Question 1)

$$x[n] = -\tfrac{1}{3}\left(\tfrac{1}{2}\right)^n u[n] - \tfrac{4}{3}(2)^n u[-n-1]$$

ⓒ Find the z-transform of $x[n] = ?$

$$X(z) = -\frac{1}{3} \times \frac{1}{1-\frac{1}{2}z^{-1}} + \frac{4}{3} \times \frac{1}{1-2z^{-1}}$$

$$X(z) = \frac{4}{3(1-2z^{-1})} - \frac{1}{3(1-\frac{1}{2}z^{-1})}$$

$$X(z) = \frac{4(1-\frac{1}{2}z^{-1}) - (1-2z^{-1})}{3(1-2z^{-1})(1-\frac{1}{2}z^{-1})}$$

$$X(z) = \frac{4 - \cancel{2z^{-1}} - 1 + \cancel{2z^{-1}}}{3(1-2z^{-1})(1-\frac{1}{2}z^{-1})}$$

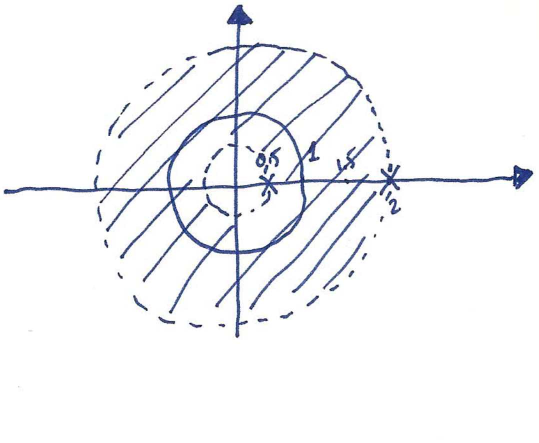
$$X(z) = \frac{3}{3} \times \frac{1}{(1-2z^{-1})(1-\frac{1}{2}z^{-1})}$$

$$\boxed{X(z) = \frac{1}{(1-2z^{-1})(1-\frac{1}{2}z^{-1})}}$$

ⓓ What are the possible choices for the region of convergence of $Y(z)$?

$$Y(z) = \frac{1-z^{-2}}{(1-\frac{1}{2}z^{-1})(1-2z^{-1})}$$

Since the input was two sided exponential sequence and the system is linear time invariant :-



$\frac{1}{2} < |z| < 2$

e What are the possible choices for the impulse response of the system ?

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 - z^{-2}}{\cancel{(1 - \frac{1}{2}z^{-1})}\cancel{(1-2z^{-1})}} \times \frac{\cancel{(1-2z^{-1})}\cancel{(1-\frac{1}{2}z^{-1})}}{1}$$

$$H(z) = 1 - z^{-2}$$

Inverse z transform

$$\therefore \boxed{h[n] = \delta[n] - \delta[n-2]}$$

Question 2)

If $H(z)$ is causal, it means that $h[n]$ is a right handed sequence. The ROC extends outwards from the outemost pole. to infinity. When the inverse is taken, the poles and zeros switch places. So now there is a pole at $z = \infty$. In order for the system to be causal, the ROC should extend further outwards from $z=\infty$ which is impossible. Therefore the inverse cannot be causal and stable.

Question 3)

