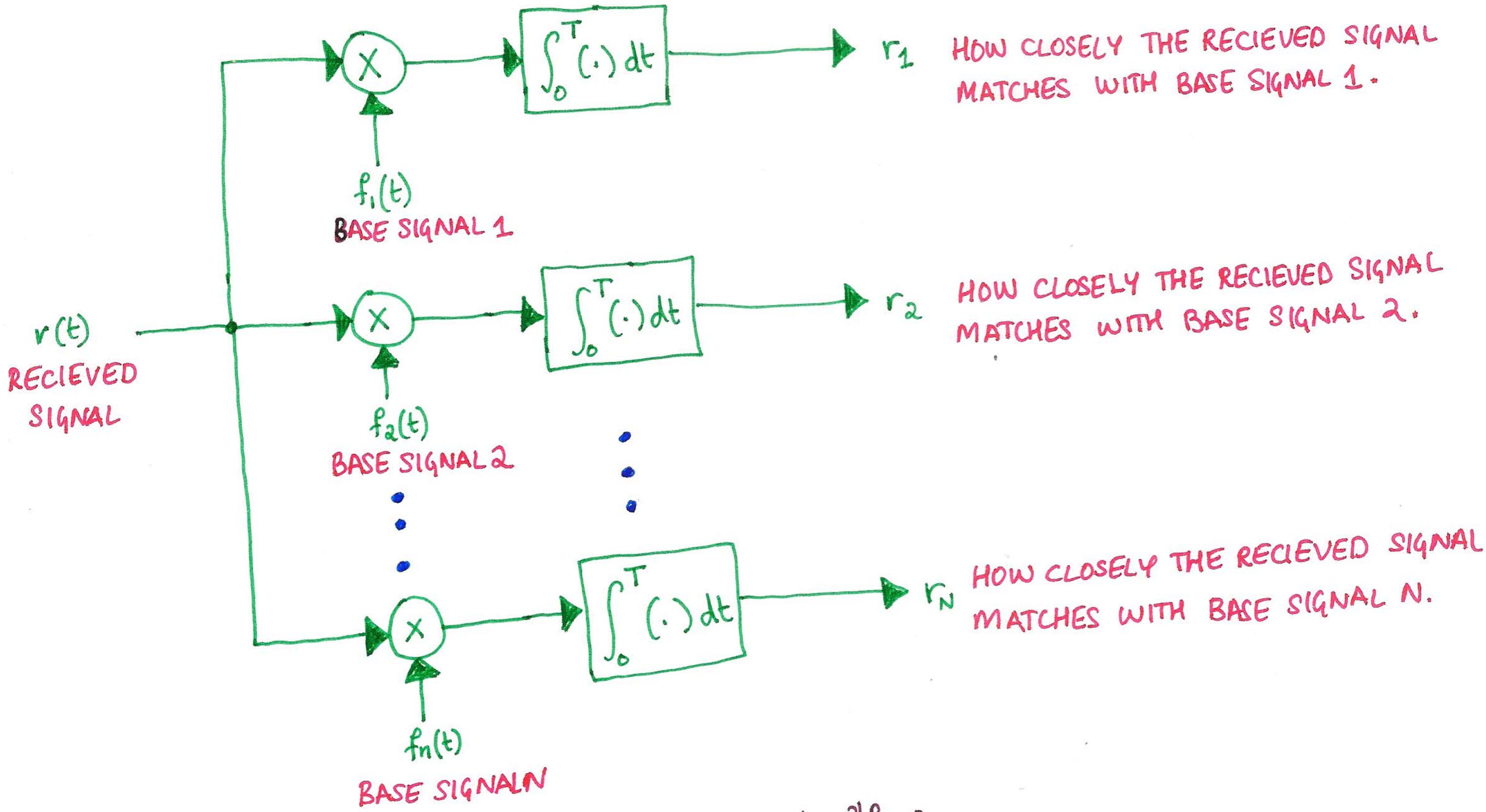


→ What are we really doing ?

Cross-corelation → $\int_{-\infty}^{\infty} s_1(t)s_2(t)\,dt$

This tells us how two signals are related and what is the common part.

