Abu Dhabi University

CEN 464 - Digital Signal Processing

Lab Report 4
Sampling, Aliasing and Reconstruction of Digital Image using MATLAB

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Section 1

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Contents

Abstract

In this lab we were introduced to Image Processing Toolbox provided in MATLAB. We were exposed to simple image cropping, various types filtering and applying 2 dimensional Fast Fourier Transform on an image.

1   Objectives of the lab

- Be able to do fourier transform in 2 dimensions to an image.

- Be able to plot fft in 3 dimensions

- Understand the effect of sampling an image

- Understand Aliasing effect and possibility of image reconstruction

2   List of equipment used

- A Computer.

- MATLAB.

- Image Processing Toolbox.

3   Question 1

a

Define the 2d sin signal, Z(x, y).

```matlab
A = 5;      % Amplitude of the input
Fx = 10;   % frequency x of one component of input
Fy = 10;   % frequency y of other component of input
phi = 0;   % the phase of the signal

NumberOfSamples = 256;      % total no of samples of my input
StepSize = 1/NumberOfSamples;  % something for the plotting

x = 0:StepSize:1-StepSize; % 256 x values from 0 to 1 in steps of 1/256
y = 0:StepSize:1-StepSize; % 256 y values from 0 to 1 in steps of 1/256

[X,Y] = meshgrid(x,y); % this is our x-y plot enviornment

Z = A*sin(2*pi*fx*X + 2*pi*fy*Y +phi); % this is our input signal

% Finding DFT
DFT = log(abs(fftshift(fft2(Z))));

%Intensity plot of DFT
imtool(DFT);
imshow(Z);
```
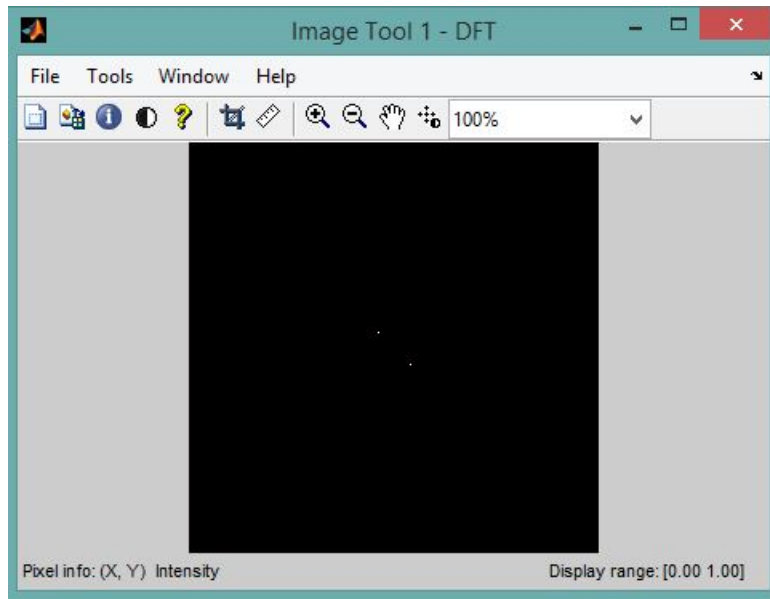
Figure 1: The figure shows the 2D representation of FFT of input image.