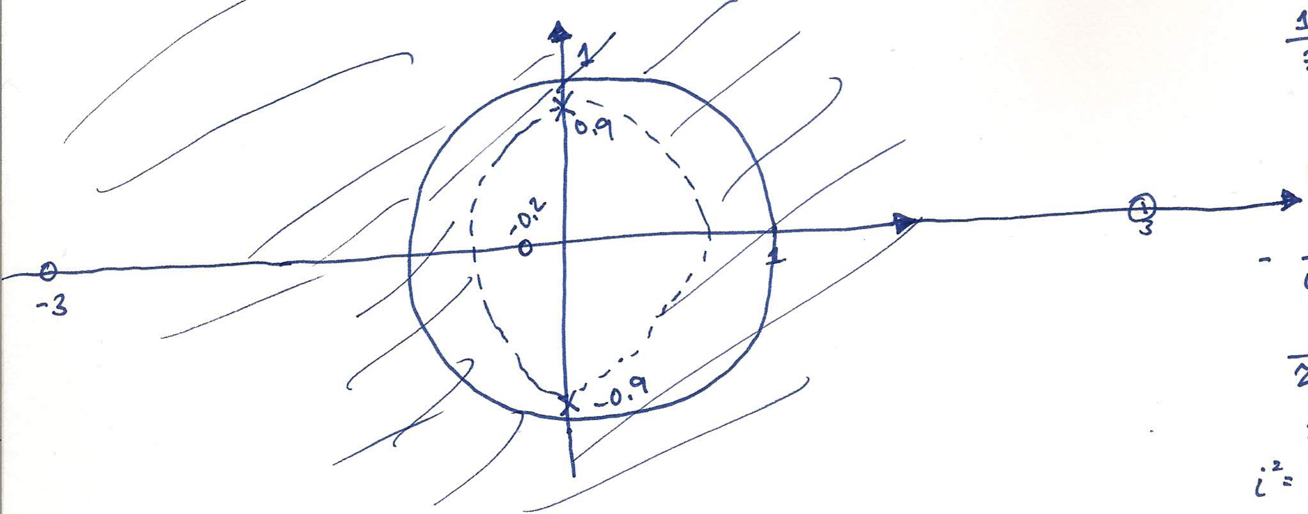
A discrete-time causal LTI system has the system function.

$$H(z) = \frac{(1+0.2z^{-1})(1-9z^{-2})}{(1+0.81z^{-2})}$$

$$-\frac{1}{0.2} = z^{-1}$$

$$\boxed{z^{+1} = -0.2}$$

a Is the system stable ?



$$z^{-2} = \frac{1}{9}$$

$$\frac{1}{z^2} = \frac{1}{9}$$

$$z^2 = 9$$

$$\boxed{z = \pm 3}$$

$$-\frac{1}{0.81} = z^{-2}$$

$$\frac{1}{z^2} = -\frac{1}{0.81}$$

$$z^2 = -0.81$$

$$i^2 = -1$$

$$z^2 = i^2 0.81$$

$$z = \pm\sqrt{0.81}\, i$$

$$\boxed{z = \pm 0.9 i}$$

→ $h[n]$ is right handed sequence. ∴ ROC emerges outwards and includes unit circle. $\boxed{\therefore \text{Stable}}$

b

$$H(z) = H_1(z)\, H_{ap}(z)$$

Find expressions for a minimum phase system $H_1(z)$ and an all-pass system $H_{ap}(z)$ such that

$$H(z) = H_1(z)\, H_{ap}(z)$$

Refining the original given $H(z)$

$$H(z) = \frac{(1 + 0.2z^{-1})(1 + 3z^{-1})(1 - 3z^{-1})}{(1 - j0.9z^{-1})(1 + j0.9z^{-1})}$$

$$H_1(z) = \frac{-9\,(1 + 0.2z^{-1})(1 + \frac{1}{3}z^{-1})(1 - \frac{1}{3}z^{-1})}{(1 - 0.9jz^{-1})(1 + 0.9jz^{-1})}$$

$$H_{ap}(z) = \frac{(z^{-1} - \frac{1}{3})(z^{-1} + \frac{1}{3})}{(1 - \frac{1}{3}z^{-1})(1 + \frac{1}{3}z^{-1})}$$

Question 4)

For each of the following system functions, state whether or not it is a minimum-phase system. Justify the following answers:

a $\quad H_1(z) = \dfrac{(1 + 2z^{-1})(1 + \frac{1}{2}z^{-1})}{(1 - \frac{1}{3}z^{-1})(1 + \frac{1}{3}z^{-1})}$