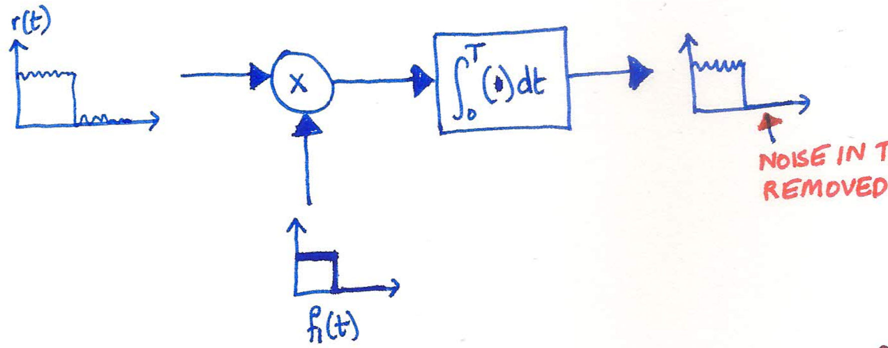
So we feed the recieved signal into corelators which compare the incoming signal to all the base signals.



Now this does two things which are positive:-

① Removes the noise which is orthogonal to base signals.

Eg.



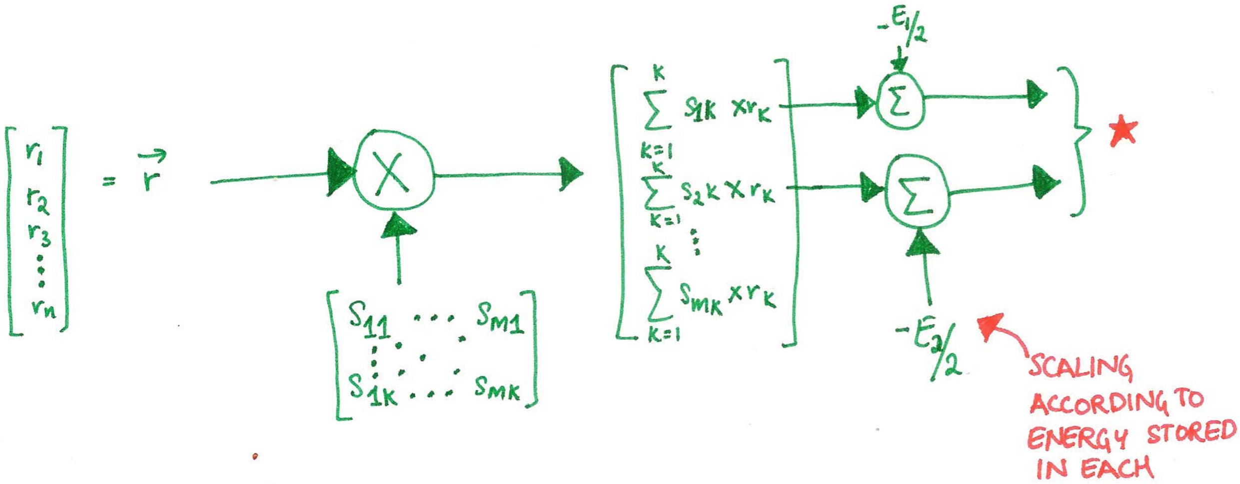
② Tells us how much weightage each base carries in the recieved signal.

Eg.

BASE 1 BASE 2 BASE 3

$$r(t) = \underbrace{A\, f_1(t) + B\, f_2(t) + C\, f_3(t)}_{\text{OUR CIRCUIT TELLS US THIS.}} \ldots$$

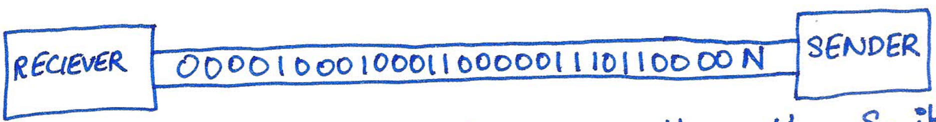
(2) Scale the compared signal ($\vec{r}$) to give Energy 1.