b

Use the intensity plot to verify that you have $10c/ph$ on the vertical and horizontal axes.

```matlab
% Calculating DFT
DFT = log(abs(fftshift(fft2(Z))));
% Intensity plot of DFT
imtool(DFT);
% Perspective plot of DFT
mesh(DFT); colormap hot;
```
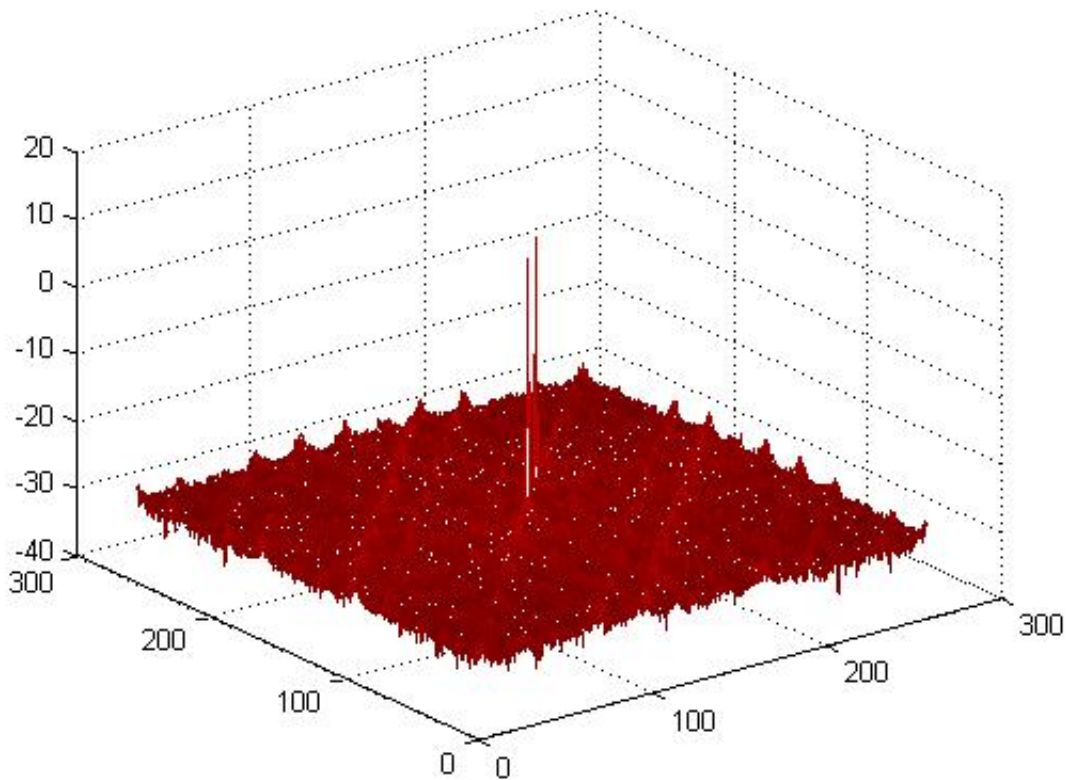
Figure 2: The figure shows the 3D representation of FFT of input image. There are two basic frequencies present. One is -10Hz and other is at +10Hz.

c

Is the signal band-limited or not? If so, what is the maximum frequency?
Yes, the signal is band limited and the maximum horizontal and vertical frequencies are +10Hz and +10Hz.

d

Do you see one or two spikes in the spectrum? Why is this the case?
We see two spikes in the spectrum because there are two frequencies in the signal +10Hz and -10Hz.

e

Has the DFT revealed that we indeed have $f_x = 10$ c/ph and $fy = 10$ c/ph?
We can calculate that there are spikes at locations (139, 139) and (119 ,119). When we subtract from the center location (129, 129) these value because of fftshift, we get two spikes at (-10, -10) and (10, 10).

f

Is our result with the DFT in agreement with what we know about the spectrum of a sine function?

Yes, a sinusoidal signal has two spikes in the frequency domain. One at +f and another at -f because of the nature of symmetry of frequency domain. This is the same case over here in 2D.

g

Use the maximum frequency and the step size from part 1 to determine the Nyquist sampling rate(after every how many pixel we sample it). Verify it is 12.8.

```
NoOfSamples = 256;
Step = 1/NoOfSamples;
f_max = 10;
f_nyquist = 2*f_max;
T_nyquist = 1/f_nyquist;
verifyValue = T_nyquist/Step
```

```
>> NoOfSamples = 256;
Step = 1/NoOfSamples;
f_max = 10;
f_nyquist = 2*f_max;
T_nyquist = 1/f_nyquist;
verifyValue = T_nyquist/Step

verifyValue =

    12.8000
```

Figure 3: The figure shows the answer generated by MATLAB

h

Oversample Z, plot the DFT of the over sampled signal, and reconstruct the original signal from the over sampled signal using the sinc interpolator given at the end of the lab. Report your observations. For example, do the DFT and visual inspection still give fx = 10 c/ph and fy = 10 c/ph. Why is this the case? Do our finding agree with the Nyquist Theorem?

```
OverSampled = Z(1:4:256, 1:4:256);
OverSampled_DFT = log(abs(fftshift(fft2(OverSampled))));
Recons_From_OverSampled = sincinterp(OverSampled , 256, 256);
subplot(4,3,10); imshow(OverSampled);
xlabel('Over Sampled');
subplot(4,3,11); imshow(OverSampled_DFT,[]);
xlabel('OverSampled Spectrum');
subplot(4,3,12); imshow(Recons_From_OverSampled);
xlabel('Reconstruction from Oversampled');
```

i

Under-sampled Z, plot the DFT of the under sampled signal, and reconstruct the original from the under sampled signal using the sinc-interpolator. What are your observations? For example, do we still have fx = 10 c/ph and fy = 10 c/ph or has the frequency changed or assumed a different alias. What is this phenomenon?

```
UnderSampled = Z(1:15:256 , 1:15:256);
UnderSampled_DFT = log(abs(fftshift(fft2(UnderSampled))))
Recons_From_UnderSampled = sincinterp(UnderSampled , 256, 25
subplot(4,3,4); imshow(UnderSampled);
xlabel('Undersampled');
subplot(4,3,5); imshow(UnderSampled_DFT, []);
xlabel('UnderSampled Spectrum');
subplot(4,3,6); imshow(Recons_From_UnderSampled);
xlabel('Reconstruction From UnderSampled')
```
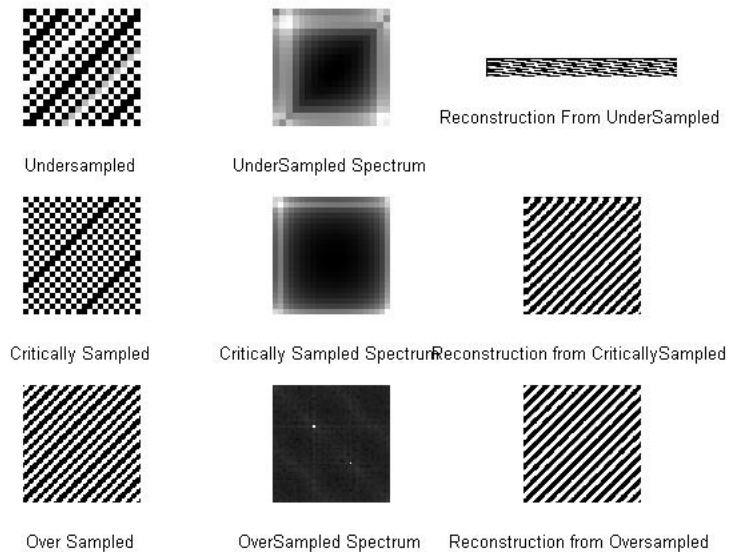

Figure 4: The figure shows the three sampling scenarios. Oversampled, where the samples were taken with steps of 4, Critically sampled, where the samples were taken every 12 pixels, and undersampled, where a pixel was chosen after every 15 pixels.

4 Question 2

a

Read a real world image into Mat lab.

```matlab
I = imread('cameraman.tif');
```

b

Create an intensity plot of the DFT of the image.

```matlab
% Making intensity plot of the DFT
DFT = log(abs(fftshift(fft2(im2double(I)))));
imshow(DFT,[]);
xlabel('Spectrum');
```

c

Is the image signal band-limited? Meaning, is there a maximum frequency after which all other frequency are zero?

Yes, the signal is band limited.

d

What does that tell us about the sampling/reconstruction of the image signal(i.e., is it possible to perfectly reconstruct the original image from the sampled one)?

Once we sample, we are restricting the bandwidth of the signal, so it is not possible to reconstruct the entire signal back because information in some bands is permanently lost.

e

Prove your finding in the previous step by sampling the signal with a step size of 2(reduce the size by 1/2) and then reconstruct the signal. Identify areas in the image where aliasing is visible?

Once we sample, we are restricting the bandwidth of the signal, so it is not possible to reconstruct the entire signal back because information in some bands is permanently lost. For example, the signal is not completely reconstructed after reducing size because the reconstruction involves guessing the pixel values which might be coincidentally be correct or entirely wrong. In essence, we cannot reconstruct the image back.

f

What is the solution to avoid aliasing?

If we follow the Nyquist sampling rate which says that the sampling frequency should be at least

twice the highest frequency of the signal, we can reconstruct the signal on the receiver side with marginal errors.

```matlab
% Read an image into MATLAB
CameraMan = imread('cameraman.tif');
% Do Fourier Transformation on image
Discrete_FT = log(abs(fftshift(fft2(im2double(CameraMan)))));

subplot(2,2,1); imshow(CameraMan);
xlabel('Cameraman Image');

subplot(2,2,2); imshow(Discrete_FT,[]);
xlabel('Freqeuncy Plot');

%Sampling the image
Sampled  = CameraMan(1:2:256, 1:2:256);
subplot(2,2,3); imshow(Sampled, []);
xlabel('Sampled Image');

%Reconstruction from sampled image using sinc
Recontructed_Image = sincinterp(im2double(Sampled),256,256);
subplot(2,2,4); imshow(Recontructed_Image,[]);
xlabel('Reconstructed Image');
```
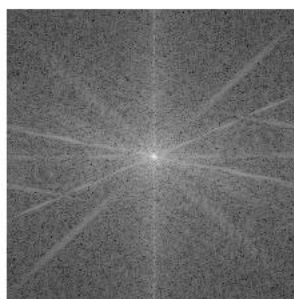
Figure 5: The figure shows the cameraman image reconstructed by MATLAB. As we can see that the image is blurry and has lost a lot of information.