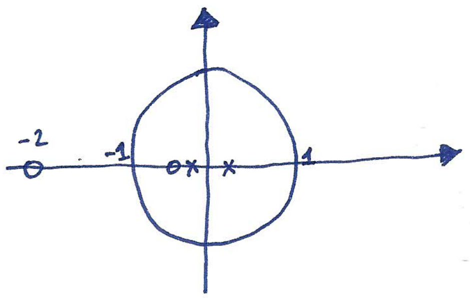
$1 + 2z^{-1} = 0$

$-2z^{-1} = 1$

$z^{-1} = -\frac{1}{2}$

$z = -2 \longrightarrow$ Zero is outside unit circle

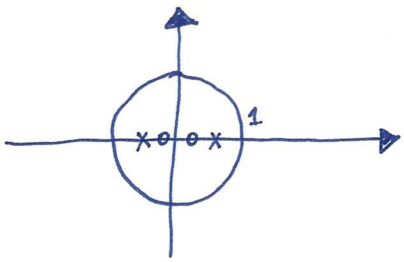
$\boxed{\therefore \text{ Not minimum phase}}$



(b) $$H_2(z) = \frac{\left(1+\frac{1}{4}z^{-1}\right)\left(1-\frac{1}{4}z^{-1}\right)}{\left(1-\frac{2}{3}z^{-1}\right)\left(1+\frac{2}{3}z^{-1}\right)}$$

All poles and zeros are inside the unit circle

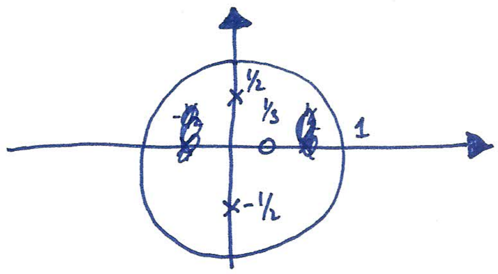
$\boxed{\therefore \text{ Minimum Phase System}}$



(c) $$H_3(z) = \frac{1-\frac{1}{3}z^{-1}}{\left(1-\frac{1}{2}jz^{-1}\right)\left(1+\frac{1}{2}jz^{-1}\right)}$$

All poles and zeros are inside the unit circle

$\boxed{\therefore \text{ Minimum Phase System}}$



(d) $$H_4(z) = \frac{z^{-1}\left(1-\frac{1}{3}z^{-1}\right)}{\left(1-\frac{1}{2}jz^{-1}\right)\left(1+\frac{1}{2}jz^{-1}\right)}$$

There is a zero at $z^{-1}=0$, The inverse of the system moves that... ~~zero~~ $z=\frac{1}{0}$ there is a zero at infinity, the inverse system will have a pole at $z=\infty$ which will make the system un-stable.

$\boxed{\therefore \text{ Not Minimum Phase}}$

Question 5)

$$H(z) = \frac{(1-1.5z^{-1}-z^{-2})(1+0.9z^{-1})}{(1-z^{-1})(1+0.7jz^{-1})(1-0.7jz^{-1})}$$

(a) Write the difference equation ?

→ Expand the Equation.

$$H(z) = \frac{1-0.6z^{-1}-2.35z^{-2}-0.9z^{-3}}{1-z^{-1}+0.49z^{-2}-0.49z^{-3}} = \frac{Y(z)}{X(z)}$$

→ Difference Equation

$$y[n]-y[n-1]+0.49y[n-2]-0.49y[n-3] = x[n]-0.6x[n-1]-2.35x[n-2]-0.9x[n-3]$$

(b) Plot the pole-zero diagram and indicate ROC for the system function ?