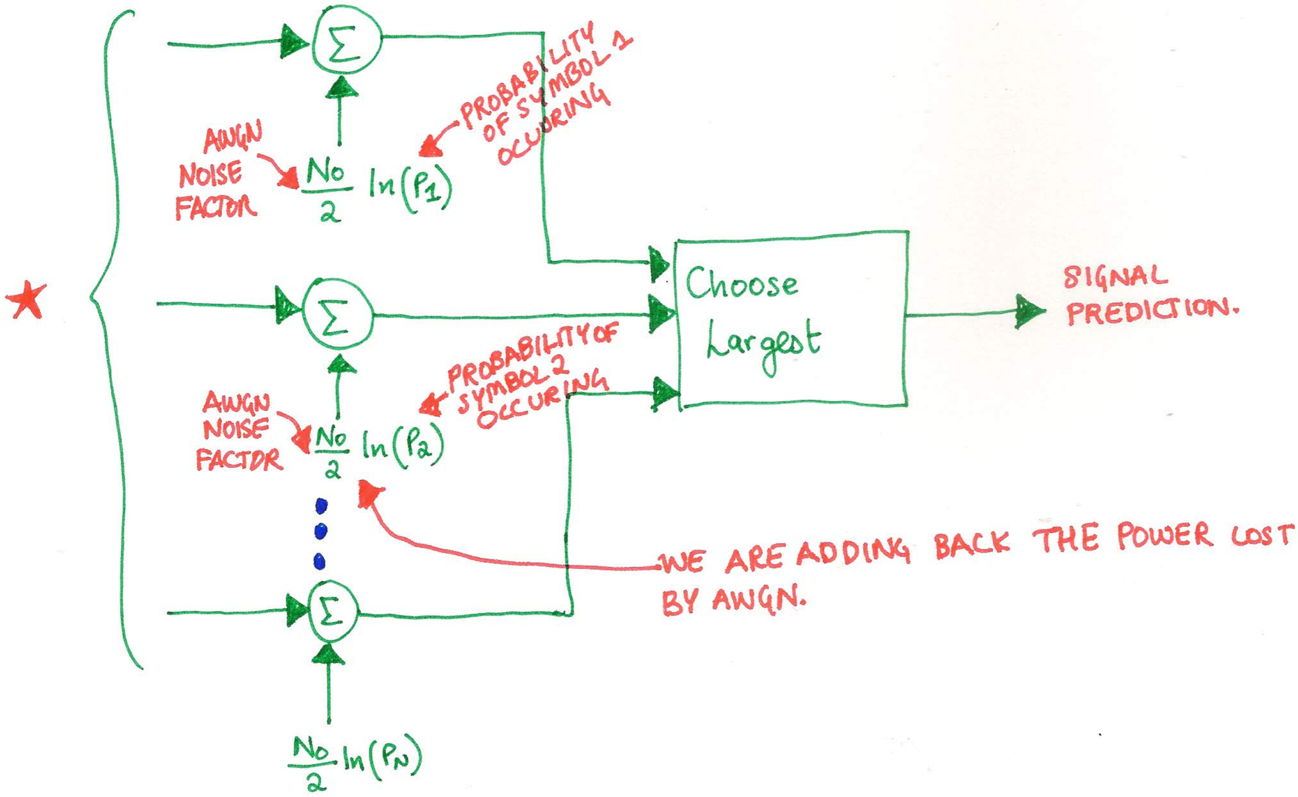
(3) To be fair and Equal, also to remove noise and predict correctly.

We need to remove noise and take into account the propability of each signal occuring

Eg. Consider the following example.

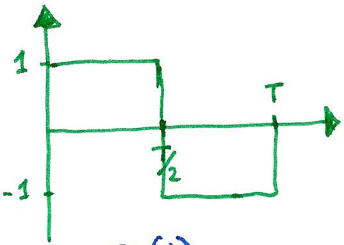


The amounts of 0's are more than 1's. So it is more likely that the next symbol N is 0 rather than a 1.

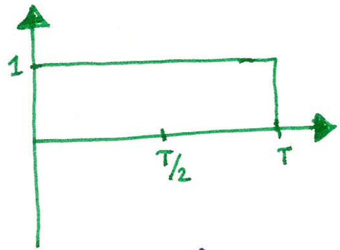


Example

Assume that binary data is transmitted via two wave forms.



$S_1(t)$



$S_2(t)$

(a) Determine the basis functions for this signal set?