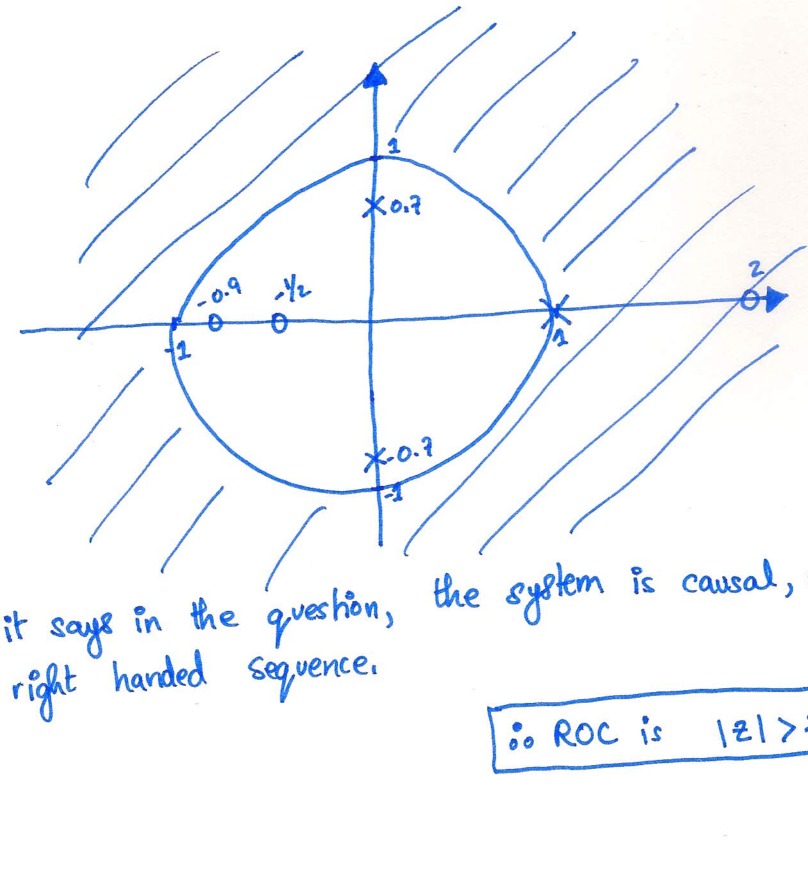
let $z^{-1} = x$

$$1-1.5x-x^2=0$$

$x = \frac{1}{2}$ $x = -2$

$\boxed{z^{-1} = \frac{1}{2}}$ $\boxed{z^{-1} = -2}$

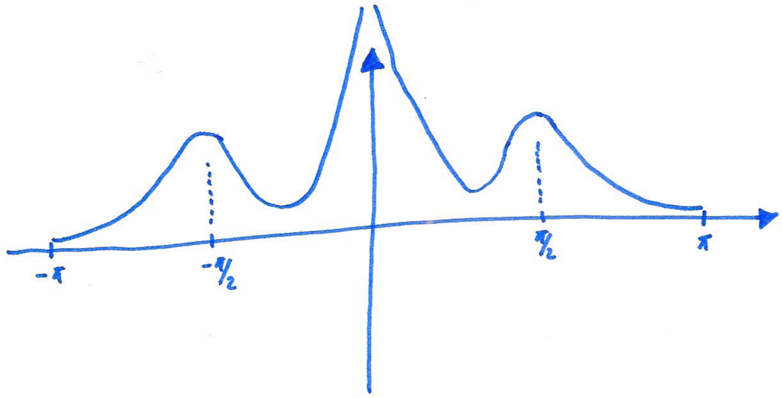
$$\therefore H(z) = \frac{(1-\frac{1}{2}z^{-1})(1+2z^{-1})(1+0.9z^{-1})}{(1-z^{-1})(1+0.7jz^{-1})(1-0.7jz^{-1})}$$



As it says in the question, the system is causal, which means that h[n] is right handed sequence.

$\boxed{\therefore \text{ROC is } |z| > 1}$

(c) Sketch $|H(e^{j\omega})|$



We move on unit circle, as the pole gets nearer, the magnitude increases.

(d)

i) The system is stable ? $\boxed{\text{FALSE}}$

ROC does not contain unit circle.

ii) The system is not stable, therefore the impulse response is not absolutely summable. Thus, the impulse Response does not approach a constant.

The impulse response approaches a constant for large n ? $\boxed{\text{FALSE}}$

iii) The magnitude of the frequency response has a peak at approximately $w = \pm \pi/4$? $\boxed{\text{FALSE}}$

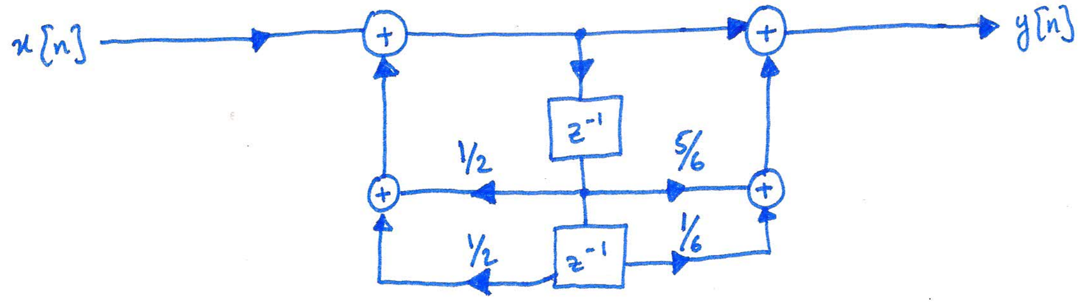
According to the magnitude response drawn up, the magnitude does not climb up at $\pi/4$ or does not have a peak at $\pi/4$.

iv) When an Inverse of the system is taken, the poles and zeros switch places, therefore the system can be stable but it can't be both stable and causal at the ~~same~~ certain ROC

The system has a stable and causal inverse ? $\boxed{\text{FALSE}}$

Question 6) Draw the signal flow diagram for the direct form II implemetation of the LTI system with system function ?

$$H(z) = \frac{1 + \frac{5}{6} z^{-1} + \frac{1}{6} z^{-2}}{1 - \frac{1}{2} z^{-1} - \frac{1}{2} z^{-2}}$$



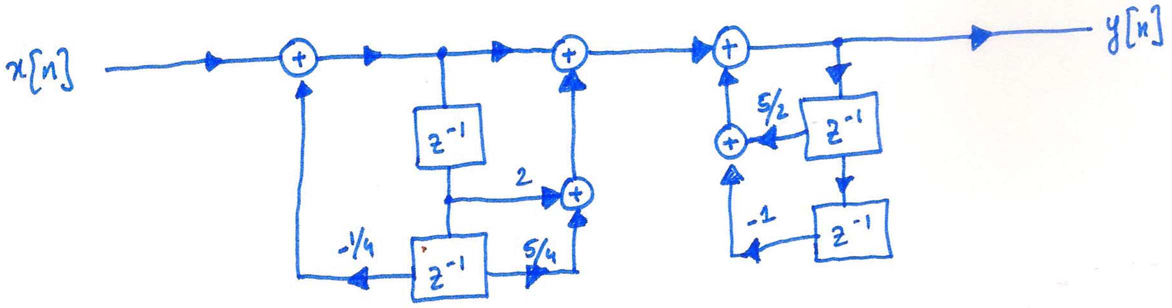
Question 7)

Draw a signal flow graph implementing the system function

$$H(z) = \frac{\left(1 + \left(1 - j/2\right) z^{-1}\right)\left(1 + \left(1 + j/2\right) z^{-1}\right)}{\left(1 + \left(j/2\right) z^{-1}\right)\left(1 - \left(j/2\right) z^{-1}\right)\left(1 - \left(1/2\right) z^{-1}\right)\left(1 - 2z^{-1}\right)}$$

by cascade form ?

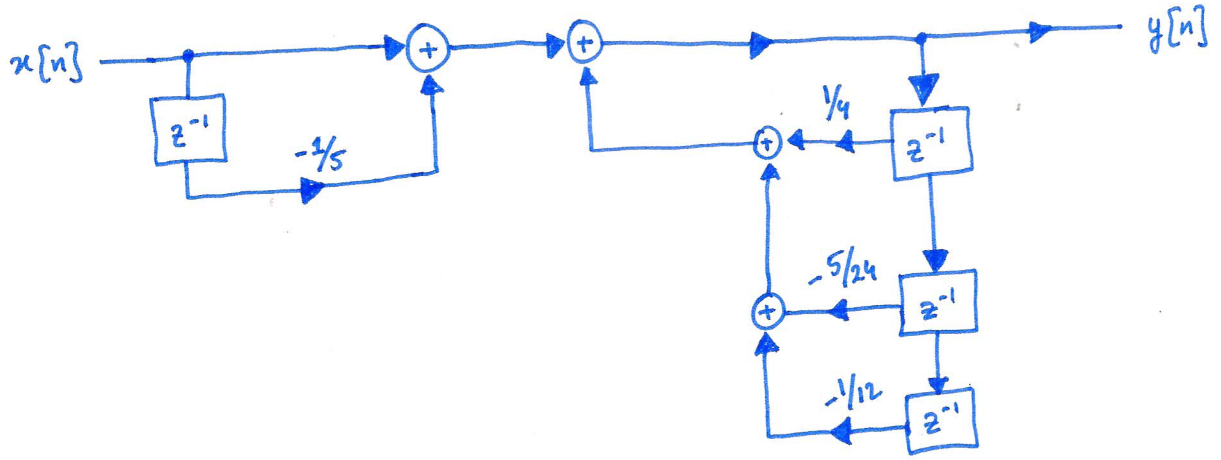
$$H(z) = \frac{1 + 2z^{-1} + \frac{5}{4} z^{-2}}{1 + \frac{1}{4} z^{-2}} \times \frac{1}{1 - \frac{5}{2} z^{-1} + z^{-2}}$$