(b) What is the dimensionality of the signal set?

(c) Draw the reduced complexity optimal receiver?

(d) Draw the resulting constellation diagram?

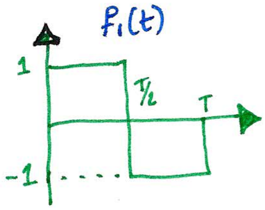
(e) Draw the decision boundaries assuming that $p_1 = p_2$?

(f) Determine the probability of error?

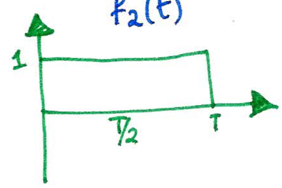
Answer

(a) Let me take the base signals as

$f_1(t)$



$f_2(t)$



(1) STEP 1: CHECK ORTHOGONALITY:

$$\int_0^T f_1(t)\, f_2^*(t)\, dt = \int_0^{T/2} (1)(1)\,dt + \int_{T/2}^{T} (-1)(1)\, dt = \Big[t\Big]_0^{T/2} + \Big[-t\Big]_{T/2}^{T}$$

$$= \frac{T}{2} - T + \frac{T}{2} = 0$$

PASSED

(2) STEP 2: CHECK NORMALIZATION:

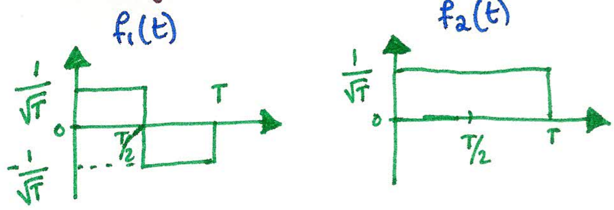
$$\text{signal 1} \Rightarrow \sqrt{\int_0^T |f_1(t)|^2\, dt} = \sqrt{\int_0^{T/2} (1)\, dt + \int_{T/2}^{T} (1)\, dt} = \sqrt{\frac{T}{2} + T - \frac{T}{2}}$$

$$= \sqrt{T}$$

ERROR! The signal 1 is not normalized

So how can we make it normalized. i.e. Energy = 1?

Let me change the base signals to:-



① STEP 1: CHECK ORTHOGONALITY:

$$\int_0^T f_1(t)\, f_2^*(t) = \int_0^{T/2}\left(\frac{1}{\sqrt{T}}\right)\left(\frac{1}{\sqrt{T}}\right)dt + \int_{T/2}^{T}\left(-\frac{1}{\sqrt{T}}\right)\left(\frac{1}{\sqrt{T}}\right)dt$$

$$= \frac{1}{T}\left(\frac{T}{2}\right) - \frac{1}{T}\left(T - \frac{T}{2}\right) = \frac{1}{T}\left(\frac{T}{2} - T + \frac{T}{2}\right) = 0$$

✔ PASSED

② STEP 2: CHECK NORMALIZATION:

Signal 1 $\Rightarrow$ $\sqrt{\int_0^T |f_1(t)|^2 dt} = \sqrt{\int_0^{T/2} \frac{1}{T} dt + \int_{T/2}^{T}\left(+\frac{1}{T}\right)dt}$

$$= \sqrt{\frac{1}{T}\left(\frac{T}{2}\right) + \frac{1}{T}\left(-\frac{T}{2} + T\right)} = \sqrt{\frac{1}{2} - \frac{1}{2} + 1} = \sqrt{1} = 1$$

Signal 2 $\Rightarrow$ $\sqrt{\int_0^T |f_2(t)|^2 dt} = \sqrt{\int_0^T \left(\frac{1}{\sqrt{T}}\right)^2 dt} = \sqrt{\frac{1}{T}(T)} = \sqrt{1} = 1$

✔ PASSED

③ STEP 3: MATRIX FORM:

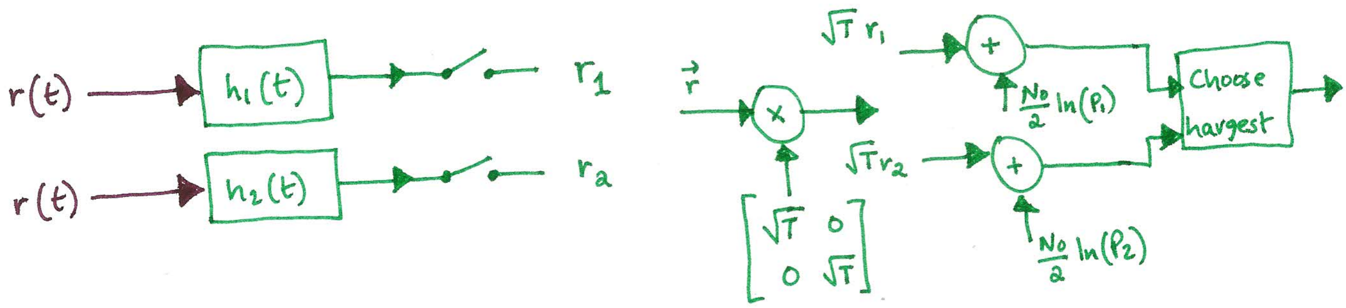
$$\begin{bmatrix} s_1(t) \\ s_2(t) \end{bmatrix} = \begin{bmatrix} \sqrt{T} & 0 \\ 0 & \sqrt{T} \end{bmatrix} \begin{bmatrix} f_1(t) \\ f_2(t) \end{bmatrix}$$

SIGNAL 2 is $\sqrt{T}$ times $f_2(t)$ base.

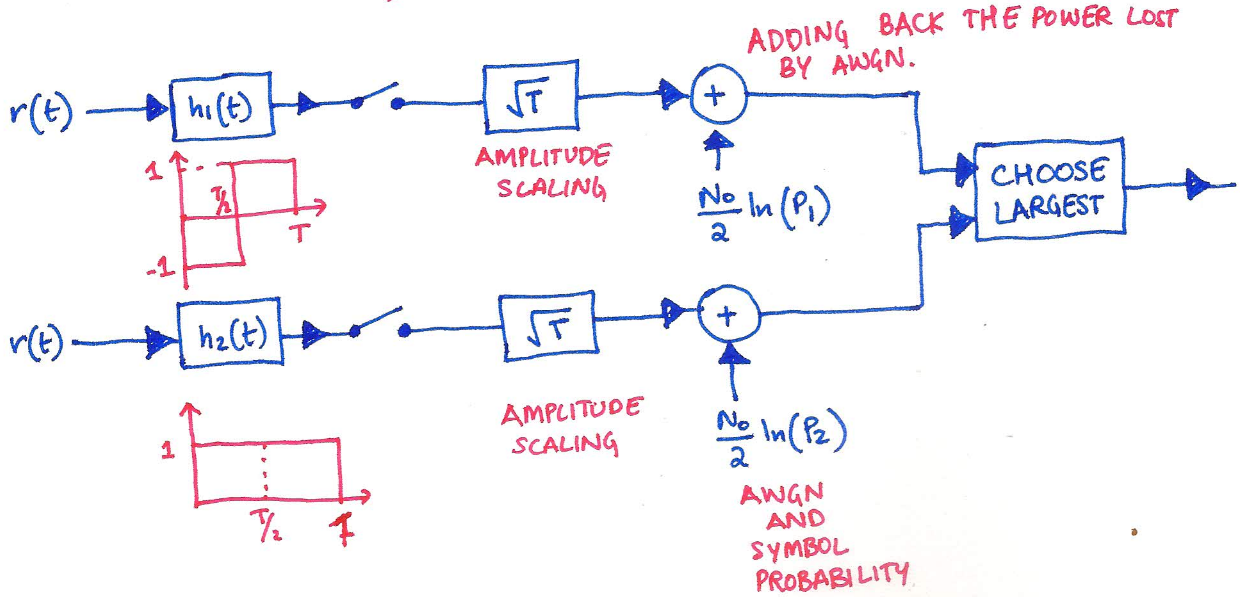
SIGNAL 1 is $\sqrt{T}$ times $f_1(t)$ base.

(b) The dimensionality of the signal set is 2.
Since all the signals in the set can be represented by just 2 base signals.

(c)

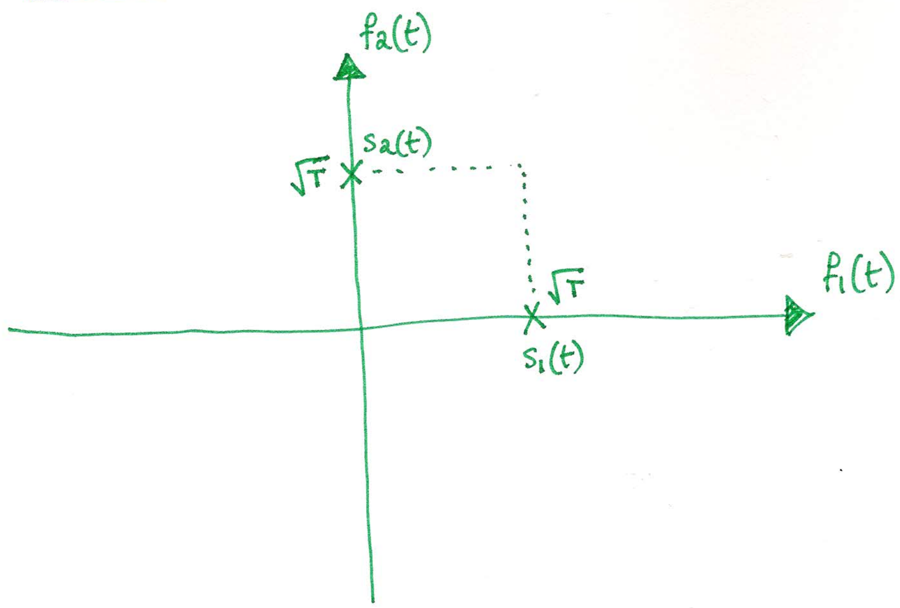


OR

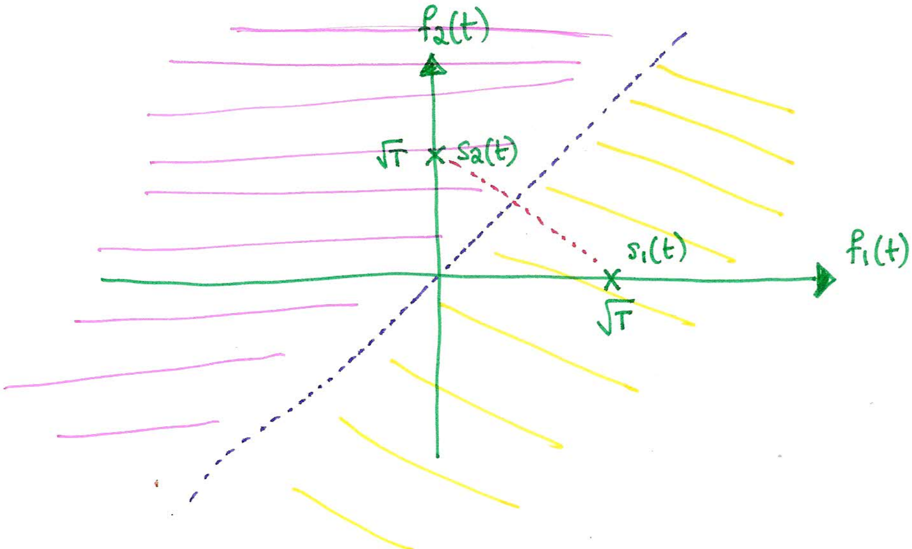


KEEP IN MIND:
When designing filters, you should do time reversal of basis functions.

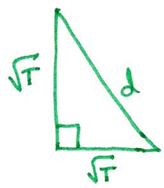
(d)



(e) Since probability of symbol 1 occuring is equal to probability of symbol 2 occuring, the distance will be equally divided



How can we find the distance between two symbols?



$$d = \sqrt{(\sqrt{T})^2 + (\sqrt{T})^2}$$

$$d = \sqrt{2T}$$

(f) The probability of Error is:-

$$P_s(\text{error}) = Q\left(\frac{d}{\sqrt{2N_0}}\right)$$

distance between two symbols.

$$\text{Probability of Symbol Error} = Q\left(\frac{\sqrt{2T}}{\sqrt{2N_0}}\right)$$

$$= Q\left(\sqrt{\frac{T}{N_0}}\right)$$

TIP:

$Q(x)$ function is the tail probability of the standard normal distribution.