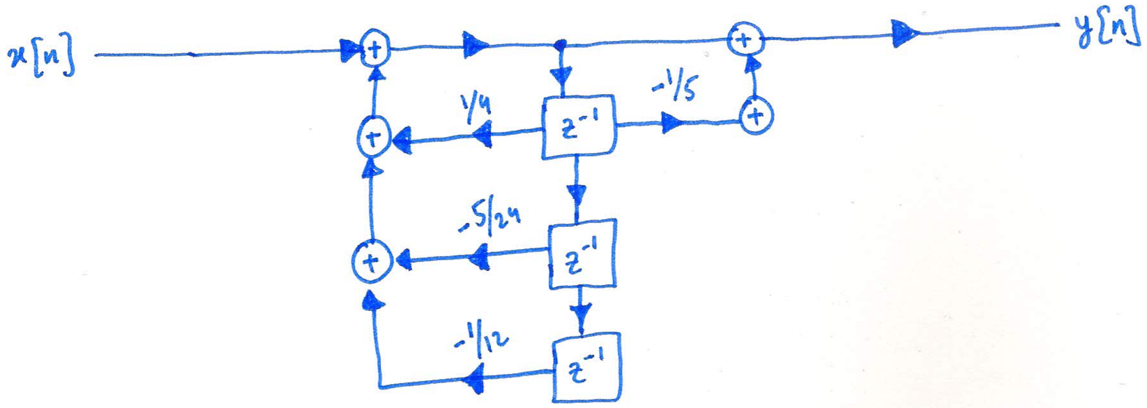
Question 8)

$$H(z) = \frac{\left(1 - \frac{1}{5} z^{-1}\right)}{\left(1 - \frac{1}{2} z^{-1} + \frac{1}{3} z^{-2}\right)\left(1 + \frac{1}{4} z^{-1}\right)}$$

(a) DIRECT FORM I :

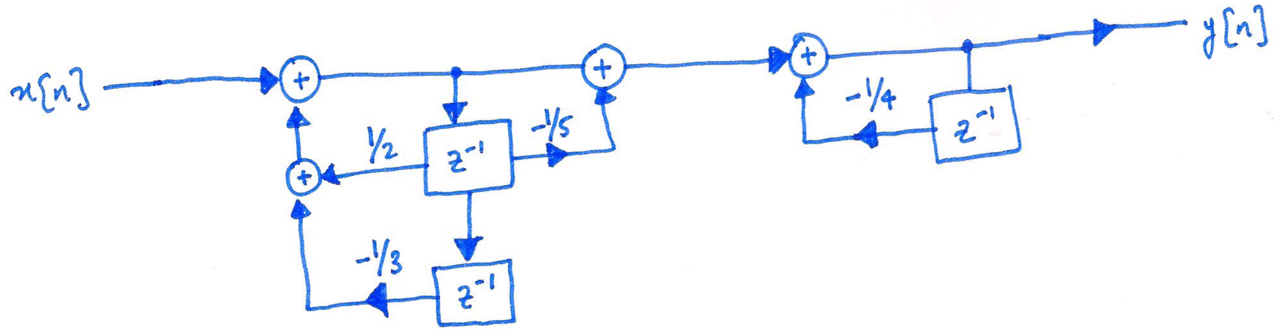


(b) DIRECT FORM II :



(c) CASCADE FORM :

$$H(z) = \frac{1 - \frac{1}{5} z^{-1}}{1 - \frac{1}{2} z^{-1} + \frac{1}{3} z^{-2}} \times \frac{1}{1 + \frac{1}{4} z^{-1}}$$



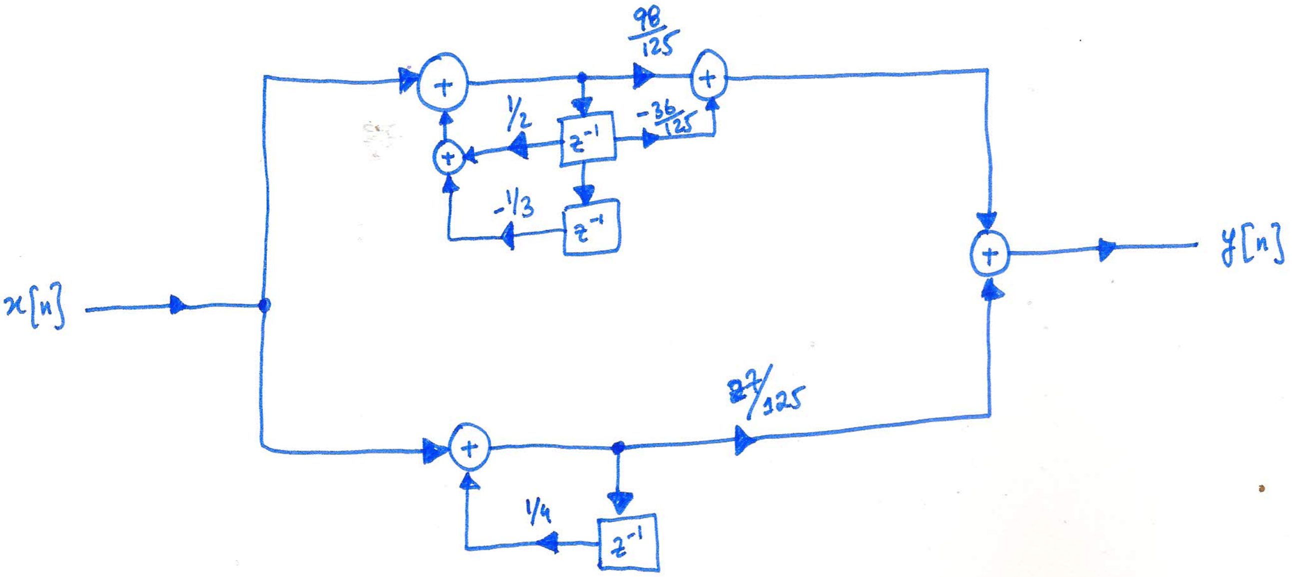
(d) PARALLEL FORM:

Using partial fractions,

$$H(z) = \frac{Az^{-1} + B}{1 - \frac{1}{2}z^{-1} + \frac{1}{3}z^{-2}} + \frac{C}{1 + \frac{1}{4}z^{-1}}$$

$$B = \frac{98}{125} \qquad C = \frac{27}{125} \qquad A = -\frac{36}{125}$$

$$H(z) = \frac{-\frac{36}{125}z^{-1} + \frac{98}{125}}{1 - \frac{1}{2}z^{-1} + \frac{1}{3}z^{-2}} + \frac{\frac{27}{125}}{1 + \frac{1}{4}z^{-1}}$$



Question 9)

(a)

$$H(z) = \frac{1}{1 - z^{-1}}\left(\frac{1 - \frac{1}{2}z^{-1}}{1 - \frac{3}{8}z^{-1} + \frac{7}{8}z^{-2}} + 1 + 2z^{-1} + z^{-2}\right)$$

$$H(z) = \frac{2 + \frac{9}{8}z^{-1} + \frac{9}{8}z^{-2} + \frac{11}{8}z^{-3} + \frac{7}{8}z^{-4}}{1 - \frac{11}{8}z^{-1} + \frac{5}{4}z^{-2} - \frac{7}{8}z^{-3}}$$

(b)

$$y[n] - \tfrac{11}{8}y[n-1] + \tfrac{5}{4}y[n-2] - \tfrac{7}{8}y[n-3] = 2x[n] + \tfrac{9}{8}x[n-1] + \tfrac{9}{8}x[n-2] + \tfrac{11}{8}x[n-3] + \tfrac{7}{8}x[n-4]$$

(c) DIRECT FORM II: