4.1

Case	Mode
1	active
2	saturation
3	active
4	saturation
5	inverted active mode
6	active
7	cut-off
8	cut-off

4.2

$i_c = I_s e^{v_{BE}/V_T}$

For Device #1

$0.2 \times 10^{-3} = I_{S1} e^{0.72/0.025}$

$I_{S1} = \underline{\underline{6.214 \times 10^{-17} A}}$

For Device #2

$12 \times 10^{-3} = I_{S2} e^{0.72/0.025}$

$I_{S2} = \underline{\underline{3.728 \times 10^{-15} A}}$

Since $I_s \propto A$, the relative junction areas is:

$\frac{A_2}{A_1} = \frac{I_{S2}}{I_{S1}} = \frac{i_{C2}}{i_{C1}} = \frac{12}{0.2} = \underline{\underline{60}}$

4.3

$A_{E2} = 10^{-6} A_{E1} \Rightarrow$

$I_{S2} = 10^{-6} I_{S1}$

$i_{C1} = I_{S1} e^{v_{BE1}/V_T}$

$i_{C2} = I_{S2} e^{v_{BE2}/V_T}$ & $i_{C1} = i_{C2} \Rightarrow$

$I_{S1} e^{v_{BE1}/V_T} = i_{C1} \approx i_{C2} = I_{S2} e^{v_{BE2}/V_T}$

$I_{S1} e^{v_{BE1}/V_T} = 10^{-6} I_{S1} e^{v_{BE2}/V_T}$

$10^6 = e^{(v_{BE2} - v_{BE1})/V_T}$

$v_{BE2} - v_{BE1} = V_T \ln(10^6) = 0.025 \ln(10^6)$
$= 0.345$

4.4

$i_{C1} = I_{S1} e^{v_{BE1}/V_T} = 10^{-12} e^{0.7/0.025} = 1.45 \text{ A}$

$i_{C2} = I_{S2} e^{v_{BE2}/V_T} = 10^{-18} e^{-0.7/0.025} = 1.45 \ \mu\text{A}$

If we set i_C to 1.45 μA in case *I* and v_{BE} are allowed to vary

$1.45 \times 10^{-6} = 10^{-12} e^{v_{BE}/0.025}$

$v_{BE} = 0.354$

4.5

$i_{C,\text{old}} = I_{S,\text{old}}\, e^{v_{BE,\text{old}}/V_T}$

$v_{BE,\text{old}} = V_T \ln\left(\frac{i_{C,\text{old}}}{I_{S,\text{old}}}\right)$

$i_{C,\text{old}} = 1 \text{ mA};\ I_{S,\text{old}} = 5 \times 10^{-15} \text{A}$

$V_T = 0.025$ Volts

$v_{BE,\text{old}} = 0.025 \ln\left(\frac{1 \times 10^{-3}}{5 \times 10^{-15}}\right) = 0.651$

$i_{C,\text{new}} = 1 \text{ mA};\ I_{S,\text{new}} = 5 \times 10^{-18} \text{A}$

$V_T = 0.025$

$v_{BE,\text{new}} = 0.025 \ln\left(\frac{1 \times 10^{-3}}{5 \times 10^{-18}}\right) = 0.823$

4.6

$i_C = I_S e^{v_{BE}/V_T}$

$10 \times 10^{-3} = I_S e^{0.76/0.025} \Rightarrow I_S = 6.273 \times 10^{-16} \text{A}$

For

$v_{BE} = 0.7 \text{ V} \Rightarrow i_C = 6.273 \times 10^{-16} e^{0.7/0.025}$
$= 0.907 \text{ mA}$

For

$i_C = 10\ \mu A \Rightarrow 10 \times 10^{-6} = 6.273 \times 10^{-16}$

$e^{v_{BE}/0.025}$

$\therefore\ v_{BE} = 0.587\ V$

Alternate way – without calculating I_S

For $v_{BE} = 0.7\ V$

$$\frac{i_C}{10\ mA} = e^{\frac{0.7 - 0.76}{0.025}}$$

$\therefore\ i_C = 0.907\ mA$

For $i_C = 10\ \mu A$

$$\frac{10 \times 10^{-6}}{10 \times 10^{-3}} = e^{\frac{v_{BE} - 0.76}{0.025}}$$

$v_{BE} = 0.587\ V$

4.7

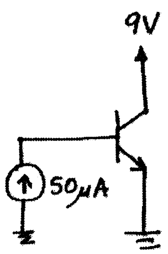
$i_c = \beta i_B$

$400 = \beta \times 7.5$

$$\beta = \frac{400}{7.5} = \underline{\underline{53.3}}$$

$$\alpha = \frac{\beta}{\beta + 1} = \frac{53.3}{54.3} = \underline{\underline{0.982}}$$

4.8



$\beta = 60$ to 300

$I_c = \beta I_B$ ranges from

$= 60 \times 50\ \mu A$ to $300 \times 50\ \mu A$

$= \underline{\underline{3\ mA \text{ to } 15\ mA}}$

$I_E = I_C + I_B$ ranges from

$= \underline{\underline{3.05\ mA \text{ to } 15.05\ mA}}$

Max Power $= 9 \times I_{C\,max} = 9 \times 15$

$= \underline{\underline{135\ mW}}$

4.9

$i_C = I_S e^{v_{BE}/V_T}$

$$i_B = \frac{i_C}{\beta}$$

$$i_E = \frac{\beta + 1}{\beta} i_C$$

$i_C = (5 \times 10^{-15}) e^{0.650/0.025} = 977\ \mu A$

i_C is constant and independent of β

$$i_B \text{ ranges from } \frac{i_C}{\beta} = \frac{977 \times 10^{-6}}{50} = 19.6\ \mu A$$

$$\text{to } \frac{i_C}{\beta} = \frac{977 \times 10^{-6}}{200} = 4.89\ \mu A$$

i_E ranges from

$$\frac{\beta + 1}{\beta} i_C = \frac{51}{50} 977 \times 10^{-6} = 998\ \mu A$$

$$\text{to } \frac{\beta + 1}{\beta} i_C = \frac{201}{200} 977 \times 10^{-6} = 983\ \mu A$$

4.10

$i_E = 1\ mA$

<u>Case I</u>: $i_B = 50\ \mu A$

$i_C = i_E - i_B = 1 \times 10^{-3} - 50 \times 10^{-6} = 950\ \mu A$

$$\beta = \frac{i_C}{i_B} = \frac{950 \times 10^{-6}}{50 \times 10^{-6}} = 19$$

$$\alpha = \frac{\beta}{\beta + 1} = \frac{19}{20} = 0.95$$

<u>Case II</u>: $i_B = 10\ \mu A$

$i_C = i_E - i_B = 1 \times 10^{-3} - 10 \times 10^{-6}\ A$

$= 990\ \mu A$

$$\beta = \frac{i_C}{i_B} = \frac{990 \times 10^{-6}}{10 \times 10^{-6}} = 99$$

$$\alpha = \frac{\beta}{\beta + 1} = \frac{99}{100} = 0.99$$

Case III: $i_B = 25\ \mu A$

$i_C = i_E - i_B = 1 \times 10^{-3}\ A - 25 \times 10^{-6}\ A$

$= 975\ \mu A$

$$\beta = \frac{i_C}{i_B} = \frac{975 \times 10^{-6}}{25 \times 10^{-6}} = 39$$

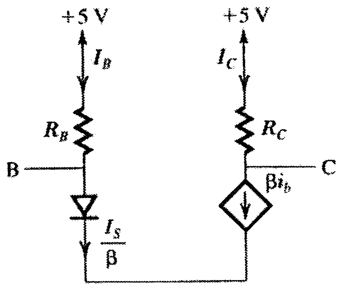
$$\alpha = \frac{\beta}{\beta + 1} = \frac{39}{40} = 0.975$$

4.11

$$I_B = \frac{I_S}{\beta} e^{V_{BE}/V_T} \Rightarrow V_{BE} = V_T \ln\left[\frac{\beta I_B}{I_S}\right]$$

$$V_{BE} = 25 \ln\left[\frac{10^{-3}}{5 \times 10^{-15}}\right] = 650\ \text{mV}$$

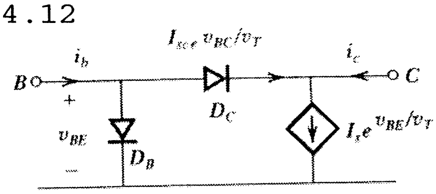
$I_B = I_C/125 = 1000/125 = 8\ \mu A$



$$R_B = \frac{V_{BB} - V_{BE}}{I_B} = \frac{5 - 0.65}{0.008} = 544\ \text{k}\Omega$$

$$R_C = \frac{V_{CC} - V_{CE}}{I_C} = \frac{5 - 1}{1} = 4\ \text{k}\Omega$$

4.12



$I_C = 0$ when $I_{SC}^{V_{BC}/V_T} = I_S e^{V_{BE}/V_T}$

$$\frac{I_{SC}}{I_S} = e^{(V_{BE} - V_{BC})/V_T} = 100$$

$$V_{CE} = V_{BE} - V_{BC} = V_T \ln\frac{I_{SC}}{I_S}$$

$= 25 \ln 100 = 115\ \text{mV}$

For $V_{ce} = 0.4\ V$ $\quad V_{BC} = 0.7 - 0.4 = 0.3\ V$

$i_{BC} = I_{SC}\, e^{0.3/V_T} = 10^{-13} e^{12} = 0.0168\ \mu A$

For $V_{CE} = 0.3\ V$ $\quad V_{BC} = 0.7 - 0.3 = 0.4\ V$

$i_{BC} = I_{SC}\, e^{0.4/V_T} = 10^{-13} e^{16} = 0.089\ \mu A$

For $V_{CE} = 0.1\ V$ $\quad V_{BC} = 0.7 - 0.1 = 0.6\ V$

$i_{BC} = I_{SC}\, e^{0.6/V_T} = 10^{-13} e^{24} = 2.65\ \text{mA}$

For $V_{BE} = 0.7\ V$

$$i_{BE} = \frac{I_S}{\beta} e^{0.7/V_T} = \frac{10^{-15}}{100} e^{28} \Rightarrow 14.5\ \mu A$$

$i_{CE} = I_{SC}\, e^{0.7/V_T} = 10^{-15} e^{28} = 1.45\ \text{mA}$

For $V_{CE} = 0.4\ V$ $\quad V_{BC} = 0.3\ V$

$i_B = i_{BE} + i_{BC} = 14.5 + 0.02 = 14.52\ \mu A$

$i_C = i_{CE} - i_{BC} = 1.45 - 0 = 1.45\ \text{mA}$

$i_C / i_B = 1.45\ \text{mA} / 14.52\ \mu A = 100$

For $V_{CE} = 0.3\ V$ $\quad V_{BC} = 0.4\ V$

$i_b = 14.5 + 0.089 = 145.89\ \mu A$

$$i_C = 1.45 - \frac{0.089}{1000} = 1.45\ \text{mA}$$

$i_C/i_b \approx 1.45\ \text{mA}/146\ \mu A = 9 \cdot 9$

For $V_{CE} = 0.1\ V$ $\quad V_{BC} = 0.6\ V$

$i_b = 14.5 + 2.65 = 4.1\ \text{mA}$

$i_C = 1.45 - 2.65 = -1.2\ \text{mA}$

V_{CE} too low for model

4.13

given: $i_C = I_S\, e^{V_{BE}/V_T} - I_{SC}\, e^{V_{BC}/V_T}$

and $i_C = \frac{I_S}{\beta} e^{V_{BE}/V_T} + I_{SC}\, e^{V_{BC}/V_T}$

and $\beta_{\text{forced}} = \left.\frac{i_C}{i_B}\right|_{\text{Sat}^n} \leq \beta$

$$\beta_{\text{forced}} = \beta \cdot \frac{I_S\, e^{(V_{CEsat} + V_{BC})/V_T} - I_{SC}\, e^{V_{BC}/V_T}}{I_S\, e^{(V_{CEsat} + V_{BC})/V_T} + I_{SC}\, e^{V_{BC}/V_T}}$$

$$= \beta \cdot \frac{I_S\, e^{V_{BC}/V_T}\,[e^{V_{CEsat}/V_T} - I_{SC}/I_S]}{I_S\, e^{V_{BC}/V_T}[e^{V_{CEsat}/V_T} + \beta I_{SC}/I_S]}$$

$$\therefore\ e^{V_{CEsat}} = \frac{-\beta\frac{I_{SC}}{I_S} - \beta\frac{I_{SC}}{I_S} \times \beta_{\text{forced}}}{\beta - \beta_{\text{forced}}}$$

$$= \frac{I_{SC}}{I_S}\left[\frac{\beta + \beta\ \beta_{\text{forced}}}{\beta - \beta\ \beta_{\text{forced}}}\right]$$

$$= \frac{I_{SC}}{I_S}\left[\frac{1 + \beta_{\text{forced}}}{1 - \beta_{\text{forced}}/\beta}\right] \qquad \text{QED}$$

For $\beta_{\text{forced}} = 50$

$$V_{CEsat} = 25 \ln\left[100 \cdot \frac{1 + 50}{1 - 50/100}\right]$$

$$= 25\ \ln[10200] = 230.8\text{ mV}$$

For $\beta_{forced} = 10$

$$V_{CEsat} = 25\ \ln\left[100 \cdot \frac{1+10}{1-10/100}\right]$$

$$= 25\ \ln[122.2] = 177.7\text{ mV}$$

For $\beta_{forced} = 5$

$$V_{CEsat} = 25\ \ln\left[100 \cdot \frac{1+5}{1-5/100}\right]$$

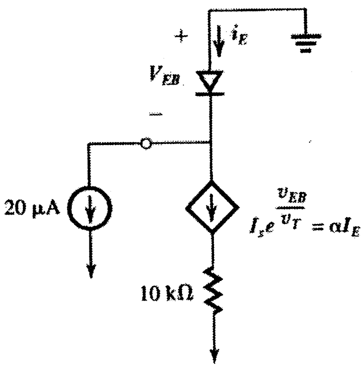
$$= 25\ \ln[631.6] = 161.2\text{ mV}$$

For $\beta_{forced} = 1$

$$V_{CEsat} = 25\ \ln\left[100 \cdot \frac{1+1}{1-1/100}\right]$$

$$= 25\ \ln[202] = 132.7\text{ mV}$$

4.14



$\beta = 40$

$\alpha_F = \frac{40}{41}$

$I_S = 10^{-13}$ A

$i_E = \frac{I_S}{\alpha} e^{V_{EB}/V_T} = I_S e^{V_{EB}/V_T} + 0.02 \times 10^{-3}$ A

$I_S\ e^{V_{EB}/V_T}\left(\frac{1}{\alpha} - 1\right) = 0.02 \times 10^{-3}$ A

$10^{-13}\ e^{V_{EB}/0.025}\left(\frac{41}{40} - 1\right) = 0.02 \times 10^{-3}$ A

$V_{EB} = 0.570\text{ V} \Rightarrow V_B = -0.570$ V

$i_E = \frac{I_S}{\alpha} e^{V_{EB}/V_T} = \frac{10^{-13}}{40} e^{\frac{0.57}{0.025}}$

$= 0.82$ mA

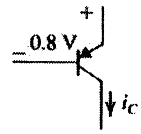
$i_C = \alpha i_E \Rightarrow$

$V_C = -10 + \alpha i_E \times 10$

$= -10 + \frac{40}{41} \times 0.82 \times 10$

$= -2$ V

4.15



$\because i_C = I_S\, e^{V_{EB}/V_T}$

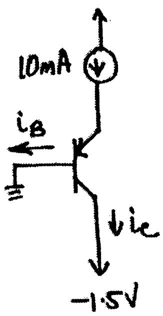
Use $\frac{i_C}{1\text{A}} = e^{\frac{v_{EB} - 0.8}{0.025}}$

to calculate v_{EB} for a particular i_C

For $i_C = 10$ mA $\quad v_{EB} = 0.685$ V

For $i_C = 5$ A $\quad v_{EB} = 0.840$ V

4.16



$\beta = 10$

$i_C = \alpha i_E = \frac{10}{11} \times 10 = \underline{\underline{9.09\text{mA}}}$

$i_B = i_E - i_C = \underline{\underline{0.91\text{ mA}}}$

$i_C = I_S\ e^{V_{EB}/V_T}$

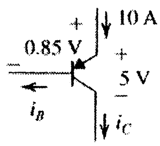
$9.09 \times 10^{-3} = 10^{-16}\ e^{V_{EB}/0.025}$

$V_E = V_{EB} = \underline{\underline{0.803\text{V}}}$

For $\beta = 1000$

$i_C = \frac{\beta}{\beta + 1} i_E = \frac{1000}{1001} \times 10 = \underline{\underline{9.99\text{mA}}}$

4.17



for $\beta = 15$

$i_E = (\beta + 1) i_B$

$10 = (\beta + 1) i_B$

$i_B = \frac{10}{16} = 0.625$ A

Calculating I_{S1}

$i_C = \frac{\beta}{\beta+1} i_E = I_{S1} e^{v_{EB}/v_T}$

$\frac{15}{16} \times 10 = I_{S1} e^{0.85/0.025}$

$I_{S1} = 1.608 \times 10^{-14}$ A

Compare this to

$I_{S2} = i_C e^{-v_{EB}/v_T}$

$= 10^{-3} e^{-0.7/0.025}$

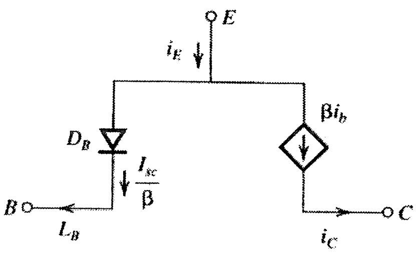
$= 6.914 \times 10^{-16}$

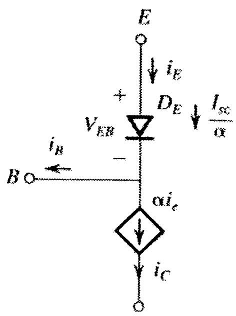
$\because I_S \propto$ area

$\frac{\text{Area1}}{\text{Area2}} = \frac{I_{S1}}{I_{S2}} = \frac{1.608 \times 10^{-14}}{6.914 \times 10^{-16}}$

= 23.3 times larger

4.18

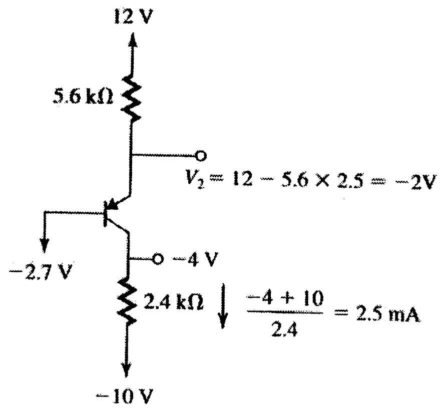




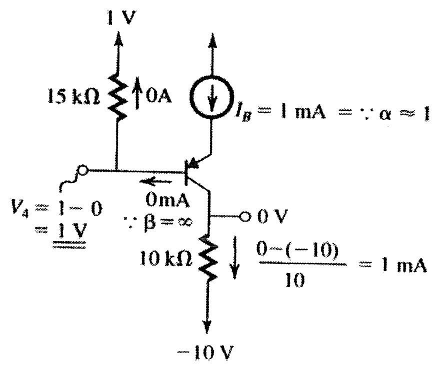
4.19

(a) $I_1 = \frac{10.7 - 0.7}{10} = 1$ mA

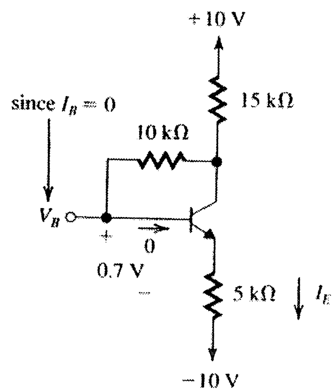
(b)



(c)



(d)



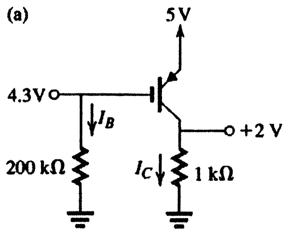
$I_E = I_C$

$\frac{V_B - 0.7 + 10}{5} = \frac{10 - V_E}{15}$

$$15V_6 + 139.5 = 50 - 5V_6$$
$$V_6 = -4.475\text{ V}$$
$$I_5 = \frac{V_6 - 0.7 + 10}{5}$$
$$= \frac{-4.475 - 0.7 + 10}{5} = 0.965\text{ mA}$$

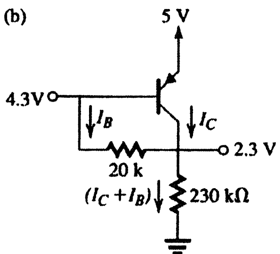
4.20

(a)



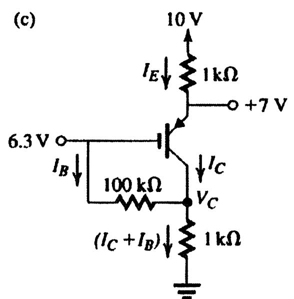
$$\frac{I_c}{I_B} = \beta = \frac{\left(\frac{2}{1\text{ K}}\right)}{\left(\frac{4.3}{200\text{ K}}\right)} = \frac{2\text{ m}}{0.0215\text{ m}} = 93$$

(b)



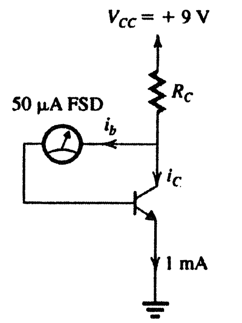
$$(I_c + I_B) = \frac{2.3}{230} = 10\text{ mA}$$
$$I_B = \left(\frac{4.3 - 2.3}{20\text{ K}}\right) = 0.1\text{ mA}$$
$$\frac{I_c}{I_B} = \left(\frac{10\text{ m} - 0.1\text{ m}}{0.1\text{ m}}\right) = \beta = 99$$

(c)



$$I_E = \left(\frac{10 - 7}{1\text{ K}}\right) = 3\text{ mA}$$
$$I_E = I_c + I_B = 3\text{ mA}$$
$$V_c = 3\text{ m}(1\text{ K}) = 3\text{ V}$$
$$I_B = \frac{6.3 - 3}{100\text{ K}} = 33\ \mu\text{A}$$
$$\frac{I_E}{I_B} = \beta + 1 = \frac{3\text{ m}}{33\ \mu} = 90.9$$
$$\beta = 89.9$$

4.21



For F.S.D $i_b = 50\ \mu\text{A}$

$i_C = 1000 - 50 = 950\ \mu\text{A}$

Since $R_m = 0\ \Omega$ $V_{CE} = V_{BE} = 0.7$ V

$\therefore$ active mode

$$R_C = \frac{V_{CC} - V_C}{I_C} = \frac{9 - 0.7}{1\text{mA}} = 8.3\text{ k}\Omega$$

$$I_C = \beta I_B \therefore \beta = \frac{950}{50} = 19$$

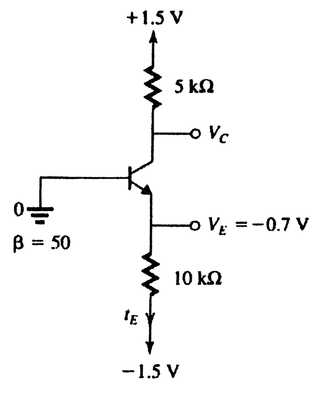
For FSD/5: $i_b = 10\ \mu\text{A}$, $i_C = 990\ \mu\text{A}$

$\Rightarrow \beta = 99$

For FSD/10: $i_b = 5\ \mu\text{A}$, $i_C = 995\ \mu\text{A}$

$\Rightarrow \beta = 199$

4.22



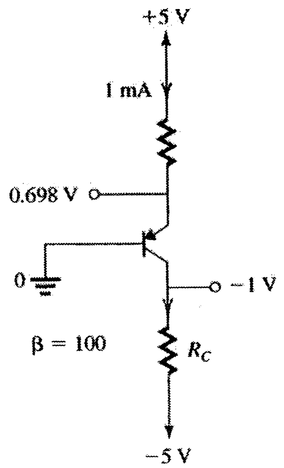
$$I_E = \frac{V_E - V_{EE}}{R_E} = \frac{0.8}{10\text{ K}} = 80\ \mu\text{A}$$

$$I_C = \frac{\beta}{\beta + 1} I_E = \frac{50}{51} \times 80 = 78\ \mu\text{A}$$

$$I_B = \frac{I_E}{\beta + 1} = \frac{80}{51} = 1.6\ \mu\text{A}$$

$$V_c = V_{cc} - I_c R_C = 1.5\text{ V} - 0.078 \times 5\text{ V}$$
$$= 1.11\text{ V}$$

4.23



$$V_{BE(1\text{mA})} - V_{BE(0.1\text{mA})}$$
$$= 25\ln\left[\frac{1}{0.1}\right]$$

$$\therefore\ V_{BE(1\text{mA})} = 640\text{ mV} + 57.9\text{ mV}$$
$$= 698\text{ mV}$$

$$I_C = \frac{100}{101} I_E = 0.99\text{ mA}$$

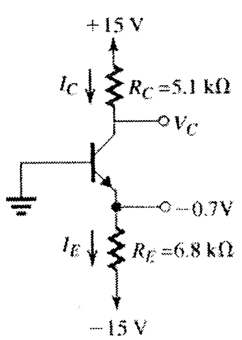
$$R_C = \frac{-1 - (-5)}{0.99} = 4.04\text{ k}\Omega$$

$$R_E = \frac{5 - 0.698}{1} = 4.3\text{ k}\Omega$$

V_C can be raised untill $\approx +0.4$ V

$$R_C = \frac{5 + 0.4}{0.99} = 5.45\text{ k}\Omega$$

4.24



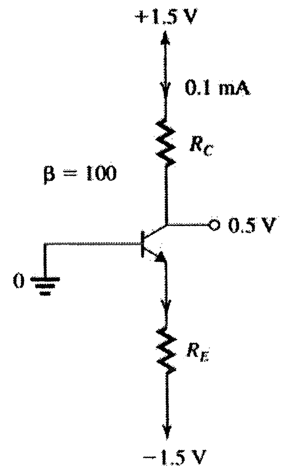
$$\alpha \approx 1$$

$$I_E \approx I_c = \frac{(-0.7 - (-15))}{6.8\text{ k}\Omega} = 2.1\text{ mA}$$

$$V_c = 15 - 5.1\text{ K}(2.1\text{ m})$$

$$V_c = 4.3\text{ V}$$

4.25



$$\Delta V_{BE} = V_T \ln\left[\frac{I_{C7}}{I_{C1}}\right]$$
$$= 25\ln[0.1] = -57.6\text{ mA}$$

$$V_{BE(0.1)} = 742\text{ mV}$$

$$R_C = \frac{1.5 - 0.5}{0.1} = 10\text{ k}\Omega$$

$$V_E = -0.742\text{ V}$$

$$R_E = \frac{-0.742 + 1.5}{\frac{\beta + 1}{\beta} I_C}$$
$$= \frac{100}{101} \cdot \frac{0.758}{0.1} = 7.5\text{ k}\Omega$$

4.26

(a) $V_B = \underline{0V}$

$V_E = V_B - 0.7 = \underline{-0.7V}$

$$I_E = \frac{-0.7 + 3}{2.2} = \underline{1.05\text{mA}}$$

$I_C = \frac{30}{31} I_E = \underline{\underline{1.02\,mA}}$

$V_C = 3 - 1.02 \times 2.2 = \underline{\underline{0.756\,V}}$

$I_B = \frac{I_C}{\beta} = \frac{1.02}{30} = \underline{\underline{0.034\,mA}}$

(b) $V_B = \underline{0\,V}$

$V_E = V_B + 0.7 = \underline{0.7\,V}$

$I_E = \frac{3 - V_E}{1} = \frac{3 - 0.7}{1} = \underline{2.3\,mA}$

$I_C = \alpha I_E = \frac{30}{31} \times 2.3 = \underline{\underline{2.23\,mA}}$

$V_C = -3 + 1 \times I_C = -3 + 2.23$

$= -\underline{0.77\,V}$

$I_B = \frac{I_C}{\beta} = \frac{2.23}{30} = \underline{\underline{0.0743\,mA}}$

(c) $V_B = \underline{3\,V}$

$V_E = V_B + 0.7 = \underline{3.7\,V}$

$I_E = \frac{9 - V_E}{1.1} = \frac{9 - 3.7}{1.1} = \underline{\underline{4.82\,mA}}$

$I_C = \alpha I_E = \frac{30}{31} \times 4.82 = \underline{\underline{4.66\,mA}}$

$V_C = I_C \times 0.56 = \underline{2.62\,V}$

$I_B = \frac{I_C}{\beta} = \frac{4.66}{30} = 0.\underline{\underline{155\,mA}}$

(d) $V_B = \underline{3\,V}$

$V_E = 3 - 0.7 = \underline{2.3\,V}$

$I_E = V_E / 0.47 = 2.3 / 0.47 = \underline{\underline{4.89\,mA}}$

$I_C = \alpha I_E = \frac{30}{31} \times 4.89 = 4.73\,mA$

$V_C = 9 - 1 \times I_C = 9 - 4.73 = \underline{\underline{4.22\,V}}$

$I_B = I_C/\beta = \frac{4.73}{30} = \underline{\underline{0.158\,mA}}$

4.27

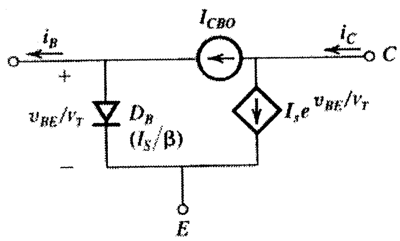
I_{CBO} doubles for every 10°C rise in temperature.

Thus if $I_{CBO} = 20\,nA$ at 25°C

At 85°C $I_{CBO} = 2^{\frac{85-25}{10}} \times 20$

$= \underline{\underline{1280\,nA}}$

4.28



$$i_B = \frac{I_S}{\beta} e^{\frac{v_{BE}}{V_T}} - I_{CBO} \qquad (1)$$

$$i_C = I_S e^{\frac{v_{BE}}{V_T}} + I_{CBO} \qquad (2)$$

$$i_E = I_S\left(1 + \frac{1}{\beta}\right) e^{v_{BE}/V_T} \qquad (3)$$

for β opencircuited, $i_B = 0$ and (1) gives

$$\frac{I_S}{\beta} e^{v_{BE}/V_T} = I_{CBO} \Rightarrow e^{v_{BE}/V_T} = \frac{\beta I_{CBO}}{I_S}$$

substitute into (2) & (3) $\Rightarrow$

$$i_C = (\beta + 1) I_{CBO}$$

$$i_E = (\beta + 1) I_{CBO}$$

4.29

Given $i_E = 0.5\text{mA}$, $V_{EB} = 0.692\text{V}$ } at 20°C

(a) The junction temperature rises to 50°C

$$V_{EB} = 0.692 - 2\times10^{-3}(50-20)$$
$$= \underline{\underline{0.632\text{V}}}$$

(b) The Base-Emitter Voltage is fixed $V_{EB} = 0.7\text{V}$ at ALL TEMPERATURES

At 20°C ~ $i_E = 0.5\text{mA}$ at $V_{EB} = 0.692\text{V}$
Thus for $V_{EB} = 0.7\text{V}$ we have

$$\frac{i_E}{0.5\times10^{-3}} = e^{\frac{0.7-0.692}{0.025}}$$

$$i_E = \underline{\underline{0.689\text{mA}}}$$

Now if $T = 50°\text{C}$ & $V_{EB} = 0.7\text{V}$

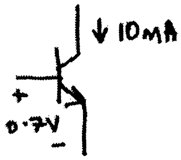
from (a) we see that at 50°C, $I_E = 0.5\text{mA}$, $V_{EB} = 0.632\text{V}$

Therefore for $V_{EB} = 0.7\text{V}$

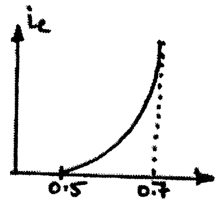
$$\frac{i_E}{0.5\times10^{-3}} = e^{\frac{0.7-0.632}{0.025}}$$

$$i_E = \underline{7.59\text{mA}}$$

4.30



$$\frac{i_c}{10\text{mA}} = e^{\frac{v_{BE}-0.7}{0.025}}$$
$$= e^{\frac{0.5-0.7}{0.025}}$$
$$i_c = \underline{\underline{3.35\ \mu\text{A}}}$$



Notice the current drops significantly at $v_{BE} = 0.5\text{V}$

4.31

V_{BE} changes by −2 mV/°C for a particular current. Given that at 25° C $V_{BE} = 0.7$ V and $i_C = 10$ mA
Thus

@ −25° C $\quad V_{BE} = 0.7 - 2\times10^{-3}(-50)$
$= 0.8$ V and $i_C = 10$ mA

@ 125° C $\quad V_{EB} = 0.7 - 2\times10^{-3}(100)$
$= 0.5$ V and $i_C = 10$ mA

4.32

$$r_o = \frac{1}{3\times10^{-5}} = \underline{\underline{33.3\text{k}\Omega}}$$

$$V_A = r_o I_c = 33.3\times10^{3}\times3\times10^{-3}$$
$$= \underline{\underline{100\text{V}}}$$

$$r_o = \frac{V_A}{I_c} = \frac{100}{30} = \underline{\underline{3.3\text{k}\Omega}}$$

4.33

$r_o = V_A / I_C = 200/I_C$

@ $I_C = 1mA$ $r_o = \underline{200 k\Omega}$

@ $I_C = 100 \mu A$ $r_o = \frac{200}{0.1} = \underline{2.0 M\Omega}$

4.34

$V_{BE} = 0.72$ V $\cdot i_C = 1.8$ mA $V_{CE} = 2$ V

$i_C = 2.4$ mA $V_{CE} = 14$ V

$$r_o = \frac{\Delta V_{CE}}{\Delta i_C} = \frac{14-2}{2.4-1.8} = 20 \text{ k}\Omega$$

Near saturation $V_{CE} = 0.3$V

$$\therefore \frac{\Delta V_{CE}}{\Delta i_C} = \frac{0.3-2}{i_C - 1.8} = 20 \text{ k}\Omega$$

$i_C = 1.72$ mA

Calculating V_{CE} for $i_C = 2.0$ mA

$$\frac{\Delta V_{CE}}{\Delta i_C} = r_o$$

$$\frac{V_{CE}-2}{2-1.8} = 20 \Rightarrow V_{CE} = 6 \text{ V}$$

Take the ratio of currents to find the early voltage (with Eq 5.36)

$$\frac{2.4}{1.8} = \underbrace{e^{\frac{V_{BE}-V_{BE}}{V_T}}}_{=1}\left(\frac{1+14/V_A}{1+2/V_A}\right)$$

$$2.4 + \frac{4.8}{V_A} = 1.8 + \frac{25.2}{V_A}$$

$V_A = 34$ V

$$r_o = \frac{V_A}{I_C}$$

where I_C is the current near saturation $\leftrightarrow$ active boundary. As calculated above $I_C = 1.72$ mA

$$r_o = \frac{34 \text{ V}}{1.72 \text{ mA}} = 19.8 \text{ k}\Omega$$ compared to the above calculation of 20 kΩ.

4.35

Large signal or DC β:

$$h_{FE} = \frac{i_C}{i_B} = \frac{1.2mA}{8\mu A} = \underline{150}$$

Small signal $h_{fe} = \frac{0.1mA}{0.8\mu A} = \underline{125}$

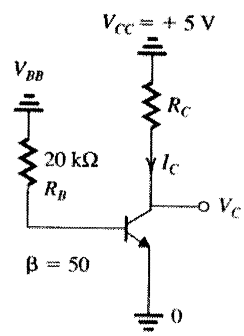
$$r_o = \frac{V_A}{I_C} = \frac{100V}{1.2mA} = 83.3k\Omega$$

$$\Delta i_C = h_{fe}\Delta i_B + \frac{\Delta v_{CE}}{r_o}$$

$$= 125 \times 2\mu A + 2/83.3k\Omega = 0.274mA$$

$$\therefore i_C = 1.2mA + \Delta i_C = \underline{1.474mA}$$

4.36



(a) active region

$$I_C = \frac{V_{CC}-V_C}{R_C} = \frac{5-1}{1 \text{ K}} = 4 \text{ mA}$$

$$I_B = \frac{I_C}{\beta} = \frac{4}{50} = 0.08 \text{ mA}$$

$$V_{BB} = 0.7 + \frac{20 \times 4}{50} = +2.3 \text{ V}$$

(b) edge of saturation $v_C = 0.3$ V

$I_C = \frac{5-0.3}{1} = 4.7$ mA

$I_B = I_C/\beta = 4.7/50 = 0.094$ mA

$V_{BB} = 0.094 \times 20 + 0.7 = 2.58$ V

(c) deep saturation $v_C = 0.2$ V $\beta_F = 10$

$I_C = (5 - 0.2)/1 = 4.8$ mA

$I_B = I_C/\beta_{forced} = 4.8/10 = 0.48$ mA

$V_{BB} = 0.48 \times 20 + 0.7 = +10.3$ V

4.37

Assume active:

$V_E = 3$ V, $V_B = 2.3$ (Assume $V_{BE} = 0.7$ V)

$I_B = \frac{2.3}{10\text{ K}} = 2.3$ mA

$I_c = 2.3$ m(50) = 115 mA

$V_c = 115$ m(1 K) = 115 V, $V_c < V_B$ (not true!)

saturation. Use $V_{ECSAT} = 0.2$ V

$+3 - V_{ECSAT} - V_c = 0$

$V_c = 3 - 0.2 = 2.8$ V

$V_B = 2.3$

$V_E = 3$ V

$V_c > V_B < V_E$ ∴ SATURATED

$$\beta_{forced} = \frac{I_{CSAT}}{I_B} = \frac{\left(\frac{2.8}{1\text{ K}}\right)}{\left(\frac{3}{10\text{ K}}\right)} = \frac{2.8\text{ m}}{0.3\text{ m}} = 9.33$$

Transistor will operate at edge of saturation when $V_c = V_B = 2.8$ V

$$\therefore R_B = \frac{V_B}{I_B} = \frac{2.8}{3\text{ m}} = 9.3\text{ k}\Omega$$

4.38

(a) $V_B = 2$V

$V_E = 2 - 0.7 = \underline{1.3V}$

$I_E = \frac{V_E}{1} = 1.3$mA

$I_C \cong 1.3$ mA

$V_c = 5 - 1.3 = \underline{3.7V}$

(b) $V_B = 1$V

$V_E = 1 - 0.7 = \underline{0.3V}$

$I_E \cong I_c = 0.3$mA

$V_c = 5 - 0.3 = \underline{4.7V}$

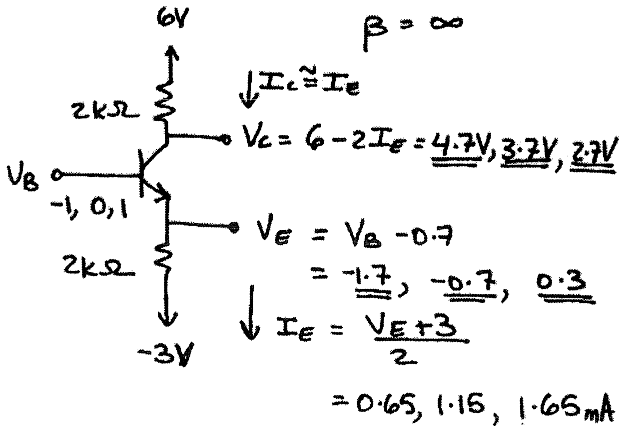
(c) $V_B = 0$V - cutoff

$V_E = \underline{0V}$

$I_E = 0$A

$V_c = \underline{5V}$

4.39



- Want V_B when $I_E = \frac{1}{10} \times 1.15$mA $= 0.115$mA

$V_E = -3 + 0.115 \times 2 = -2.77$ V

$V_B = V_E + 0.7 = \underline{-2.07\text{ V}}$

- Want V_B at the edge of conduction

At the edge of conduction assume $V_{BE} = 0.5$ V

∴ $V_B - 0.5 - 2I_E + 3 = 0$ ← $I_E = 0$ at edge of conduction

$V_B = \underline{-2.6V}$

$V_E = V_B - 0.5 = \underline{-3V}$

$I_C \cong 0A \quad \therefore V_C = \underline{6V}$

At saturation assume $V_{CE} = 0.2V$

$V_{CB} = -0.5V$

$\therefore I_E = \frac{V_B - 0.7 + 3}{2} \cong I_C = \frac{6 - (V_B - 0.5)}{2}$

$\therefore V_B + 2.3 = 6.5 - V_B$

$V_B = \underline{2.1V}$

$V_E = 2.1 - 0.7 = \underline{\underline{1.4V}} \quad V_C = V_B - 0.5 = \underline{\underline{1.6V}}$

-want V_B at $\beta_{forced} = 2$, $V_{CE} = 0.2V$

$V_{CB} = -0.5V$

$\beta_{forced} = \frac{I_{csat}}{I_B} = 2$

$I_E = I_B + I_{csat} = \frac{I_{csat}}{2} + I_{csat}$

$= 3/2\, I_{csat}$

$V_E = V_B - 0.7 = -3 + 2I_E$

$= -3 + 3I_{csat}$

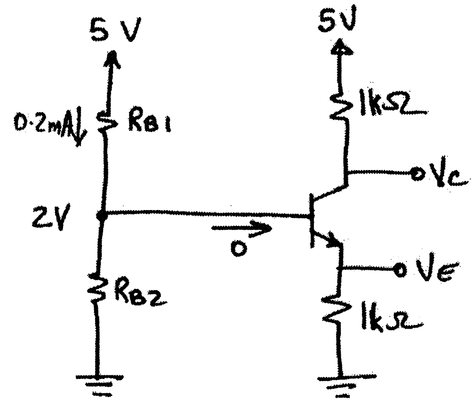
$I_{csat} = \frac{2.3 + V_B}{3}$

$I_C = \frac{V_{CC} - (V_B - 0.5)}{2} = I_{csat}$

$6.5 - V_B = \frac{2}{3}(2.3) + \frac{2}{3}V_B$

$V_B = \frac{6.5 - \frac{2(2.3)}{3}}{1\,2/3} = \underline{\underline{2.98V}}$

4.40



For $\beta = \infty$

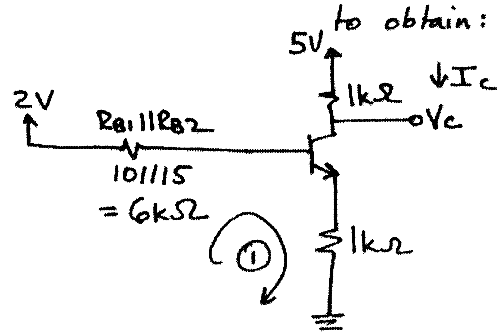
$\frac{5}{R_{B1} + R_{B2}} = 0.2 \quad \& \quad \frac{R_{B2}}{R_{B1} + R_{B2}} 5 = 2$

$R_{B1} + R_{B2} = 25k\Omega$ ↗ sub

$\Rightarrow \frac{R_{B2}}{25} \times 5 = 2$

$R_{B2} = \underline{\underline{10k\Omega}} \quad R_{B1} = \underline{\underline{15k\Omega}}$

Now for $\beta = 100$, use thevenin's to obtain:



Loop ① $2 - 6\left(\frac{I_E}{\beta + 1}\right) - 0.7 - I_E(1) = 0$

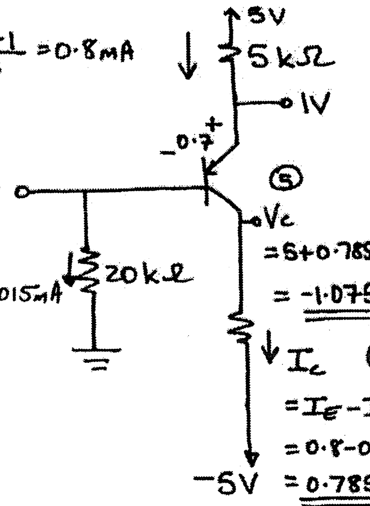
101

$I_E = 1.29mA$

$I_C = \frac{100}{101} I_E = \frac{100}{101} \times 1.29 = \underline{\underline{1.28\,mA}}$

$V_C = 5 - 1.28(1) = \underline{\underline{3.72\,V}}$

4.41



① $I_E = \frac{5-1}{5} = 0.8\,mA$

② $V_B = 1 - 0.7 = 0.3\,V$

③ $I_B = \frac{0.3}{20} = 0.015\,mA$

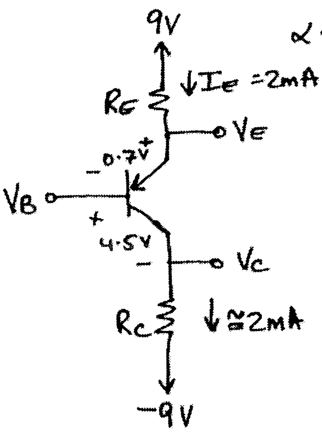
④ $I_C = I_E - I_B = 0.8 - 0.015 = \underline{\underline{0.785\,mA}}$

⑤ $V_C = -5 + 0.785(5) = \underline{\underline{-1.075\,V}}$

⑥ $\beta = \frac{I_C}{I_B} = \frac{0.785}{0.015} = \underline{\underline{52.3}}$

⑦ $\alpha = \frac{I_C}{I_E} = \frac{0.785}{0.8} = \underline{\underline{0.98}}$

4.42



$\alpha \cong 1$

$9 - 2R_E - 0.7 - 4.5 - 2R_C + 9 = 0$

$6.4 - R_E - R_C = 0$

LET $V_B = 0V$

$R_E = \frac{9-0.7}{2} = \underline{\underline{4.15\,k\Omega}}$

$R_C = \frac{-4.5+9}{2} = \underline{\underline{2.25\,k\Omega}}$

Using 5% resistor values

$R_E = 3.9\,k\Omega \quad R_C = 2.2\,k\Omega$

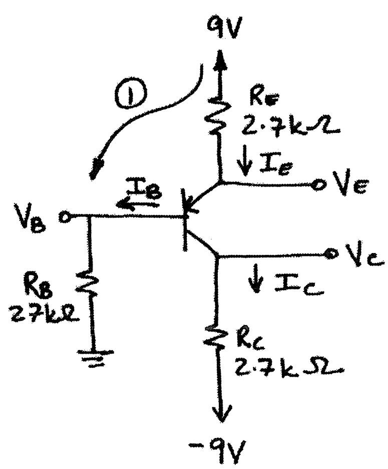
$I_E = \frac{9-0.7}{3.9} = \underline{\underline{2.12\,mA}}$

$V_C = -9 + 2.12 \times 2.2 = -4.3\,V$

$\therefore V_{BC} = \underline{\underline{4.3\,V}}$

4.43

$\beta = 30$



Loop ① $9 - 2.7 I_E - 0.7 - \frac{I_E}{31} R_B = 0$

$I_E = 2.3243\,mA$

$V_B = R_B \times I_E/31 = \underline{\underline{2.02\,V}}$

$V_E = 9 - 2.7 I_E = \underline{\underline{2.72\,V}}$

$V_C = -9 + \frac{30}{31} I_E (2.7) = \underline{\underline{-2.93\,V}}$

FOR $R_B = 270\,k\Omega$

Loop ① $9 - 2.7 I_E - 0.7 - \frac{R_B}{31} I_E = 0$

$I_E = 0.7274\,mA$

$V_B = R_B \times \frac{I_E}{31} = \underline{6.34V}$

$V_E = 9 - 2.7 I_E = \underline{7.04V}$

$V_C = \frac{30}{31} I_E(2.7) - 9 = \underline{-7.10V}$

To return the voltages to the ones first calculated we have

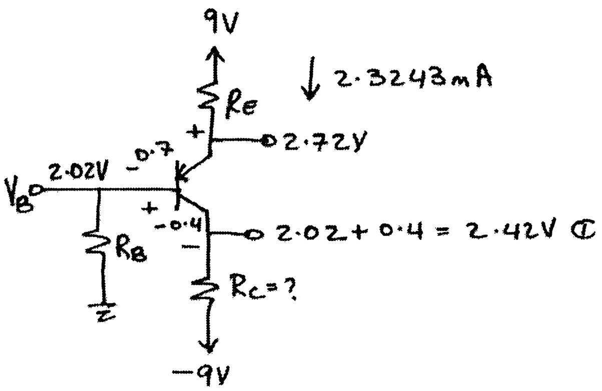
Loop ① $\sim I_E = 2.3243$ mA

$9 - 2.7I_E - 0.7 - \frac{270}{\beta+1} I_E = 0$

$\beta = \underline{309}$

4.44

Using the values from the first part of P5.76 and for the edge of saturation $V_{BC} > -0.4V$



CIRCUIT AT THE EDGE OF SATURATION

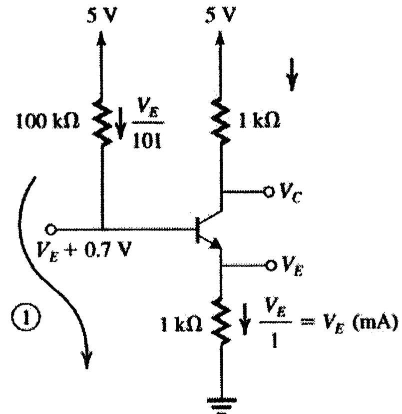
$I_C = \frac{30}{31} I_E = \frac{30}{31} \times 2.3243$

$R_C = \frac{2.42 + 9}{30/31 \times 2.3243} = \underline{5.08 k\Omega}$

4.45

$\beta = 100$

(a) $R_B = 100$ kΩ – ∵ R_B is large assume active mode.



$\frac{100}{101} I_E = \frac{100}{101} V_E$ (mA)

Loop (1)

$5 - \frac{V_E}{101} \times 100 - 0.7 - V_E \times 1 = 0$

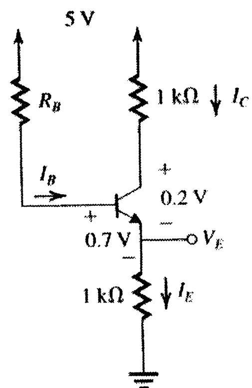
$V_E = 2.16$ V

$V_B = V_E + 0.7 = 2.86$ V

$V_C = 5 - 1 \times \frac{100}{101} V_E = 2.86$ V

Thus the BJT is in active mode as assumed.

(b) $R_B = 10$ kΩ - assume saturation



$I_B = \frac{5 - (V_E + 0.7)}{R_B}$

$I_C = \frac{5 - (V_E + 0.2)}{1}$

$I_E = \frac{V_E}{1} = I_B + I_C$

$\therefore V_E = \frac{4.3 - V_E}{10} + 4.8 - V_E$

$10V_E + V_E + 10V_E = 4.3 + 48$

$V_E = 2.49\text{ V}$

$V_C = 2.49 + 0.2 = 2.69\text{ V}$

$V_B = V_E + 0.7 = 3.19\text{ V}$

Check : $I_C = \frac{5 - 2.69}{1} = 2.31\text{ mA}$

$I_B = \frac{5 - 3.19}{10} = 0.181\text{ mA}$

$\frac{I_C}{I_B} = \frac{2.31}{0.181} = 12.76 < 100$

Hence, we are in saturation as assumed!

(c) $R_B = 1\text{ k}\Omega$ - expect saturation, use circuit in (b)

$I_B = \frac{5 - (V_E + 0.7)}{R_B} = \frac{4.3 - V_E}{1}$

$I_C = \frac{5 - (V_E + 0.2)}{1} = \frac{4.8 - V_E}{1}$

$I_E = I_B + I_C = V_E$

$4.3 - V_E + 4.8 - V_E = V_E$

$V_E = 3\text{ V}$

$V_B = 3.7\text{ V}$

$V_C = 3.2\text{ V}$

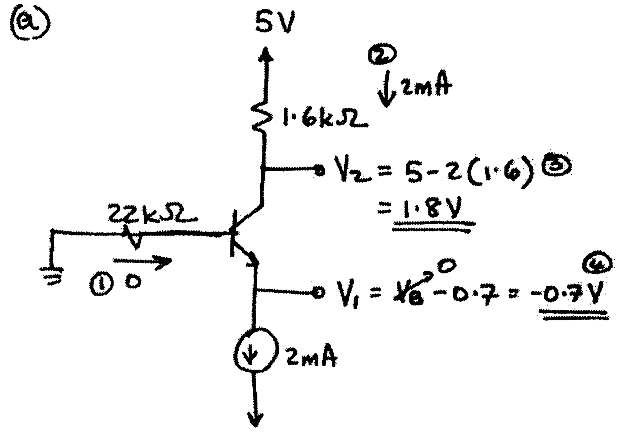
Check $I_B = 4.3 - 3 = 1.3\text{ mA}$

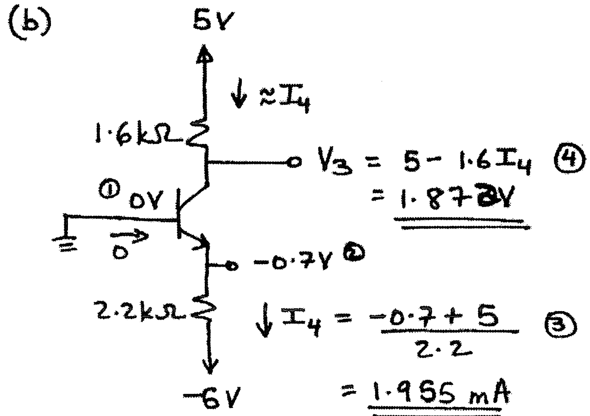
$I_C = 4.8 - 3 = 1.8\text{ mA}$

$\frac{I_C}{I_B} = \frac{1.8}{1.3} = 1.4 < 100$

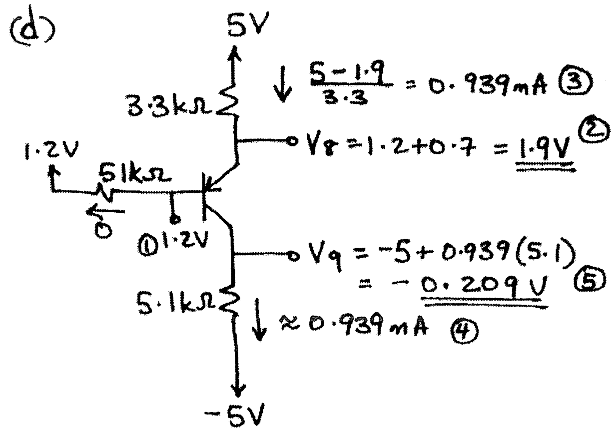
$\therefore$ Saturation as assumed

4.46

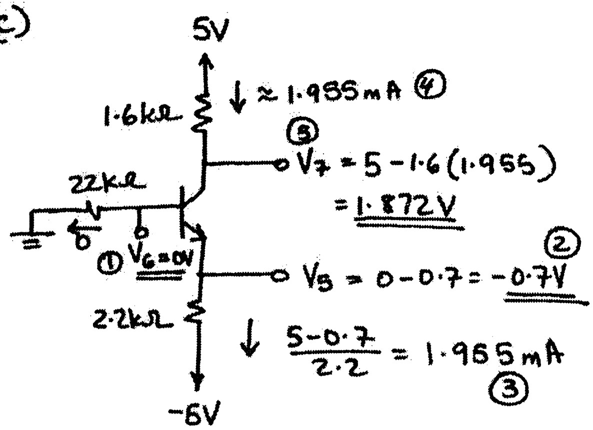




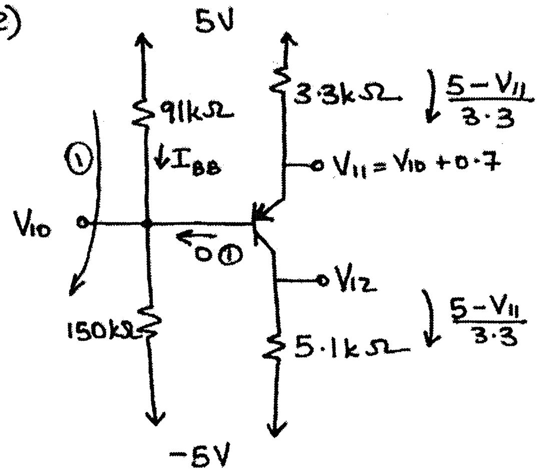
see below for part (c)



(c)



(e)



Loop ①

$$5 - 91 I_{BB} - 150 I_{BB} + 5 = 0$$

$$I_{BB} = \frac{10}{91 + 150}$$

$$V_{10} = -5 + 150 I_{BB}$$

$$= -5 + \frac{150}{91+150} \times 10$$

$$= \underline{1.224V}$$

$$V_{11} = V_{10} + 0.7 = \underline{1.924V}$$

$$\therefore I_C \approx I_E = \frac{5 - V_{11}}{3.3}$$

$$V_{12} = -5 + \left(\frac{5 - V_{11}}{3.3}\right) 5.1 = \underline{-0.246V}$$

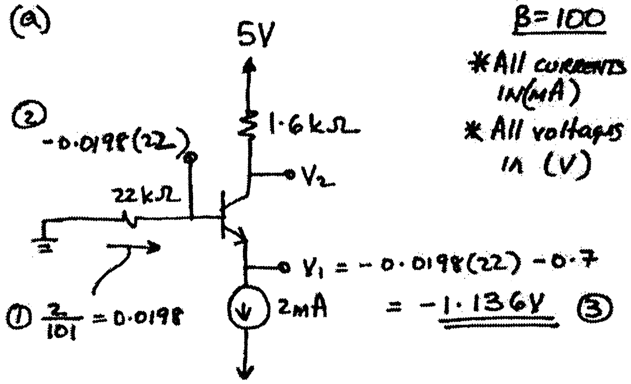
4.47

(a)

$\beta = 100$

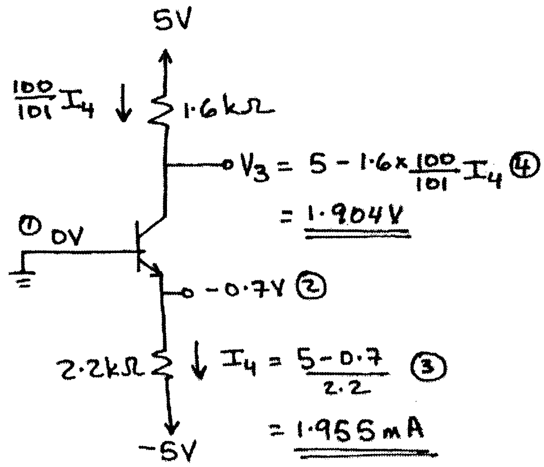
*All currents in (mA)

*All voltages in (V)

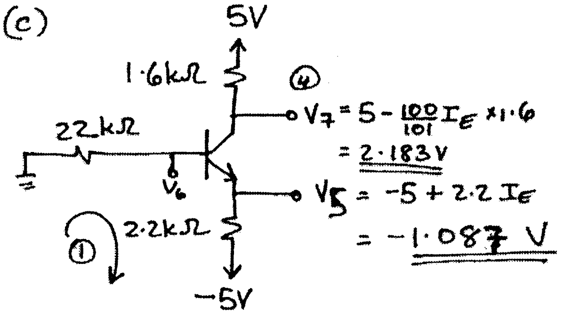


$$④\ V_2 = 5 - 2\left(\frac{100}{101}\right) 1.6 = \underline{1.832V}$$

(b)



(c)



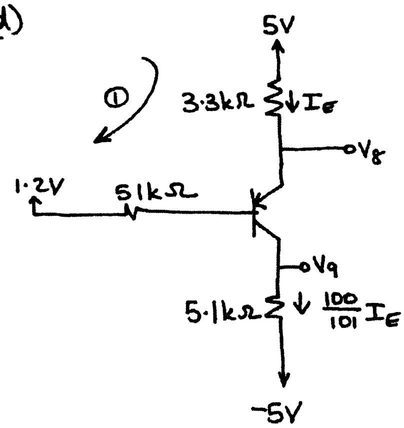
$$\text{Loop } ①\quad 0 - \frac{I_E}{101} 22 - 0.7 - 2.2 I_E + 5 = 0$$

$$I_E = 1.778 mA$$

$$③\ V_6 = V_5 + 0.7 = \underline{-0.387\ V}$$

CONT

(d)



Loop ①

$$5 - 3.3I_E - 0.7 - \frac{I_E}{101} 51 - 1.2 = 0$$

$$I_E = 0.8147 \text{ mA}$$

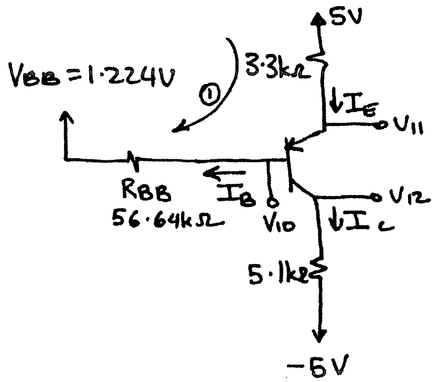
$$V_8 = 5 - 3.3 I_E = \underline{\underline{2.3114 \text{ V}}}$$

$$V_9 = -5 + 5.1 \times \frac{100}{101} I_E = \underline{\underline{-0.8862 \text{V}}}$$

(e) Use Thévenin's theorem to simplify the bias network:

$$V_{BB} = -5 + \frac{150}{150+91} \times 10 = 1.224 \text{ V}$$

$$R_{BB} = 150 \| 91 = 56.64 \text{ k}\Omega$$



Loop ①

$$5 - 3.3I_E - 0.7 - \frac{I_E}{101} R_{BB} - 1.224 = 0$$

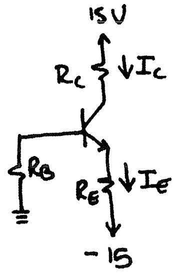
$$I_E = 0.7967 \text{ mA}$$

$$V_{11} = 5 - 3.3I_E = \underline{\underline{2.371 \text{V}}}$$

$$V_{12} = \frac{100}{101} I_E \times 5.1 - 5 = \underline{\underline{-0.977 \text{V}}}$$

$$V_{10} = V_{11} - 0.7 = \underline{\underline{1.67 \text{V}}}$$

4.48



Nominal $\beta = 100$.
Thus,
nominal $\alpha = \frac{100}{101} = 0.99$

nominal $I_E = 1$mA
nominal $I_C = 0.99$mA
nominal $V_C = 5$V

Thus, $R_C = \frac{15-5}{0.99} = 10.1 \text{k}\Omega \overset{\text{use}}{\Rightarrow} \underline{10\text{k}\Omega}$

$$I_E = 1 = \frac{15 - 0.7}{R_E + \frac{R_B}{\beta+1}}$$

$$= \frac{14.3}{R_E + \frac{R_B}{101}}$$

$$\Rightarrow R_E + \frac{R_B}{101} = 14.3 \quad (1)$$

As β varies from 50 to 150, need to limit the variation of I_E to ±10% of 1mA. One can reason that the maximum variation in I_E occurs for $\beta = 50$ (as opposed to $\beta = 150$). To see this more that when β decreases from 100 to 50 the base current doubles while a change in β from

CONT.

100 to 150 causes the base current to decrease to 2/3 its nominal value. Thus our decision will be based on imposing the 10% limit for $\beta = 50$.

$$0.9 = \frac{14.3}{R_E + \frac{R_B}{\beta+1}} = \frac{14.3}{R_E + \frac{R_B}{51}}$$

$$R_E + \frac{R_B}{51} = 15.89 \quad (2)$$

$$(2) - (1) \Rightarrow \quad R_B\left(\frac{1}{51} - \frac{1}{101}\right) = 1.59$$

$$\Rightarrow R_B = 163.8\,k\Omega \overset{\text{use}}{\Rightarrow} \underline{\underline{164\,k\Omega}}$$

Sub into (1) gives

$$R_E = 12.7\,k\Omega \overset{\text{use}}{\Rightarrow} \underline{\underline{13\,k\Omega}}$$

To find the expected range of I_C & V_C corresponding to β variation from 50 to 150 we use

$$I_C = \alpha \frac{14.3}{R_E + \frac{R_B}{\beta+1}}$$

for $\beta = 50$ $\quad I_C = \frac{50}{51} \frac{14.3}{13 + \frac{164}{51}} = \underline{\underline{0.864\,mA}}$

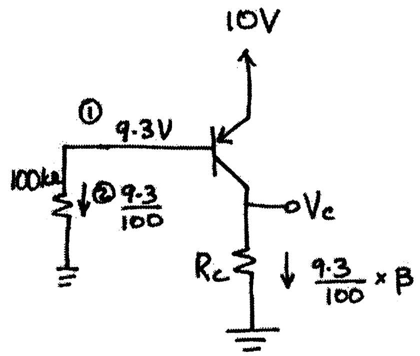
$$V_C = 15 - 0.864 \times 10 = \underline{\underline{6.36\,V}}$$

for $\beta = 150$ $\quad I_C = \frac{150}{151} \times \frac{14.3}{13 + \frac{164}{151}}$

$$= \underline{\underline{1.008\,mA}}$$

$$V_C = 15 - 1.008 \times 10 = \underline{\underline{4.92\,V}}$$

4.49



For $V_C = 5\,V = \frac{9.3}{100} \times \beta \times R_C \qquad \beta = 50$

$$R_C = \frac{500}{9.3 \times 50} = \underline{\underline{1.08\,k\Omega}}$$

For $\beta = 100$

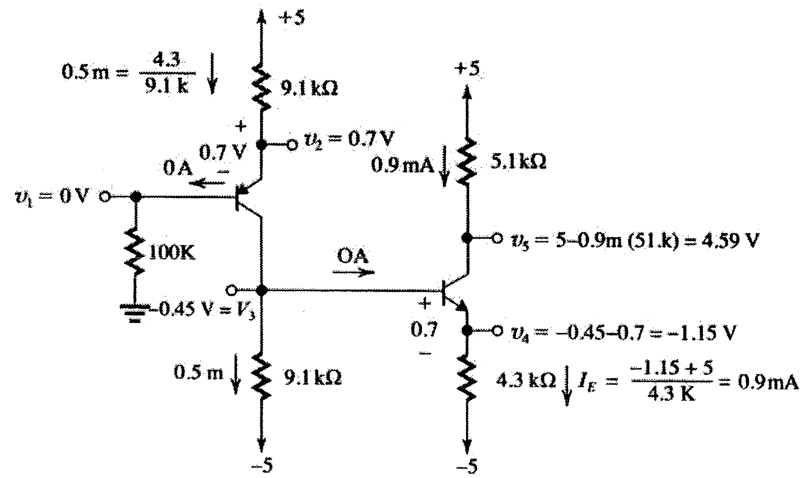
$$V_C = \frac{9.3}{100} \times \beta \times R_C = \frac{9.3}{100} \times 100 \times 1.08$$

$$= \underline{\underline{10.04\,V}} \leftarrow V_{BC} = 9.3 - 10.04 = -0.74$$

Since $V_{BC} < -0.4\,V$ the transistor $\underline{\underline{\text{saturates}}}$!

4.50

(a) $\beta = \infty$



$$+5 - I_{E1}(9.1\text{ K}) - 0.7 - I_{B1}(100\text{ K}) = 0$$

$$I_{B1} = \frac{I_{E1}}{\beta + 1}$$

$$4.3 = I_{E1}\left(9.1\text{ K} + \frac{100\text{ K}}{101}\right)$$

$$I_{E1} = \frac{4.3}{10,090} = .43\text{ mA}$$

$$V_2 = 5 - 9.1\text{ K}(.43\text{ m}) = 1.36\text{ V}$$

$$V_1 = 1.36 - 0.7 = .66\text{ V}$$

$$I_{C1} = \alpha I_{E1} = .426\text{ m}$$

$$-5 + 9.1\text{ K}(I_{C1} + I_{B2}) - 0.7 - I_{E2}(4.3\text{ K}) + 5 = 0$$

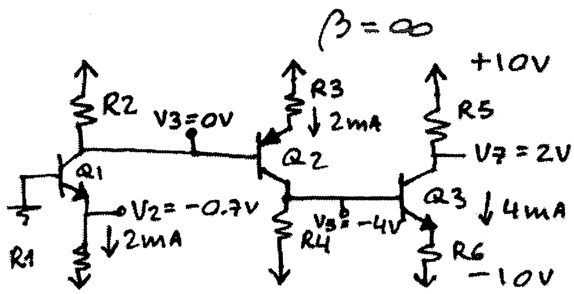
$$9.1\text{ K}(.426\text{ m}) + \frac{9.1\text{ K } I_{E2}}{101} - 0.7 - I_{E2}(4.3\text{ K}) = 0$$

$$I_{E2} = \frac{3.2}{4210} = .75\text{ mA}$$

$$V_4 = -5 + I_{E2}(4.3\text{ K}) = -1.8\text{ V}$$

$$V_3 = V_4 + 0.7 = -1.08\text{ V}$$

4.50



$R_1 = \frac{9.3}{2} = \underline{\underline{4.7K\Omega}}$

$R_2 = \frac{10}{2} = 5 \rightarrow \underline{\underline{5.1K\Omega}}$

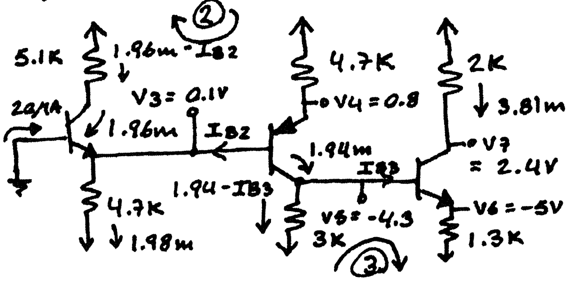
$R_3 = \frac{9.3}{2} = \underline{\underline{4.7K\Omega}}$

$R_4 = \frac{6}{2} = \underline{\underline{3K\Omega}}$

$R_5 = \frac{8}{4} = \underline{\underline{2K\Omega}}$

$R_6 = \frac{10-4.7}{4} = \underline{\underline{1.3K\Omega}}$

$\beta = 100$



② $(1.96 - I_{B2}) \times 5.1$
$= (\beta+1) I_{B2} \times 4.7 + 0.7$

$I_{B2} = 0.0194\,mA$

$I_{E2} = 1.96\,mA$

$V_3 = \underline{\underline{0.1V}}$ $\quad V_4 = \underline{\underline{0.8V}}$

③ $(1.94 - I_{B3}) \times 3$
$= 0.7 + 1.3 \times (\beta+1) \cdot I_{B3}$

$I_{B3} = 0.038\,mA$

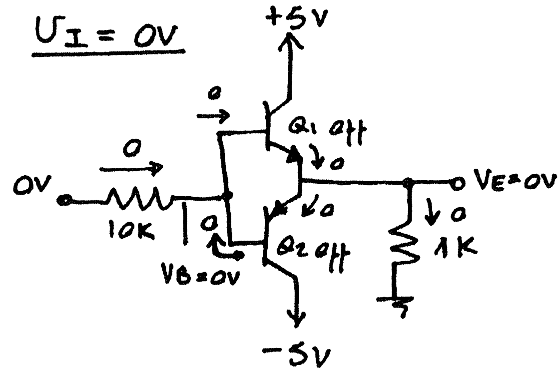
$I_{E3} = 3.85\,mA$

$V_5 = -\underline{\underline{4.3V}}$ $\quad V_6 = \underline{\underline{-5V}}$

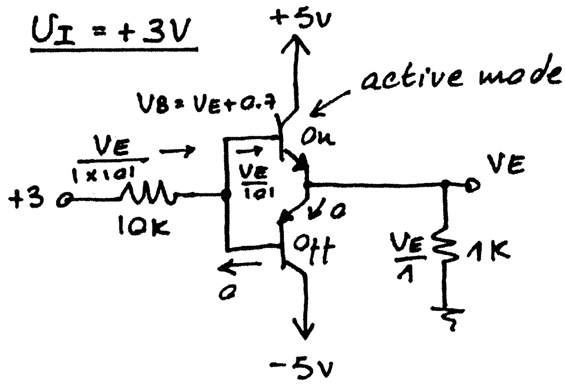
$V_7 = \underline{\underline{2.4V}}$

4.51

$\underline{V_I = 0V}$



$\underline{V_I = +3V}$

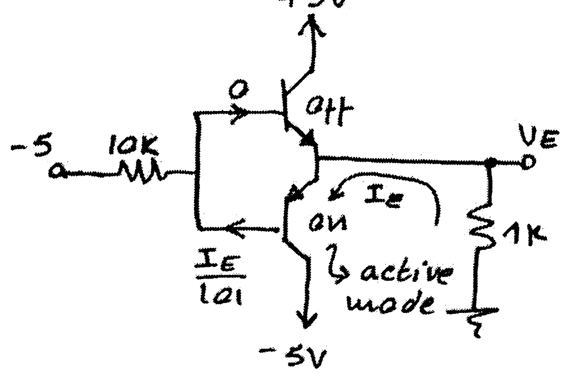


$3 = \frac{V_E}{101} \times 10 + 0.7 + V_E$

$\Rightarrow V_E = \underline{\underline{2.09V}}$

$V_B = \underline{\underline{2.79V}}$

$v_I = -5V$

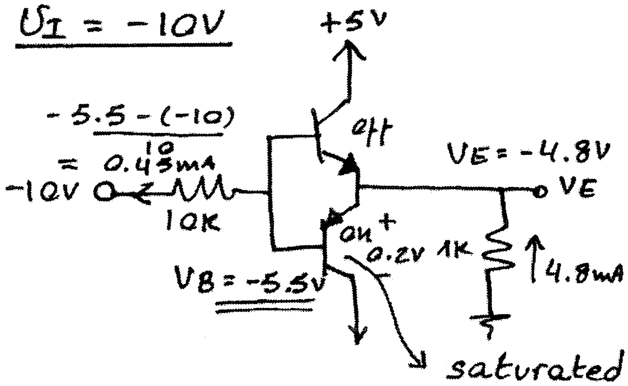


$$I_E = \frac{5 - 0.7}{1 + 10/101} = 3.91 mA$$

$V_E = -3.91V$

$V_B = -4.61V$

$v_I = -10V$



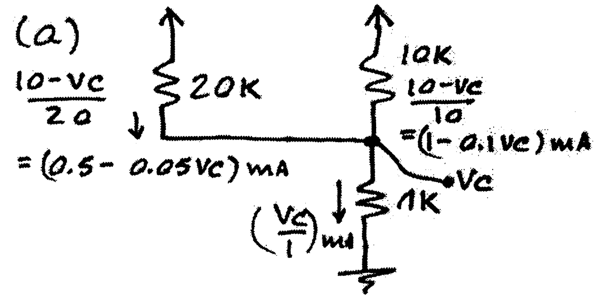
$$\frac{I_C}{I_B} = \frac{4.35}{0.45} = 9.7 < 100$$

thus, Q_2 is saturated as assumed

$V_E = -4.8V$ $V_B = -5.5V$

4.52

(a)



$$(0.5 - 0.005V_C) + (1 - 0.1V_C) = V_C$$

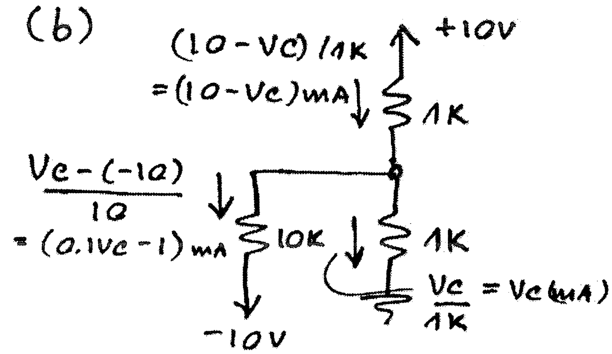
$V_C = 1.3V$

$$I_C = \frac{10 - 1.3}{10} = 0.87mA$$

$$I_B = \frac{10 - 1.3}{20} = 0.435mA$$

thus $\beta_{forced} = \frac{0.87}{0.435} = 2$

(b)



$$10 - V_C = (0.1V_C + 1) + (V_C)$$

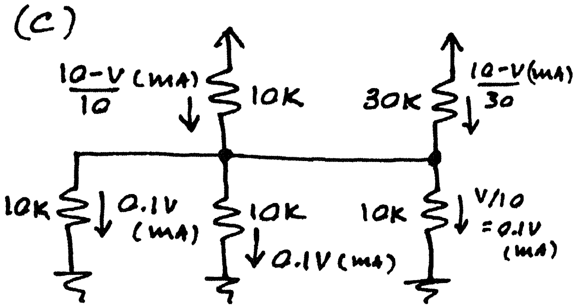
$\Rightarrow V_C = +4.29V$

$I_C = 4.29mA$

$$I_B = \frac{4.29 + 10}{10} = 1.43mA$$

$$\beta_{forced} = \frac{4.29}{1.43} = 3$$

(c)



Node equation:

$$\frac{10-V}{10} + \frac{10-V}{30} = 0.1V + 0.1V + 0.1V$$

$$30 - 3V + 10 - V = 9V$$

$$40 = 13V$$

$$\Rightarrow V = \underline{3.08\,V}$$

thus, $V_{C3} \simeq V_{C4} \simeq 3.08V$

$$I_{B3} = 0.1V = 0.308\,mA$$

$$I_{E3} = \frac{10-3.08}{10} \simeq 0.692\,mA$$

$$I_{C3} = 0.692 - 0.308 = 0.384\,mA$$

$$\beta_{3\,forced} = \frac{0.384}{0.308} = \underline{\underline{1.25}}$$

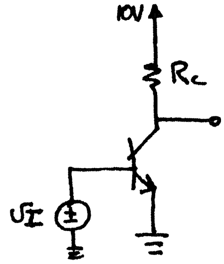
$$I_{C4} = \frac{10-3.08}{30} = 0.231\,mA$$

$$I_{E4} = 0.1V = 0.308\,mA$$

$$I_{B4} = 0.308 - 0.231 = 0.077\,mA$$

$$\beta_{4\,forced} = \frac{0.231}{0.077} = \underline{\underline{3}}$$

4.53



$v_C = 1V + \Delta V$

assume $v_{CEsat} = 0.2V$

$$A_V = \frac{-I_C R_C}{V_T} = -\frac{10-1}{0.025} = \underline{\underline{-360\,\tfrac{V}{V}}}$$

At the verge of saturation $v_{CEsat} = 0.3V$

$\therefore v_C = 1 + \Delta V = 0.3.$

$$\Delta v = \underline{\underline{-0.7\,V}}$$

$\therefore v_O = 0.3V \quad i_C = \frac{10-0.3}{R_C}$

$$\frac{i_{C2}}{i_{C1}} = \frac{9.7/R_C}{(10-1)/R_C} = e^{\Delta v/V_T}$$

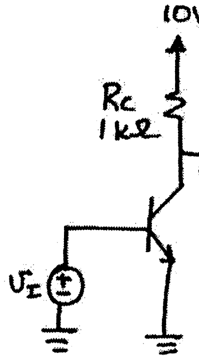
$\therefore$ Maximum Input signal

$$\Delta v = 0.025 \ln \frac{9.7}{9} = \underline{\underline{1.87\,mV}}$$

If we assume linear operation right to saturation we can use the gain A_v to calculate the maximum input swing. Thus for an output swing $\Delta v_o = 0.8$ we have

$$\Delta v_i = \frac{-\Delta v_o}{A_v} = \frac{-0.7}{-360} = \underline{\underline{1.94\,mV}}$$

4.54



$V_{CE} = 10 - 10 I_C$ ← DC collector voltage

$$A_V = -\frac{I_C R_C}{V_T}$$

$$= -\frac{I_C \times 10}{0.025}$$

$$= -400 I_C \quad (1)$$

- Assuming the output voltage $v_O = 0.3V$ is the lowest v_{CE} to stay out of saturation.

$$\therefore v_O = 0.3 = 10 - i_C R_C$$

$$= 10 - I_C R_C + \Delta v_O$$

$$\Delta v_O = -10 + 0.3 + I_C \times 1 \quad (2)$$

- Max output voltage before the transistor is cut off

$$V_{CE} + \Delta v_O = V_{CC}$$

$$\Delta v_O = V_{CC} - V_{CE}$$

$$= 10 - 10 + 10 I_C$$

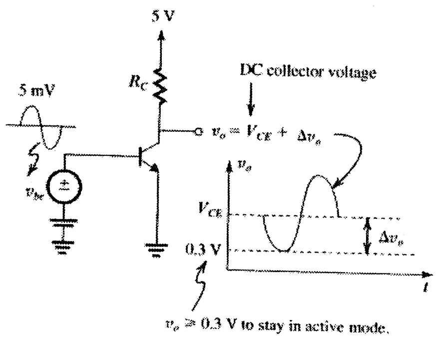
$$= 10 I_C \quad (3)$$

Use (1) to calculate the gain and (2), (3) to calculate the output limits in order to stay in active mode for a particular bias current I_C.

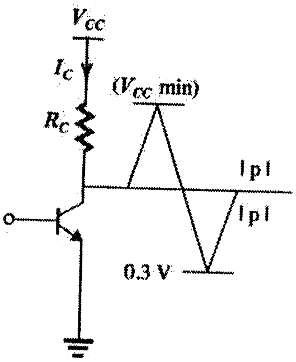
I_C (mA)	A_V (V/V)	ΔV_O (V)
1	−40	−8. to 1
2	−80	−7. to 2
5	−200	−4.7 to 5
8	−320	−1.7 to 8
9	−360	−0.7 to 9

4.55

Since we are assuming linear operation we don't have to go to $i_C = I_S e^{V_{BE}/V_T}$ equation.



$$A_V = -\frac{I_C R_C}{v_T} = -\frac{V_{CC} - V_{CE}}{v_T}$$



$V_{CC} = 0.3 + |P| + I_C R_C$

$|A_V| = (-)g_m R_C = \frac{I_C R_C}{V_T} \geq \frac{p}{V_T}$

$\therefore V_{CC}$ min

4.56

On the verge of Satuation

$V_{CE} - \Delta v_O = 0.3$ V

for linear operation $\Delta v_O = A_V v_{be}$

$V_{CE} - |A_V v_{be}| = 0.3$

$(5 - I_C R_C) - A_V \times 5 \times 10^{-3} = 0.3$

$5 - |A_V V_T| - |A_V \times 5 \times 10^{-3}| = 0.3$

$|A_V(0.025 + 0.005)| = 5 - 0.3$

$|A_V| = 156.67$ Note A_v is negative.

$\therefore A_V = -156.67$ V / V

Now we can find the dc collector voltage. Refer to the sketch of the output voltage, we see that

$|\Delta v_O| = |(A_V \times 0.005)|$

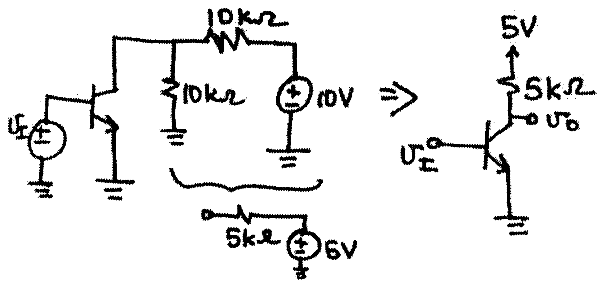
$\therefore V_{CE} = 0.3 + |A_V| 0.005 = 1.08$ V

$= V_{CEsat} + P + |A_V| V_T$

$I_C R_C = |A_V| V_T$

	A_v(V/V)	P(V)	$A_v V_T$	$V_{cc} = \lvert A_V \rvert_T + P + 0.3$
(a)	−20	0.2	0.5	1.0 → 1.0 V
(b)	−50	0.5	1.25	2.05 → 2.5 V
(c)	−100	0.5	2.5	3.3 → 3.5 V
(d)	−100	1.0	2.5	3.8 → 4.0 V
(e)	−200	1.0	5.0	6.3 → 6.5 V
(f)	−500	1.0	12.5	13.8 → 14 V
(g)	−500	2.0	12.5	14.8 → 15 V

4.57

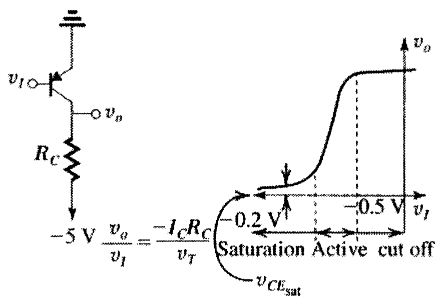


$$\frac{v_o}{v_I} = -\frac{I_C R_L}{V_T} = -\frac{0.5 \times 5}{0.026}$$

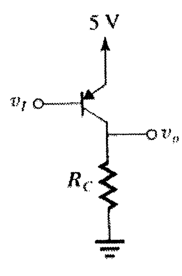
$$= -100 \text{ V/V}$$

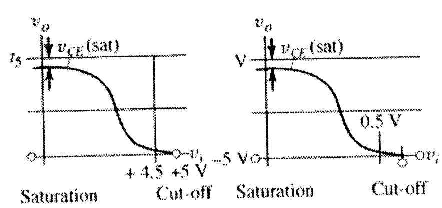
4.58

(a)



(b)





4.59

Including the Early effect we note that:

$$i_C = I_S e^{v_{BE}/V_T}\left(1 + v_{CE}/V_A\right)$$

Also, note $I_C = I_S e^{v_{BE}/V_T}$ Eq (5.38b) is the value of the collector current with the Early voltage neglected.

Starting with the voltage at the collector we have:

$$v_O = V_{CC} - i_C R_C$$
$$= V_{CC} - R_C I_S e^{v_{BE}/V_T}\left(1 + \frac{v_{CE}}{V_A}\right)$$

Take derivative to get gain Av

$$A_v = \frac{\partial v_O}{\partial v_I}$$
$$= -R_C I_S\left[\frac{e^{v_{BE}/V_T}}{V_T}\left(1 + \frac{v_{CE}}{V_A}\right) + \frac{e^{v_{BE}/V_T}}{V_A}\frac{\partial v_{CE}}{\partial v_I}\right]$$

$$A_v = -\frac{R_C I_S}{V_T} e^{v_{BE}/V_T}\left[1 + v_{CE}/V_A + \frac{V_T}{V_A}\frac{\partial v_{CE}}{\partial v_I}\right]$$

$$= \frac{-R_C I_C}{V_T}\left[1 + \frac{v_{CE}}{V_A} + \frac{V_T}{V_A} A_v\right]$$

$$-A_v\left[\frac{1}{\frac{R_C I_C}{V_T}} + \frac{V_T}{V_A}\right] = 1 + \frac{v_{CE}}{V_A} = \frac{V_A + v_{CE}}{V_A}$$

$$-A_v\left[\frac{V_A + R_C I_C}{\frac{R_C I_C V_A}{V_T}}\right] = \frac{V_A + V_{CE}}{V_A}$$

$$-A_v \Big/ \frac{R_C I_C}{V_T} = \frac{V_A}{V_A + R_C I_C} \times \frac{V_A + v_{CE}}{V_A}$$

$$= \frac{V_A + V_{CE}}{V_A + R_C I_C} \quad \div \text{ top \& bottom by } V_A + V_{CE}$$

$$= \frac{1}{\frac{V_A}{V_A + v_{CE}} + \frac{R_C I_C}{V_A + V_{CE}}}$$

This term is ≅ 1 ∵ $V_A >> v_{CE}$

$$\therefore A_v \cong \left[\frac{-R_C I_C/V_T}{\left(1 + \frac{R_C I_C}{V_A + v_{CE}}\right)}\right]$$

Q.E.D.

For $V_{CC} = 5\,V$ $V_{CE} = 2.5\,V$ $V_A = 100\,V$

Ignoring the Early Voltage:

$$A_v = \frac{-I_C R_C}{V_T} = \frac{V_{CC} - V_{CE}}{V_T} = \frac{5 - 2.5}{0.025} = 100\,\frac{V}{V}$$

With the Early voltage

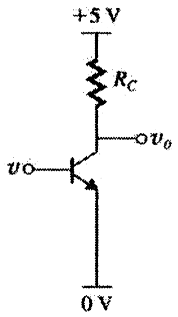
$$A_v \cong \frac{-I_C R_C/V_T}{1 + \frac{R_C I_C}{V_A + v_{CE}}}$$

But $v_{CE} = 0.5V$ & $\frac{I_C R_C}{V_T} = 100$ as shown above.

$$\therefore A_v = \frac{-100}{1 + \frac{2.5}{100 + 2.5}}$$

$$= -97.7\,V/V$$

4.60



For $V_O = 2\text{V}$, $R_C = 1\ \text{k}\Omega$

$I_C = \frac{5-2}{1} = 3\ \text{mA}$

$A_V = \frac{-I_C R_C}{V_T} = -120\ \text{V/V}$

$\Delta V_O = -120 \times 5 = -600\ \text{mV}$

$\Delta V_{BE} = V_T \ln[I_2 / I_1]$

$\frac{I_2}{I_1} = \epsilon^{\Delta v_{BE}/v_T} = \epsilon^{5/25}$

(a) $I_2 = I_1 e^{5/25} = 3 \times 1.22 = 3.66\ \text{mA}$

$\Delta V_O = (I_2 - I_1)R_C = 0.66 \times 1 = 0.660\ \text{V}$

$A_V = -660/5 = -132\ \text{V/V}$

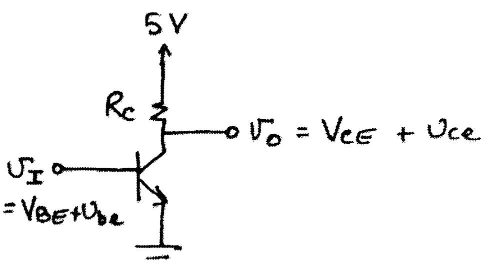
(b) $I_3 = I_1 e^{-5/25} = 3 \times 0.82 = 2.46\ \text{mA}$

$\Delta V_O = (I_3 - I_1)R_C = 0.544\ \text{V}$

$A_V = -544/5 = -109\ \text{V/V}$

ΔV_{BE}	ΔV_O (exp)	ΔV_O (linear)
+5 mV	−660 mV	−600 mV
−5 mV	+544 mV	+600 mV

4.61



(a) For maximum gain you would bias at the largest current since $A_V = -I_C R_C / V_T$. This also means you would bias at the edge of saturation $A_V = -\frac{V_{CC} - V_{CEsat}}{V_T}$

$= -\frac{5 - 0.3}{0.025}$

$= -188\ \text{V/V}$

However any signal swing at the output would automatically drive it into saturation.

(b) for $A_V = -100\ \text{V/V}$

$A_V = \frac{V_{CC} - V_{CE}}{V_T} = \frac{5 - V_{CE}}{V_T} = 100$

$V_{CE} = 2.5\text{V}$

(c) For a dc collector current of 0.5mA

$R_C = \frac{5-2.5}{0.5} = 5\ \text{k}\Omega$

(d) $I_S = 10^{-15}\text{A} \Rightarrow$

$I_C = I_S e^{V_{BE}/V_T}$

$0.5 \times 10^{-3} = 10^{-15} e^{V_{BE}/0.025}$

$V_{BE} = 0.673\text{V}$

(e) If we assume linear operation we can use A_v to find the output change for $v_{be} = 5\,mV$

$v_{ce} = A_v v_{be} = -100 \times 0.005$

$= -0.5\ V$ ~ peak sine wave.

∴ the output is a 0.5V p sine wave

(f) for $v_{ce} = 0.5$

$i_c = \frac{0.5}{5} = 0.1\,mA$ peak

This current is superimposed on I_c.

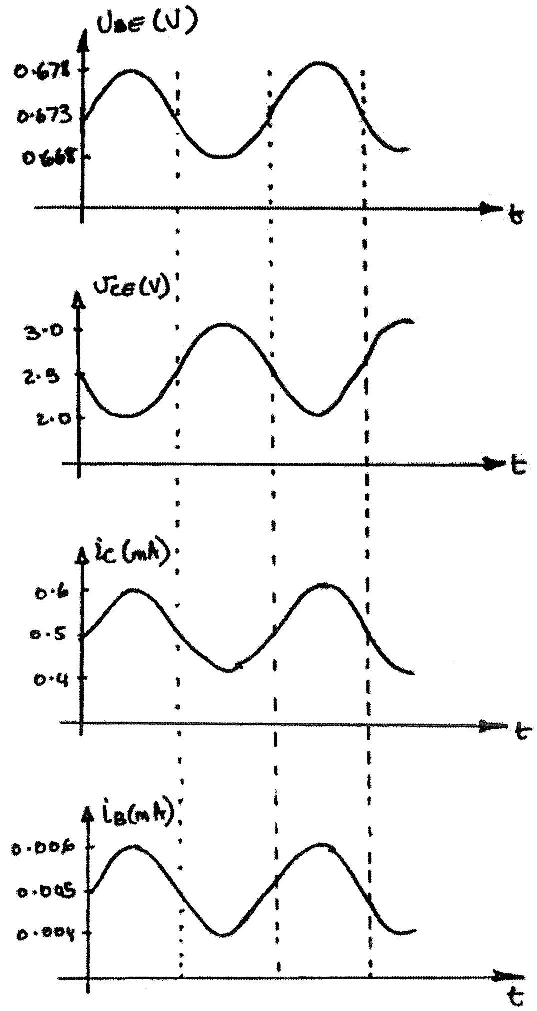
(g) $I_B = I_c/\beta = \frac{0.5}{100} = 0.005\,mA$

$i_b = \frac{i_c}{\beta} = \frac{0.1}{100} = 0.001\,mA\ p$

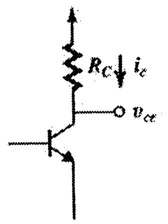
(h) $r_{in} = \frac{v_{be}}{i_b} = \frac{0.005}{0.001 \times 10^{-3}}$

$= 5\,k\Omega$

(i) See sketches that follow:



4.62



$$A_V = \frac{v_{ce}}{v_{be}} = \frac{-I_C R_C}{V_T}$$

But $V_{ce} = -i_c R_C$

$$\therefore -\frac{i_C R_C}{v_{be}} = -\frac{I_C R_C}{V_T}$$

Now $g_m = \frac{\text{Output current}}{\text{Input voltage}} = \frac{i_C}{v_{be}}$

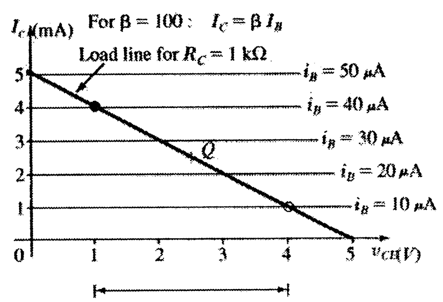
$$\therefore g_m R_C = \frac{I_C R_C}{V_T}$$

$$g_m = I_C / V_T$$

$$g_m = \frac{1\text{ mA}}{25\text{ mV}} = 40\text{ ms}$$

for $I_C = 1$ mA

4.63



Peak-to-peak V_c swing $= 4 - 1 = 3$ V

For Q point at $V_{CC}/2 = 2.5$ V

$V_{CE} = 2.5$ V : $I_C = 2.5$ mA

$I_B = 25$ μA

$$I_B = \frac{V_{BB} - 0.7}{R_B} = 25\ \mu\text{A}$$

$$\Rightarrow V_{BB} = I_B R_B + 0.7 = 2.5 + 0.7 = 3.2\text{ V}$$

4.63

(a) Using the exponential characteristic :

$$i_C = I_{ce}\, e^{v_{be}/V_T} - I_C$$

giving $\frac{i_C}{I_C} = e^{v_{be}/V_T} - 1$

(b) Using small-signal approximation :

$$i_C = g_m V_{be} = \frac{I_C}{V_T} \cdot V_{be}$$

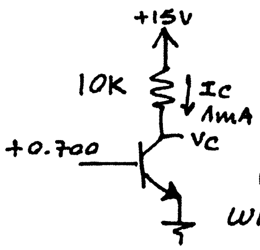
Thus, $\frac{i_C}{I_C} = \frac{V_{be}}{V_T}$

See table below

For signals at ± 5 mV, the error introduced by the small-signal approximation is 10 %

The error increases to above 20% for signals at ± 10 mV.

v_{be} (mV)	i_c/I_c Expan.	i_c/I_c small signal.	% Error
+1	+0.041	+0.040	−2
−1	−0.03P	−0.040	+2
+2	+0.083	+0.080	−4
−2	−0.077	−0.080	+4
+5	+0.221	+0.200	−9.5
−5	−0.181	−0.200	+10.3
+8	+0.377	+0.320	−15.2
−8	−0.274	−0.320	+16.8
+10	+0.492	+0.400	−18.7
−10	−0.330	−0.400	+21.3
+12	+0.616	+0.480	−22.1
−12	−0.381	−0.480	+25.9



$V_C = 15 - 1 \times 10$
$= 5V$

$v_{BE} = 0.7 + v_{be}$
where $v_{be} = 0.005V$

$i_c \cong I_C \left(1 + \frac{v_{be}}{V_T}\right)$ Eq. (5.83)

$i_c \simeq I_C + i_c$ where:

$i_c = \frac{1m \times 0.005}{25m} = 0.2m$

$i_c = 1mA + 0.2mA$

$v_C = V_{CC} - i_C R_C$ Eq. (5.101)

$\Rightarrow v_c - \underbrace{i_c R_C}_{0.2m \times 10K}$

$v_c = 5V - 2V$

$\text{gain} = \frac{-2V}{0.005V} = -400\ V/V$

while $-g_m \cdot R_C = -\frac{1m}{25m} \cdot 10K = -400\ V/V$

4.65

$g_m = \frac{I_C}{V_T} = \frac{1.2mA}{25mV} = 48\frac{mA}{V}$

$r_\pi = \frac{\beta}{g_m} = \frac{120}{48 \times 10^{-3}} = 2.5K\Omega$

$r_e = \frac{r_\pi}{\beta + 1} = \frac{2500}{121} = 20.6\Omega$

For a bias current of 120μA i.e. 10 times lower:

$g_m = \frac{48}{10} = 4.8\ mA/V$

$r_\pi = 10 \times 2.5 = 25K\Omega$

$r_e = 10 \times 20.6 = 206\Omega$

4.66

$I_C = 2mA \Rightarrow g_m = \frac{2mA}{25mV}$

$g_m = 80mA/V$

$r_e = \frac{V_T}{I_E}, \quad I_E = I_C \frac{(\beta+1)}{\beta}$

$I_E = 2mA \times \frac{51}{50} = 2.04mA$

$r_e = \frac{25m}{2.04m} = 12.25\Omega$

$r_\pi = \frac{\beta}{g_m} = \frac{50}{80 \times 10^{-3}} = 625\Omega$

gain : $-g_m \times R_C$

For $R_C = 5K\Omega$ and $\hat{v}_{be} = 5mV$

$\hat{v}_o = -80m \times 5K \times 5mV$
$= -2V$

4.67

$g_m = 50\frac{mA}{V} = \frac{I_C}{V_T}$

$\Rightarrow I_C = g_m \times V_T = 50m \times 25m = 1.25mA$

$r_\pi = 2K = \frac{\beta}{g_m} \Rightarrow \beta = 2K \times 50m$

$\underline{\underline{\beta = 100}} \longrightarrow \alpha = \frac{100}{101} = 0.99$

$I_E = \frac{I_C}{\alpha} = \frac{1.25mA}{0.99} = \underline{\underline{1.26mA}}$

$i_C(t) = I_C + g_m v_{be}(t)$

$= 1m + 40.10^{-3} \times 0.005 \sin\omega t$

$= \underline{\underline{1 + 0.2 \sin\omega t \ , mA}}$

$v_C(t) = 5 - R_C\, i_C(t)$

$= \underline{\underline{2 - 0.6 \sin\omega t \ , V}}$

$i_B(t) = i_C(t) / \beta$

$= \frac{1 + 0.2 \cdot \sin\omega t}{100}, mA$

$= \underline{\underline{10 + 2 \sin\omega t \ , \mu A}}$

Voltage gain $= \frac{-0.6}{0.005} = \underline{\underline{-120}}\ V/V$

4.68

g_m varies from: $1.2 \times 60 = 72\frac{mA}{V}$

to $0.8 \times 60 = 48\frac{mA}{V}$

β varies from 50 to 200

$R_{in}|_{base} = r_\pi = \beta / g_m$

Largest value: $r_\pi = \frac{\beta_{max}}{g_{m\,min}} = \frac{200}{48m}$

$= \underline{4.2K\Omega}$

Smallest value: $r_\pi = \frac{\beta_{min}}{g_{m\,max}} = \frac{50}{72m}$

$= \underline{\underline{694\Omega}}$

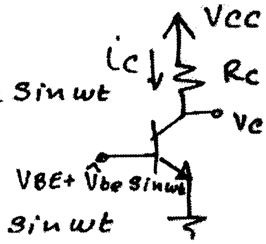
4.69

$V_C = 2V \Rightarrow I_C = \frac{V_{CC} - V_C}{R_C}$

$I_C = \frac{5-2}{3K} = 1mA$

$g_m = \frac{I_C}{V_T} = \frac{1m}{25m} = 40\frac{mA}{V}$

4.70



$i_C = I_C + g_m \hat{v}_{be} \sin\omega t$

$v_C = V_{CC} - I_C R_C - g_m \hat{v}_{be} R_C \sin\omega t$

To maintain BJT in active region, $v_C \geq v_{BE}$, thus

$V_{CC} - I_C R_C - g_m R_C \hat{v}_{be} \geq V_{BE} + \hat{v}_{be}$

To obtain the largest possible output signal we design such that this constraint is satisfied with the equality sign, that is:

$V_{CC} - R_C I_C - g_m R_C \hat{v}_{be} = V_{BE} + \hat{v}_{be}$

substituting $g_m = \frac{I_C}{V_T}$, gives.

$V_{CC} - R_C I_C - R_C I_C \frac{\hat{v}_{be}}{V_T} = V_{BE} + \hat{v}_{be}$

$\Rightarrow R_C I_C \left(1 + \frac{\hat{v}_{be}}{V_T}\right) = V_{CC} - V_{BE} - \hat{v}_{be}$

CONT.

$$R_C I_C = \frac{(V_{CC} - V_{BE} - \hat{V}_{be})}{\left(1 + \frac{\hat{V}_{be}}{V_T}\right)} \quad \text{Q.E.D.}$$

$$\text{Voltage gain} = -g_m \cdot R_C = -\frac{I_C}{V_T} \cdot R_C = -\frac{V_{CC} - V_{BE} - \hat{V}_{be}}{V_T + \hat{V}_{be}}$$

For $V_{CC} = 5V$, $V_{BE} = 0.7V$ and $\hat{V}_{be} = 5mV$

$$R_C I_C = \frac{5 - 0.7 - 0.005}{1 + \frac{0.005}{0.025}} = 3.6V$$

Thus,

$$V_C = 5 - 3.6 = +1.4V$$

Amplitude of output signal is

$$= 1.4 - (V_{BE} + \hat{V}_{be})$$
$$= 1.4 - 0.7 - 0.005$$
$$= 0.695 \text{ V}$$

$$\text{Voltage gain} = -\frac{0.695}{0.005} = -139 \frac{V}{V}$$

Check

$$\text{Voltage gain} = -\frac{(5 - 0.7 - 0.005)}{0.025 + 0.005} = -143 \text{ V/V}$$

The difference is caused by decimal rounding-up of $R_C I_C$.

Otherwise:

$$\text{Voltage gain} = -\frac{0.716}{0.005} = -143 \text{ V/V}$$

4.71

	a	b	c	d	e	t	s
α	1.000	0.990	0.98	1	0.890	0.90	0.841
β	∞	100	50	∞	100	9	16
I_C (mA)	1.00	0.89	1.00	1.00	0.48	4.5	17.5
I_E (mA)	1.00	1.00	1.02	1.00	0.25	5	18.6
I_B (mA)	0	0.010	0.020	0	0.002	0.5	1.10
g_m (mA/V)	40	39.6	40	40	0.01	180	700
$r_e (\Omega)$	25	25	24.5	25	100	5	1.34
$r_\pi (\Omega)$	00	2.5 k	1.255	00	10.1 k	50	227

4.72

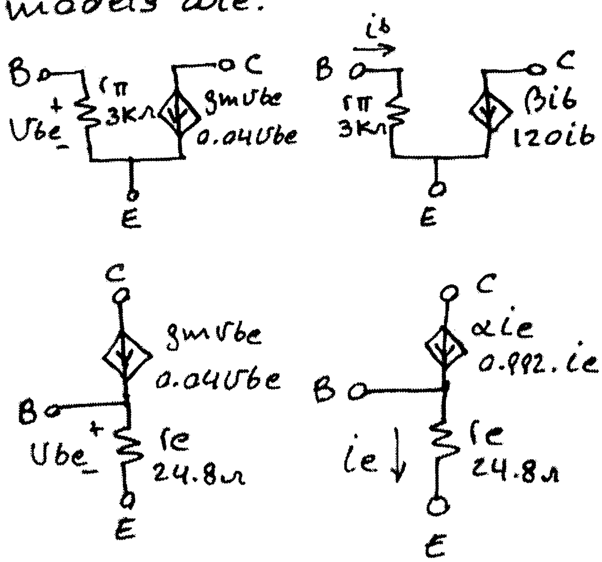
$I_C = 1mA$, $\beta = 120$ $\alpha = 0.992$

$$g_m = \frac{I_C}{V_T} = \frac{1}{25} = 40\frac{mA}{V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{120}{40 \times 10^{-3}} = 3K\Omega$$

$$r_e = \frac{V_T}{I_E} = \frac{\alpha}{g_m} = \frac{0.992}{40 \times 10^{-3}} = 24.8\,\Omega$$

The four equivalent circuit models are:



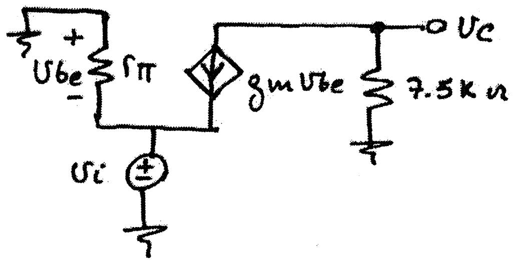
4.73

β very high $\rightarrow \alpha = 1$

$I_C = I_E = 0.5mA$

$V_C = 5 - 7.5 \times 0.5 = +1.25V$

$g_m = \frac{I_C}{V_T} = \frac{0.5m}{25m} = 20\frac{mA}{V}$



Observe that $v_{be} = -v_i$
the output voltage v_C is found from:

$v_C = -g_m v_{be} \times 7.5K$

Thus the voltage gain is

$\frac{v_C}{v_i} = g_m \times 7.5K$

$= 20 \times 7.5 = 150 V/V$

4.74

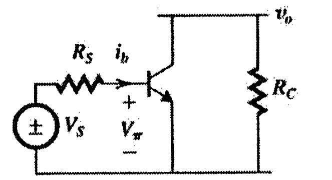
$\frac{v_C}{v_{be}} = -g_m R_C \Rightarrow v_{be} = \frac{1}{50 \times 2}$

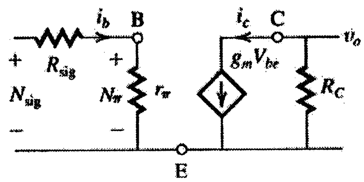
$= 10mV$ pk-to-pk

$i_b = \frac{v_{be}}{r_\pi} = \frac{10 \times 10^{-3}}{\beta / g_m} = \frac{0.01}{100/0.05}$

$i_b = 0.005mA$ pk-to-pk

4.75



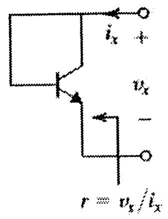


$$\frac{v_O}{v_{sig}} = \frac{v_1}{v_1} = \frac{r_\pi}{r_\pi + R_{sig}}(-)g_m R_C$$

$$= \frac{-r_\pi g_m}{r_\pi + R_{sig}} R_C$$

$$= \frac{-\beta R_C}{r_\pi + R_{sig}}$$

4.76



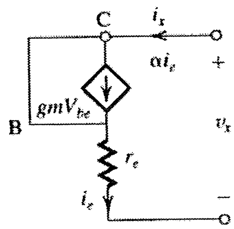
Apply V_x

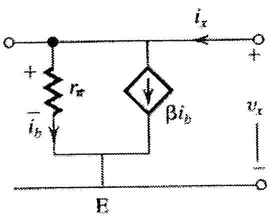
then $v_B = V_x$

$i_x = i_b + i_C$

$v_x = (i_b + i_c)r_e$

$= i_x$ re





$\therefore r = \frac{v_x}{i_e} = r_e$

(or)

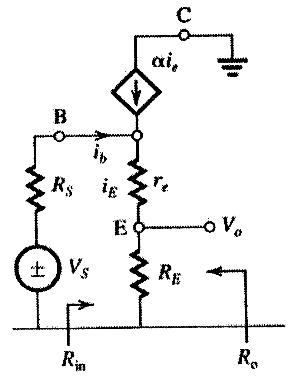
$i_x = \beta i_b + i_b$

$= (\beta + 1) i_b$

$= (\beta + 1)\frac{v_x}{r_\pi}$

$r = \frac{v_x}{i_x} = \frac{r_\pi}{\beta + 1} = r_e$

4.77



Neglecting r_o

$R_{IN} = \frac{v_{be}}{i_b}$

$= \frac{i_e(r_e + R_E)}{i_e/(\beta + 1)}$

$= (\beta + 1)(r_e + R_E)$

$v_O = -\alpha\, i_e R_E$

$i_e = \frac{v_{be}}{r_e + R_E}$

$\therefore \frac{v_O}{v_{be}} = \alpha\frac{R_E}{r_e + R_E}$

$A_V = \frac{v_O}{v_{be}} = -\frac{\alpha}{r_e}\frac{R_E}{1 + R_E/r_e} = -\frac{g_m R_C}{1 + g_m R_E}$

4.78

$\beta = 200 \rightarrow \alpha = 0.995$

$I_C - \alpha I_E = 0.995 \times 10 \text{ mA} = 9.95 \text{ mA}$

$V_C = 9.95 \text{ mV} \times 100 = 0.995 \text{ V}$

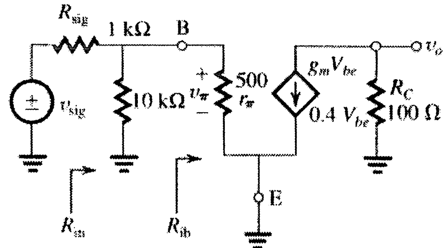
$I_B \cong \frac{10 \text{ m}}{200} = 0.05 \text{ mA}$

$V_B = 1.5 - 10 \text{ k}\Omega \times 0.05 \text{ mA}$

$= 1 \text{ V}$

$\Rightarrow V_{BC} = +0.005$

$\rightarrow$ Active region



4.79

$$g_m = \frac{I_C}{V_T} = \frac{9.95}{25\text{ m}} = 0.4\text{ A/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{200}{0.4} = 500\ \Omega$$

$$R_{ib} = r_\pi = 500\ \Omega$$

$$R_{in} = 10\text{ k}\Omega \parallel r_\pi = 476\ \Omega$$

$$v_{be} = v_{sig} \times \frac{R_{in}}{R_{sig} + R_{in}} = v_{sig} \times 0.32$$

also:

$$v_O = -g_m v_{be} \cdot R_C$$
$$= -g_m R_C \times 0.32\ v_{sig}$$
$$= -0.4 \times 100 \times 0.32\ v_{sig}$$
$$= -12.8\ v_{sig}$$

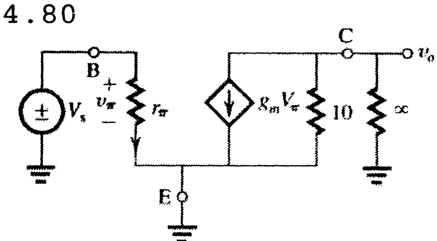
$$\Rightarrow \text{gain } \frac{v_O}{v_S} = -12.8 \simeq -13\ \frac{v}{V}$$

If $v_O = \pm 0.4$ V

$$\hat{v}_S = \frac{\hat{v}_O}{13} = 30\text{ mV}$$

$$\hat{v}_{be} = 0.32 \times 30\text{ m} = 9.8\text{ mA}$$

4.80



$$V_S = V_\pi \Rightarrow \frac{V_O}{V_S} = -g_m r_o$$

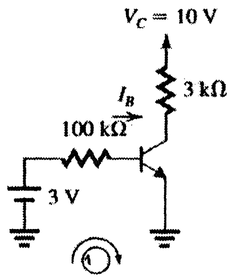
but: $r_o = \dfrac{V_A}{I_C} = \dfrac{V_A}{V_T \cdot g_m}$

$$\Rightarrow \frac{V_O}{V_S} = -\frac{V_A}{V_T}$$

if $V_A = 25\text{ V} \Rightarrow \dfrac{V_O}{V_S} = -1000\ \dfrac{\text{V}}{\text{V}}$

if $V_A = 250\text{ V} \Rightarrow \dfrac{V_O}{V_S} = -10,000\ \dfrac{\text{V}}{\text{V}}$

4.81



DC Analysis:

$$(1) I_B = \frac{3 - 0.7}{100}$$

$$I_B = 0.023\text{ mA}$$

Saturation begins to occur when $V_C \le 0.7$ V

$$\therefore I_C \ge \frac{10 - 0.7}{3} = 3.1\text{mA}$$

$$I_C = \beta I_B \rightarrow \beta \ge \frac{3.1}{0.023} = 135$$

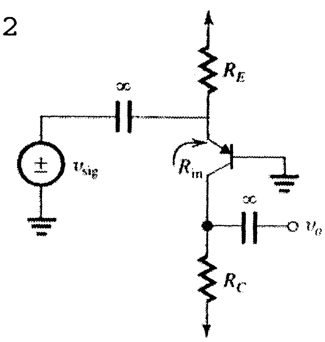
$\beta = 25$:

$$r_e = \frac{V_T}{I_E} = \frac{V_T}{(\beta + 1) I_B} = \frac{25 \times 10^{-3}}{26 \times 0.023 \times 10^{-3}}$$

$$r_e = 41.8\ \Omega$$

$$g_m = \frac{\alpha}{r_e} = \frac{25/26}{41.8} = 23\ \frac{\text{mA}}{\text{V}}$$

4.82



$$R_{in} = r_e \parallel R_E$$
$$= r_e$$
$$= 75\ \Omega$$

$$I_E = \frac{25\text{ mV}}{75\ \Omega} = (0.33\text{ mA})$$

$$R_E = \frac{10 - 0.7}{0.33} = 28\text{ k}\Omega$$

$$n = 2.8$$

$$R_C = 14\text{ k}\Omega$$

$$\frac{V_O}{V_i} = \frac{\alpha R_C}{r_e} = \frac{14}{0.075} = 187\text{ V/V}$$

4.83

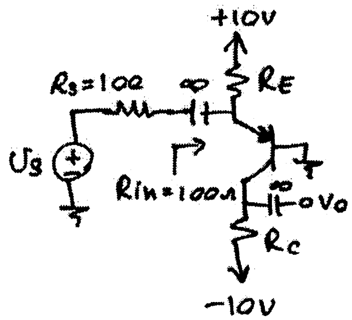
$R_{in} = R_E \| r_e \quad r_e \simeq 100\Omega$

thus, $\frac{V_T}{I_E} = 100 \rightarrow I_E = 0.25mA$

$V_E = 0.7V$

$R_E = \frac{10-0.7}{1m}$

$= \underline{\underline{9.3 K\Omega}}$



Selection of a value for Rc:

The voltage gain is directly proportional to Rc,

$\frac{v_o}{v_s} = \frac{v_e}{v_s} \cdot \frac{v_o}{v_e}$

$= \frac{R_{in}}{R_s + R_{in}} \cdot \frac{\alpha R_c}{r_e}$

$\simeq \frac{100}{100+100} \cdot \frac{R_c}{0.1}$

$= 5R_c$, R_c in $K\Omega$.

For an emitter-base signal as large as 10mV, the signal at the collector will be $g_m R_c \times 0.010$ volts. Thus the maximum collector voltage in the positive direction will be:

$V_C|_{max} = V_C + 0.01\, g_m \cdot R_c$

$= -10 + I_C R_c + 0.01 \times \frac{1}{0.1} \times R_c$

$= -10 + 0.25 R_c + 0.1 R_c$

$= -10 + 0.35 R_c$

To prevent saturation, $V_C|_{max} \leq V_B$ which is 0V.

Thus to obtain maximum gain while allowing an emitter-base signal as large as 10mV and at the same time keeping the transistor in the active mode we select Rc from:

$-10 + 0.35 R_c = 0$

$\Rightarrow R_c = \underline{\underline{28.6 K\Omega}}$

Voltage gain $= \frac{v_o}{v_s} = 5 R_c = 143 V/V$

4.84

For large β; the DC base current will be ~0

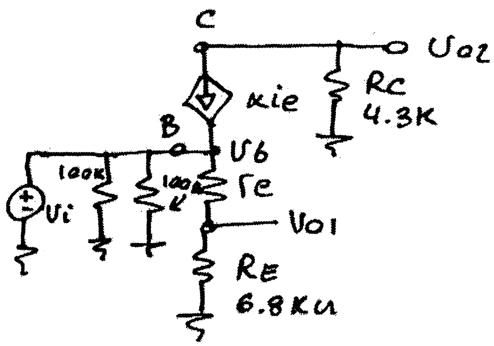
Thus the DC voltage at the base can be found directly using the voltage-divider rule

$$V_B = 15 \cdot \frac{100}{100+100} = 7.5\,V$$

If: $V_{BE} = 0.7$

$V_E = 7.5 - 0.7 = 6.8\,V$

$\rightarrow I_E = \frac{6.8\,V}{6.8\,K\Omega} = 1\,mA$



$v_b = v_i$

$\rightarrow \frac{v_{o1}}{v_i} = \frac{R_E}{R_E + r_e}$ Q.E.D.

Also,

$$i_e = \frac{v_b}{r_e + R_E} = \frac{v_i}{r_e + R_E}$$

and,

$$v_{o2} = -\alpha i_e . R_c = -\frac{\alpha R_c v_i}{r_e + R_E}$$

Thus,

$$\frac{v_{o2}}{v_i} = \frac{-\alpha R_c}{R_E + r_e} \quad \text{Q.E.D.}$$

Substituting $r_e = \frac{V_T}{I_E} = 25\,\Omega$

and $R_E = 6.8\,K\Omega$, $R_C = 4.3\,K\Omega$ and $\alpha \simeq 1$ gives

$$\frac{v_{o1}}{v_i} = \frac{6.8}{0.025 + 6.8} = 0.996\ V/V$$

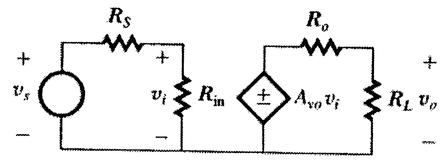
$$\frac{v_{o2}}{v_i} = \frac{-4.3}{6.8 + 0.025} = 0.63\ V/V$$

If the node labeled v_{o2} is connected to ground:

$R_E = 0$

$$\frac{v_{o2}}{v_i} = -\frac{\alpha R_c}{r_e}$$

4.85



Given : R_s = 100 KΩ A_L = 2 kΩ & R_L = 1 kΩ

Find : R_{in}, A_{VO}, R_L

a) $|V_i(e)| \geq 0.9|v_S(t)|$

$$\vartheta_{i(t)} = \frac{R_{IN}}{R_{IN} + R_S} v_S(t)$$

$$\left|\frac{R_{IN}}{R_{IN} + R_S} v_S(e)\right| \geq 0.9|v_S(t)|$$

$$\frac{R_{in}}{R_{IN} + R_S} \geq 0.9$$

b) $v_O(t) = \frac{R_L}{R_L + R_O} A_{V_O}\, v_i(t)$

$$v_O'(t) = \frac{R_i^1}{R_i^1 + R_O} A_{V_o}\, v_i(t)$$

$$|v_O'(t)| \geq 0.9|v_O(t)|$$

$$\frac{R_L'}{R_L' + R_O} \geq 0.9 \frac{R_L}{R_L + R_O} \Rightarrow$$

$$R_O \leq \frac{R_L R_L'}{9R_L - 10R_L'} = \frac{(10^3)(2 \times 10^3)}{9(2 \times 10^3) - 10(1 \times 10^3)}$$

$= 250\,\Omega$

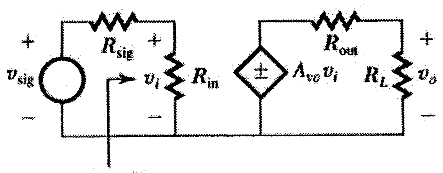
c)Taking the limiting values for R_{in} & R_o

$$10 = A_V\left(\frac{R_{IN}}{R_{IN}+R_S}\right)\left(\frac{R_L}{R_O+R_L}\right)$$

$$A_V\left(\frac{900\times10^3}{900\times10^3+100\times10^3}\right)\left(\frac{2\times10^3}{250+2\times10^3}\right)$$

$A_{VO} = 12.5$

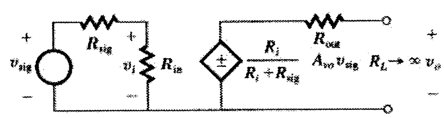
4.86



$R_i = R_{in}$

$$v_i = \frac{R_{in}}{R_{in}+R_{Sig}}v_{sig} = \frac{R_i}{R_i+R_{sig}}v_{sig}$$

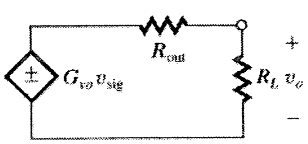
Setting $R_L \to \infty$ and substitution for v_i



$$v_O = \frac{R_i}{R_i+R_{sig}}A_{VO}\ v_{sig} \Rightarrow G_{VO}$$

$$= v_O/v_{sig} = \frac{R_i}{R_i+R_{sig}}Av_O$$

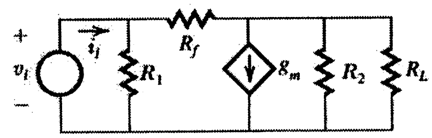
Connecting the load



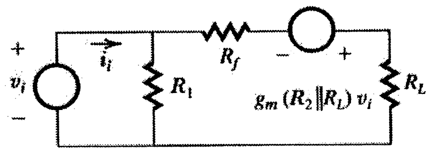
$$v_O = \frac{R_L}{R_L+R_{out}}G_{VO}\ v_{sig} \Rightarrow G_V = v_O/v_{sig}$$

$$= \frac{R_L}{R_L+R_{out}}G_{VO}$$

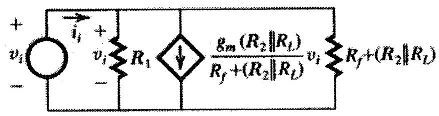
4.87



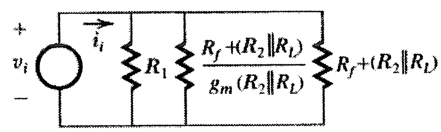
R_I and R_L are in Parallel. Also do a source transformation



Combine Rf and $R_\alpha \| R_L$ and do another source transformation



The dependent current source is equivalent to a resistor



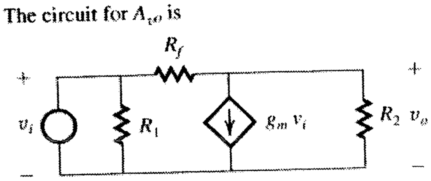
$$R_{in} = v_i/i_1 = R_1 \| \frac{R_f+(R_2\|R_L)}{g_m(R_2\|R_L)} \| (R_f+[R_2\|R_L])$$

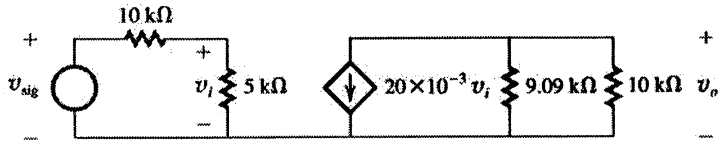
Consider the form

$$(R/e)\|R: \frac{RR}{A/a+R} = \frac{R}{1+a}$$

$$R_{in} = R_1 \| \left[\frac{R_f+(R_2\|R_L)}{1+g_m(R_2+R_L)}\right]$$

The circuit for A_{vo} is



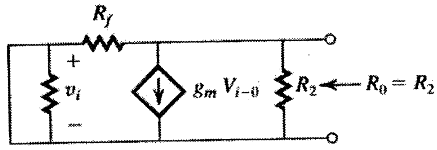


$$\frac{v_O - v_i}{R_f} + g_m v_i + \frac{1}{R_2} v_O$$

$$\left[\frac{1}{R_f} + \frac{1}{R_2}\right] V_O = \left(\frac{1}{R_f} - g_m\right) v_i$$

$$A_{VO} = v_O / v_i = \frac{1 - g_m R_f}{1 + R_f / R_2}$$

The circuit for R_o



for values given

$R_{in} = 99.90, A_{vo} = -9.9989, R_o = 100$

The dependence on R_f is

$$R_{in} = 100 \frac{1100 R_f + 10^5}{1100 R_f + 1.21 \times 10^6}$$

$$A_{VO} = -10\left(\frac{R_f - 10}{R_f + 100}\right)$$

If R_f decreases the gain becomes sensitive to R_f

If $R_f \to \infty$ $R_{in} = 100, A_{vo} = -10$

with R_f

$$G_{VO} = \frac{R_{IN}}{R_{IN} + R_{avg}} A_{VO} = \frac{-99.9}{99.9 + 100}(-9.9989)$$

$$= -4.997 \text{ V/V}$$

Without R_f

$$G_{VO} = \left(\frac{100}{100 + 100}\right)(-10) = -5$$

4.88

$R_C = 10\text{ k}\Omega$, $V_A = 50$ V, $\beta = 100$, $I_C = 0.5$ mA

$$g_m = \frac{I_C}{V_T} = \frac{0.5 \times 10^{-3}}{0.025} = 20 \times 10^{-3}\text{ S}$$

$$r_O = \frac{V_A}{I_A} = \frac{50}{0.5 \times 10^{-3}} = 100\text{ k}\Omega$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{20 \times 10^{-3}} = 5 \times 10^3\ \Omega$$

$$R_O = R_C \| r_O = (10 \times 10^3 \| 100 \times 10^3)$$

$$= 9.09\text{ k}\Omega$$

$$R_{in} = r_\pi = 5 \times 10^3$$

The circuit is now (see figure above)

$$A_v = \frac{v_O}{v_i} = -g_m(R_O \| R_L)$$

$$= -(20 \times 10^{-3})(9.09 \times 10^3 \| 10 \times 10^3)$$

$A_v = -95.23$

$$G_V = v_O / v_{sig} = \frac{R_{in}}{R_{in} + R_{sig}}$$

$$A_V = \left(\frac{5 \times 10^3}{5 \times 10^3 + 10 \times 10^3}\right)(-95.2) = -31.74$$

max signal v_{sig} is

$$\max \frac{|v_O(t)|}{|G_V|} = \frac{5 \times 10^{-3}}{31.74} = 157.5\ \mu\text{V}$$

4.89

$$|G_V| = \beta \frac{R_C \| R_L \| r_O}{R_{sig} + r_\pi}$$

IF $r_O \to \infty$ then $R_C \| R_L \| r_O \to R_C \| R_L$

Let $R_L' = R_C \| R_L$

$$|G_V| = \beta \frac{R_L'}{R_{sig} + r_\pi}$$

$$|G_V| = \frac{R_L'}{\frac{R_{sig}}{\beta} + \frac{r_\pi}{\beta}}$$

But $r_\pi/\beta = 1/g_m$

$$|G_V| = \frac{R_L'}{\frac{R_{sig}}{\beta} + \frac{r_1}{g_m}}$$

$R_L' = 10\text{ k}\Omega$; $R_{sig} = 10\text{ k}\Omega$; $\beta = 100$;

$I_c = 1$ mA

$g_m = I_C/V_T$, so

$$|G_V| = \frac{R_L'}{\frac{R_{sig}}{\beta} + \frac{V_T}{I_C}}$$

a) $$|G_V| = \frac{10^4}{\frac{10^4}{100} + \frac{0.025}{10^{-3}}} = 80 \text{ V/V}$$

b) If β ranges from $50 \to 150$

For $\beta = 50$:

$$|G_V| = \frac{10^4}{\frac{10^4}{50} + \frac{0.025}{10^{-3}}} = 44.44 \text{ V/V}$$

For $\beta = 150$:

$$|G_V| = \frac{10^4}{\frac{10^4}{150} + \frac{0.025}{10^{-3}}} = 109.09 \text{ V/V}$$

c) What is β range if $G_4 \le |G_v| \le 96$

at $|G_v| = 64$:

$$\frac{10^4}{\frac{10^4}{\beta} + \frac{0.025}{10^{-3}}} = 64 \Rightarrow \beta = 76.19$$

at $|G_v| = 96$:

$$\frac{10^4}{\frac{10^4}{\beta} + \frac{0.025}{10^{-3}}} = 96 \Rightarrow \beta = 126.32$$

d) Suppose the nominal G_v is G_{V-nom} and I_C is variable

$\beta = 50 \Rightarrow G_V = 0.8\ G_{V-nom}$

$\beta = 150 \Rightarrow G_V = 1.2\ G_{V-nom}$

Then

$$\frac{10^4}{\frac{10^4}{50} + \frac{0.025}{I_C}} = 0.8\ G_{V-nom}$$

$$\frac{10^4}{\frac{10^4}{150} + \frac{0.025}{I_C}} = 1.2\ G_{V-nom}$$

Take ratio

$$\frac{\frac{10^4}{150} + \frac{0.025}{I_C}}{\frac{10^4}{50} + \frac{0.025}{I_C}} = \frac{0.8}{1.2} \Rightarrow I_C = 0.125 \text{ mA}$$

$$\frac{10^4}{\frac{10^4}{\beta_{nom}} + \frac{0.025}{I_C}} = G_{V-nom}$$

$G_{V-nom} = 31.25\ \beta_{nom} = 83.33$

4.90

$$|G_V| = \beta\frac{R_C \| R_L \| r_O}{R_{sig} + r_\pi} = \beta\frac{(R_C \| R_L) \| r_O}{R_{sig} + r_\pi}$$

$$r_O = \frac{V_A}{I_C}$$

$$|G_V| = \frac{(R_C \| R_L) \| \frac{V_A}{I_C}}{\frac{R_{sig}}{\beta} + \frac{r_\pi}{\beta}};$$

$$\frac{r_\pi}{\beta} = \frac{1}{g_m} = \frac{V_T}{I_C}$$

thus, $$|G_V| = \frac{(R_C \| R_L) \| \frac{V_A}{I_C}}{\frac{R_{sig}}{\beta} + \frac{V_T}{I_C}}$$

$R_C \| R_L = 10\ \Omega$, $R_{sig} = 10\text{ k}\Omega$, $V_A = 25$ V, and $V_T = 0.025$ V

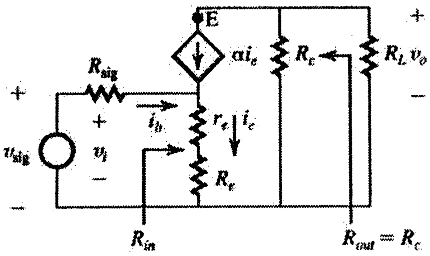
$$|G_V| = \frac{(10^4) \| 25/I_C}{\frac{10^4}{100} + \frac{0.025}{I_C}}$$

$$= \frac{25 \times 10^6 I_C}{(10^4 I_C + 25)(10^4 I_C + 2.5)}$$

I_C (ref)	$\lvert G_v \rvert$
0.1	27.47
0.2	41.15
0.5	55.56
1.0	57.14
1.25	55.55

The values of I_C that result in $|G_v| = 50$ are:
1×0.925 mA and 0.324 mA.
The 0.324 mA would be preferred since a lower power is required.

4.91



$$i_e = v_i/(r_e + R_e)$$

$$i_b = i_e - \alpha i_e = (1-\alpha)i_e = (1-\alpha)\frac{v_i}{r_e + R_e}$$

$$\alpha = \frac{\beta}{\beta+1} = \frac{100}{100+1} = 0.99$$

$$i_b = \frac{1}{(\beta+1)}\frac{v_i}{r_e + R_e}$$

$$R_{in} = v_i/i_b = (\beta+1)(r_e + R_e)$$

$$r_e = \frac{V_T}{I_E} = \frac{V_T}{I_C/\alpha} = \alpha\frac{V_T}{I_C}$$

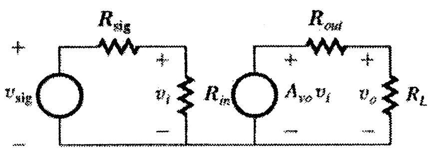
$$= (0.99)\left(\frac{0.025}{0.5\times 10^{-3}}\right) = 49.5\ \Omega$$

$$R_{in} = (100+1)(49.5+150) = 20150\ \Omega$$

$$A_{VO} = -\alpha i_e R_C = -\alpha R_C\frac{1}{r_e + R_e}$$

$$A_{VO} = -(0.99)(10\times 10^3)/(49.5+150) = -49.62$$

now model becomes



$$v_O = \frac{R_L}{R_L + R_{out}}A_{VO}\,v_i$$

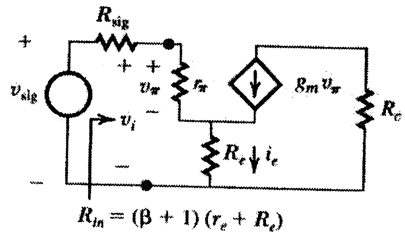
$$v_O = \frac{R_L}{R_L + R_{out}}A_{VO}\frac{R_{in}}{R_{in} + R_{sig}}v_{sig}$$

$$G_V = v_O/v_{sig}$$

$$= \frac{10\text{ k}\Omega}{10\text{ k}\Omega + 10\text{ k}\Omega}(-49.62)\frac{.20150\ \Omega}{20150\ \Omega + 10000\ \Omega}$$

$$= -16.582$$

4.92



$$R_{in} = (\beta+1)(r_e + R_e)$$

$$r_e = \frac{V_T}{I_C}$$

$$R_{in} = (\beta+1)\left(\frac{V_T}{I_C} + R_e\right)$$ multiply both sides

by I_C and rearrange :

$$-(\beta+1)R_e I_C + R_{in}I_C = (\beta+1)V_T$$

Given $\beta = 100$; $R_{in} = 20\text{ k}\Omega$; $V_T = 0.025$ V

Equation becomes

$$-101\,R_e I_c + (2\times 10^4)\,I_c = (101)(0.025) = 2.525$$

(Eq A)

Our unknowns are I_c & R_e This is one equation.

$$i_e = v_\pi/r_\pi + g_m v_\pi = (1/r_\pi + g_m)v_\pi$$

$$= \left(\frac{1}{\beta} + 1\right)g_m v_\pi$$

$$= \left(\frac{1}{\beta} + 1\right)\frac{I_C}{V_T}\,v_\pi$$

$$v_{sig} = R_e i_e + v_\pi + R_{sig}\frac{v_\pi}{r_\pi}$$

$$= R_e\left(\frac{1}{\beta} + 1\right)\frac{I_C}{V_T}v_\pi + v_\pi + \frac{R_{sig}}{\beta}\frac{I_C}{V_T}v_\pi$$

$$v_{sig} - v_{sig} = \left[\frac{1}{\beta} + 1\right]\frac{v_\pi}{V_T}R_e I_C + \frac{R_{sig}v_\pi}{\beta V_T}I_C$$

$$0.1 - 0.005 = \left[\frac{1}{100} + 1\right]\left[\frac{5\times 10^{-3}}{0.025}\right]\bullet$$

$$R_e I_C + \frac{(5000)(5\times 10^{-3})}{(100)(0.025)}I_C$$

$0.005 = 0.202\,R_e I_c + 10\,I_c$ (Eq B)

Equations A and B can be solved simultaneously

$I_C = 1.25$ mA

$R_e I_C = 0.00064$

$\Rightarrow R_e = 0.22264/1.25\times10^{-3}$

$= 178.11$

$$G_V = \frac{v_O}{v_{sig}} = \frac{v_O}{v_\pi}\cdot\frac{v_\pi}{v_{sig}}$$

$$v_O/v_\pi = -R_C g_m = -R_C\frac{I_C}{V_T}$$

$$= -(5\times10^3)\left(\frac{1.25\times10^{-3}}{0.025}\right) = -250$$

$$G_V = (-250)\left(\frac{5\times10^{-3}}{0.1}\right) = -12.5$$

4.93

$$|G_V| = \frac{\beta R_C}{R_{sig}+(\beta+1)(r_e+R_e)}$$

$$r_e = \frac{V_T}{I_E}$$

$$|G_V| = \frac{\beta R_C}{R_{sig}+(\beta+1)(V_T/I_E+R_e)}$$

$R_{sig} = 10$ kΩ ; $R_c = 10$ kΩ ; $\beta = 100$;

$V_T = 0.025$ V;

$I = 1$ mA

$$\alpha = \frac{\beta}{\beta+1} = \frac{100}{101} = 0.99$$

$I_E = I_c/\alpha = 1.01\times10^{-3}$A

If $R_e = 0$

$$|G_V| = \frac{(100)(10\times10^3)}{10\times10^3+(101)[0.025/(1.01\times10^{-3})]} = 80$$

Suppose $|G_v|$ has a nominal value $G_{V\text{-}num}$ and $0.8\,G_{V\text{-}num}$ corresponds to $\beta = 50$. Let R_e be a variable (note that $\alpha = 0.98$):

$$\frac{\beta R_C}{R_{sig}+(\beta+1)[0.025/(1.02\times10^{-3})+R_e]}$$

$$= 0.8G_{V-nom}$$

$$\frac{(50)(10^4)}{10^4+(51)(0.025/1.02\times10^{-3}+R_e)} = 0.8G_{V-nom}$$

at $\beta = 150$ $\quad G_v = 1.2\,G_{v\text{-}nom}$

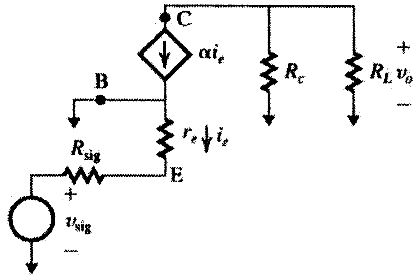
$$\frac{(150)(10^4)}{10^4+(151)(0.025/1.01\times10^{-3}+R_e)} = 1.2G_{V-nom}$$

These two equations can be solved simultaneously for R_e & $G_{v\text{-}nom}$

$R_e = 179.3$ V

$G_{v\text{-}nom} = -30.625$

4.94



$$v_{be}(t) = r_e i_e$$

$$v_O(t) = -\alpha i_e(R_C\|R_L)$$

$$v_{be}(t) = -r_e\frac{v_O(t)}{\alpha(R_C\|R_L)}$$

$$|v_O(t)| = \frac{\alpha(R_C\|R_L)}{r_e}|v_{be}(t)|$$

$$= \frac{\alpha(R_C\|R_L)}{V_T}I_E|v_{be}(t)|$$

Suppose $\alpha \approx 1$

$$|v_O(t)| = \frac{(10\text{ k}\Omega)\|(10\text{ k}\Omega)}{0.025}(0.25\text{ mA})(10\times10^{-3})$$

$$|v_O(t)| = 0.5\text{ V}$$

$$G_V = v_O(t)/v_{sig(t)} = \alpha\frac{R_C\|R_L}{R_{sig}+r_e} = \alpha\frac{R_C\|R_L}{R_{sig}+V_T/I_E}$$

$$G_V = \frac{(10\text{ k}\Omega)\|(10\text{ k}\Omega)}{1\text{ k}\Omega+0.025/10^{-3}}\quad\text{Since }\alpha\approx1$$

$= 4.88$ V/V

$$|v_{sig}(t)| = |v_O(t)|/G_V$$

$$|v_{sig}(t)| = 0.5/4.88 = 0.1025\text{ V}$$

4.95

$$|v_O(t)|_{max} = (0.5\text{ V})$$

$$|i_c(t)|_{max} = \frac{|v_O(t)|_{max}}{R_L} = \frac{0.5}{2\times10^3} = 250\,\mu\text{A}$$

$$r_e = \frac{|v_{be}(t)|_{max}}{|i_C(t)|_{max}} = \frac{5\times10^{-3}}{250\,\mu\text{A}} = 20\Omega$$

$$r_e = \frac{V_T}{I_E}\Rightarrow I_E = \frac{V_T}{r_e} = \frac{0.025}{20} = 1.2\text{mA}$$

$$|i_E(t)|_{max} = I_E + |i_C(t)|_{max} = 1.5\text{ mA}$$

$$|i_E(t)|_{max} = I_E - |i_C(t)|_{max} = 1\text{ mA}$$

Suppose $\beta = 100$

$$G_V = \frac{(\beta+1)R_L}{(\beta+1)R_L+(\beta+1)r_e+R_{sig}}$$

$$= \frac{(101)(2\times10^3)}{(101)(2\times10^3)+(101)(20)+200\times10^3} = 0.499$$

$(G_V = v_O(t)/v_{sig}(t)) \Rightarrow$

$v_{sig}(t) = \frac{v_O(t)}{G_V} = \frac{0.5}{0.499}$

$|V_{sig}|_{max} = 1.00$ Volt

4.96

$I_c = 1$mA; $\beta = 100$; $R_{sig} = 20$ kΩ;
$R_L = 1000\ \Omega$

$$I_E = \frac{\beta+1}{\beta} I_C = \frac{101}{100} 10^{-3} = 1.01 \text{mA}$$

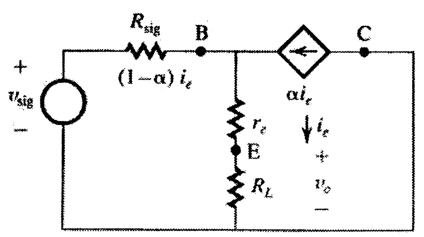
$$r_e = \frac{V_T}{I_E} = \frac{0.025}{1.01 \times 10^{-3}} = 24.752\Omega$$

$$R_{in} = (\beta+1)(r_e + R_L) = (101)(24.752 + 1000)$$
$$= 103.5 \text{ k}\Omega$$

we have:

$$v_O/v_{sig} = G_V = \frac{(\beta+1)R_L}{(\beta+1)R_L + (\beta+1)r_e + R_{sig}}$$

$$= \frac{(101)(1000)}{(101)(1000) + (101)(24.752) + 20 \times 10^3} = 0.8178$$



$$i_C(t) = v_O(t)/R_L = \frac{G_V V_{sig}}{R_L}$$

$$V_{be}(t) = r_e i_e(t) = (r_e/R_L) G_V V_{sig}(t) \Rightarrow$$

$$v_{be}(t)/v_{sig}(t) = (r_e/R_L) G_V$$

$$= (24.752/1000)(0.8178) = 0.02024$$

$$v_b(t) = v_O(t) + v_{be}(t) \Rightarrow$$

$$v_b(t)/v_{sig}(t) = G_V + (r_e/R_L)G_V = (1 + r_e/R_L)G_V$$

$$v_b(t)/v_{sig}(t) = (1 + 24.752/1000)(0.8178)$$
$$= 0.838056$$

b) $v_{be}(t)/v_{sig}(t) = 0.02024$

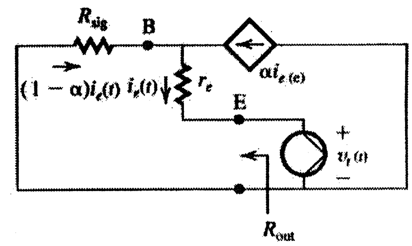
$\Rightarrow |v_{sig}(t)|_{max}$

$= |v_{be}(t)|_{max}/0.02024$

$|v_{sig}(t)|_{max} = 10 \times 10^{-3}/0.02024 = 0.494 V_{old}$

$|v_o(t)|_{max} = G_V |v_{sig}(t)|_{max} = (0.494)(0.8178)$
$= 0.404$ V

c) If R_L is removed $i_e = 0$, therefore,
$v_e = v_{sig}$, Thus
$G_{vo} = 1$,.
Now for R_{out}



$$R_{out} = -\frac{v_t(t)}{i_C(t)}$$

$$i_e(t) = \frac{v_b(t) - v_e(t)}{r_e} = \frac{v_b(t) - v_t(t)}{r_e}$$

$$v_b(t) = -i_e(t)(1-\alpha)R_{sig} \Rightarrow$$

$$i_e(t) = \frac{-i_e(t)(1-\alpha)R_{sig} - v_t(t)}{r_e};$$

$$r_e i_e(t) = -i_e(1-\alpha)R_{sig} - v_t(t)$$

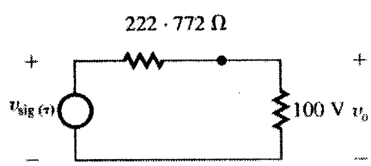
$$i_e(t) = \frac{-v_t}{r_e + (1-\alpha)R_{sig}}$$

Substituting into R_{out} expression

$$R_{out} = r_e + (1-\alpha) R_{sig} = r_e + \frac{1}{\beta+1} R_{sig}$$

$$= 24.752 + \frac{20 \times 10^3}{101} = 222.772$$

now



$$v_O(t)/v_{sig}(t) = \frac{1000}{1000 + 222.772} = 0.8178$$

This agrees with G_v

4.97

$I_c = 0.25$ mA ; $R_{sig} = 10$ kΩ ; $R_L = 1$ kΩ ;
$V_T = 0.025$

$$G_V = \frac{(\beta+1)R_L}{(\beta+1)R_L + (\beta+1)r_e + R_{sig}}$$

$$r_e = \frac{V_T}{I_E} = \frac{\beta V_T}{(\beta+1)I_C}$$

$$G_V = \frac{(\beta+1)R_L}{(\beta+1)R_L + \beta V_T/I_C + R_{sig}}$$

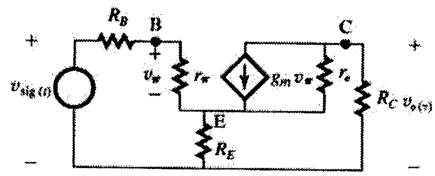
$$R_{out} = r_e + R_{sig}/(\beta+1)$$

$$= \frac{\beta V_T}{(\beta+1)I_C} + \frac{R_{sig}}{\beta+1}$$

for $\beta = 100$	$\beta = 50$	$\beta = 150$
$G_v = 0.8347$	$G_v = 0.7727$	$G_v = 0.85$
$R_{out} = 199.01\ \Omega$	$R_{out} = 298.0\ \Omega$	$R_{out} = 166.0\Omega$

4.98

Part a) Nodal equations:



Part a)

$$\frac{v_e}{R_E} + \frac{v_e - v_{sig}}{R_B + r_\pi} - g_m V_\pi + \frac{v_e - v_c}{r_o} = 0$$

$$g_m v_\pi + \frac{v_c - v_e}{r_o} + \frac{v_e}{R_C} = 0$$

$$\frac{v_\pi}{r_\pi} + \frac{v_e + v_\pi - v_{sig}}{R_B} = 0$$

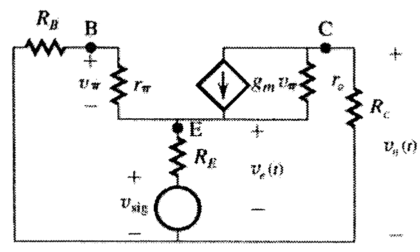
Solving:

$$\frac{v_e(t)}{v_{sig}(t)} = \frac{(g_m r_o r_\pi - R_E)R_C}{(r_\pi R_C + R_E r_\pi + r_o r_\pi + g_m R_E r_o r_\pi + R_C R_B + R_E r_o + r_o R_B + R_E R_C + R_E R_B)}$$

$$\frac{v_e(t)}{v_{sig}(t)} = \frac{R_E(g_m r_o r_\pi + R_C + r_o)}{(r_\pi R_C + R_E r_\pi + r_o r_\pi + g_m R_E r_o r_\pi + R_C R_B + R_E r_o + r_o R_B + R_E R_C + R_E R_B)}$$

$$r_o = \frac{|V_A|}{I_C}$$

Part b) Nodal equations:



$$\frac{v_e - v_{sig}}{R_E} + \frac{v_e}{R_B + r_\pi} - g_m v_\pi + \frac{v_e - v_C}{r_o} = 0$$

$$g_m V_\pi + \frac{v_e - v_e}{r_o} + \frac{v_e}{R_C} = 0$$

$$\frac{v_\pi}{r_\pi} + \frac{v_e + v_\pi}{R_B} = 0$$

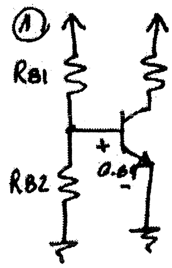
Solutions

$$\frac{V_e(t)}{V_{sig}(t)} = \frac{R_C(g_m r_o r_\pi + R_B + r_\pi)}{(r_\pi R_C + R_E r_\pi + r_o r_\pi + g_m R_E r_o r_\pi + R_C R_B + R_E r_o + r_o R_B + R_E R_C + R_E R_B)}$$

$$\frac{V_e(t)}{V_{sig}(t)} = \frac{(R_C + r_o)(R_B + r_\pi)}{(r_\pi R_C + R_E r_\pi + r_o r_\pi + g_m R_E r_o r_\pi + R_C R_B + R_E r_o + r_o R_B + R_E R_C + R_E R_B)}$$

4.99

①



$$\frac{5 \times R_{B2}}{R_{B1} + R_{B2}} = 0.690$$

→ $5R_{B2} = 0.69R_{B1} + 0.69R_{B2}$

→ $4.31R_{B2} = 0.69R_{B1}$

⟹ $\frac{R_{B1}}{R_{B2}} = \underline{\underline{6.24}}$

② Since $V_{BE} = \frac{5R_{B2}}{R_{B1} + R_{B2}}$

If both R_{B2} & R_{B1} are at 0.99 or 1.01 of their nominal value → V_{BE} will not be affected.

We must consider the cases when one resistor is at 0.99 and the other at 1.01 of their nominal value.

If: $R_{B2}' = 1.01 R_{B2}$

$R_{B1}' = 0.99 R_{B1}$

⟹ $V_{BE} = 0.702V$

If: $R_{B2}' = 0.99 R_{B2}$

$R_{B1}' = 1.01 R_{B1}$

⟹ $V_{BE} = 0.678V$

thus V_{BE} ranges from 0.678V to 0.702V

CONT.

For I_C: $I_C = I_S e^{V_{be}/V_T}$

for $V_{be} = 0.690 \rightarrow I_C = 1\,mA$

$\Rightarrow I_S = 1.032 \times 10^{-15}$

for $V_{BE} = 0.678 \rightarrow I_C = 0.619m$

$V_{BE} = 0.702 \rightarrow I_C = 1.62m$

I_C ranges from 0.618 mA to 1.62 mA.

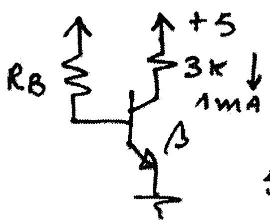
(3) If $R_C = 3K\Omega$

$V_{CE} = 5 - 3K \times 0.62m = 3.14V$

$V_{CE} = 5 - 3K \times 1.62m = 0.14V$

This circuit is too sensitive to parameter variations as shown here for a 1% resistor tolerance.

4.100



$R_B = ?$ if $\beta = 100$

$I_B \times \beta = I_C$

$\frac{5-0.7}{R_B} = \frac{1m}{100}$

$\rightarrow R_B = 430K\Omega$

$V_{CE} = 5V - 3K \cdot 1mA = 2V$

If $\beta = 50$: $I_C = \frac{5-0.7}{430K} \times 50$

$I_C = 0.50\,mA$

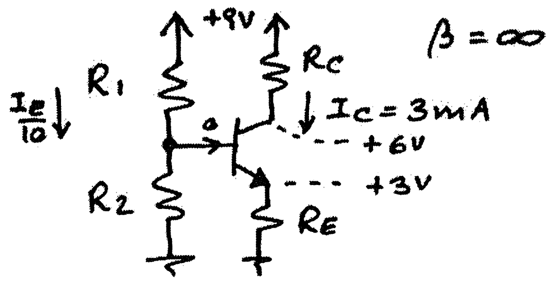
$\Rightarrow V_{CE} = 5 - 3K \times 0.5m = +3.5V$

If $\beta = 150$: $I_C = 1.5mA$

$V_{CE} = 0.5V$

This design is too sensitive to variations of β.

4.101



$\beta = \infty$

$R_C = \frac{3V}{3mA} = 1K\Omega$

$R_E = \frac{3V}{3mA} = 1K\Omega$

$V_b = 0.7 + 3 = 3.7V$

$R_1 = \frac{9 - 3.7}{I_E/10} = 17.7K\Omega$

$9V = (R_1 + R_2)\frac{I_E}{10} \rightarrow R_2 = 12.3K$

Choose suitable 5% resistors

$R_1 = 17.7K \rightarrow 18K\Omega$

$R_2 = 12.3K \rightarrow 13K\Omega$

$R_1 = R_2 = 1K$

$V_{BB} = \frac{9 \times 13}{18 + 13} = 3.77V$

For these values of R and $\beta = 90$: $R_B = 18 \parallel 13 = 7.55K\Omega$

$I_E = \frac{3.77 - 0.7}{1K + \frac{7.55K}{91}} = 2.83mA$

$\alpha = 0.989 \Rightarrow I_C = 2.80mA$

If R_E is reduced by $\sim \frac{7.55K}{91}$

$\rightarrow R_E = 910\Omega$

$\Rightarrow I_E = 3.09mA$

$I_C = 3.05mA$.

4.102

For $\beta = \infty$, $I_B = 0$, $I_E = 0.6$ mA

$R_c = \frac{3\text{ V}}{0.6\text{ m}} = 5\text{ k}\Omega = R_E$

$V_b = 0.7 + 3 = 3.7$

$R_1 = \frac{9 - 3.7}{I_E/2} = \frac{10.6}{.6\text{ m}} = 17.7\text{ k}\Omega$

$9 = (R_2 + R_1)\frac{I_E}{2} \Rightarrow R_2 = \frac{18}{I_E} - R_1 = 12.3\text{ k}\Omega$

Suitable 5%. Resistors: $R_1 = 17.4\text{ k}\Omega$

$R_2 = 12.1\text{ k}\Omega$

$\beta = 90$:

$R_B = (17.4\text{ K}) \| (12.1\text{ K}) = \frac{17.4\text{ K}(12.1\text{ K})}{29500} = 7.137\ \Omega$

$V_{BB} = \frac{9(12.1\text{ K})}{12.1\text{ K} + 17.4\text{ K}} = 3.7\text{ V}$

$I_E = \frac{3.7 - 0.7}{5\text{ K} + \frac{7137}{(90+1)}} = \frac{3}{5\text{ K} + 78.4} = .6\text{ mA}$

4.103

$$I_E = \frac{V_{BB} - V_{BE}}{R_E + \frac{R_B}{\beta+1}}$$

(a) For $\beta = 100$, varying between 50 and 150 the maximum deviation in I_E (from the nominal value obtained for $\beta = 100$) occurs at the low end of β values ($\beta = 50$). Thus, to keep I_E within $\pm 5\%$ of nominal we must impose the constraint

$$I_E(\beta=50) \geq 0.95\, I_E(\beta=100)$$

or,

$$\frac{V_{BB} - V_{BE}}{R_E + \frac{R_B}{51}} \geq 0.95\, \frac{V_{BB} - V_{BE}}{R_E + \frac{R_B}{101}}$$

or,

$$R_E + \frac{R_B}{101} \geq 0.95\left(R_E + \frac{R_B}{51}\right)$$

$$0.05 R_E \geq R_B\left(\frac{0.95}{51} - \frac{1}{101}\right)$$

$$\Rightarrow \frac{R_B}{R_E} \leq 5.73$$

Thus, the largest ratio of R_B/R_E is 5.73

(b) $I_E \cdot R_E = V_{CC}/3$

$$\rightarrow \frac{V_{BB} - V_{BE}}{R_E + \frac{R_B}{\beta+1}} \cdot R_E = \frac{V_{CC}}{3}$$

$$\frac{V_{BB} - 0.7}{1 + \frac{R_B}{R_E} \cdot \frac{1}{\beta+1}} = \frac{V_{CC}}{3}$$

$$V_{BB} = \frac{1}{3} V_{CC}\left(1 + \frac{5.73}{101}\right) + 0.7$$

$$\Rightarrow \underline{V_{BB} = 0.35\, V_{CC} + 0.7}$$

(c) $V_{CC} = 10\text{V}$

$V_{BB} = 0.35 \times 10 + 0.7 = 4.2\text{V}$

$$\rightarrow \frac{R_2}{R_1 + R_2} \times 10 = 4.2$$

$$\frac{R_2}{R_1 + R_2} = 0.42 \quad (1)$$

$$I_E \cdot R_E = \frac{1}{3} V_{CC}$$

CONT.

$2 \times R_E = \frac{1}{3} \times 10$

$\Rightarrow R_E = 1.67\,K\Omega$

$R_B = 5.73 \times 1.67 = 9.55\,K\Omega$

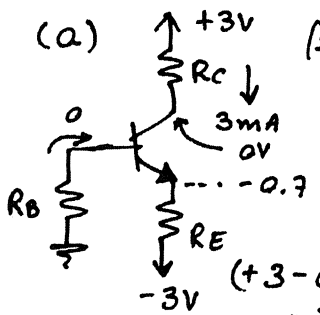
$\frac{R_1 \cdot R_2}{R_1 + R_2} = 9.55$

Substituting from ① gives

$R_1 = \frac{9.55}{0.42} = 22.7\,K\Omega$

4.104

(a)



$\beta = \infty,\ I_B = 0$

$I_E = I_C$

$3V = R_C \times 3mA$

$\rightarrow R_C = 1K\Omega$

$(+3 - 0.7) = R_E \times 3mA$

$\rightarrow R_E = 767\,\Omega$

(b) $\beta = 90 \quad \frac{V_{RE}}{10} = V_{RB}$

$I_B \cdot R_B = \frac{I_E \cdot R_E}{10}$

$\rightarrow \frac{I_E}{(\beta+1)} \cdot R_B = \frac{I_E \cdot R_E}{10}$

$\rightarrow R_B = \frac{(\beta+1)}{10} R_E$ ①

also, $0 = V_{RB} + 0.7 + V_{RE} - 3$

$2.3 = \frac{V_{RE}}{10} + V_{RE}$

$\rightarrow V_{RE} = \frac{2.3}{1.1} = 2.09V$

$2.09 = I_E \times R_E$ ②

but: $I_E = \frac{I_C}{\alpha} = \frac{3mA}{0.989} = 3.033m$

Substituting in ②:

$R_E = 689\,\Omega$

from ①:

$R_B = 6269\,\Omega$

(c) Standard 5% values:

$R_C = 1K\Omega$

$R_E = 689\,\Omega \rightarrow 680\,\Omega$

$R_B = 6269\,\Omega \rightarrow 6.2K\Omega$

(d) $\beta = \infty$: $I_B = 0$

$I_C = I_E$

$V_B = 0$

$V_E = -0.7$

$I_E = \frac{3 - 0.7}{R_E} = \frac{3 - 0.7}{680} = 3.38mA$

$V_C = 3 - 3.38m \times 1K = -0.38V$

For $\beta = 90$:

$I_E = \frac{2.3}{680 + \frac{6.2K}{91}} = 3.07mA$

$I_C = \alpha I_E = 3.04mA$

$V_B = \frac{R_B \cdot I_E}{(\beta+1)} = -0.209$

$V_E = -0.209 - 0.7 = 0.909\,V$

$V_C = 3 - I_C \cdot R_C$

$= 3 - 3.04 \times 1 = -0.04V$

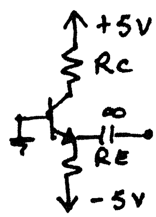
4.105

$V_E = -0.7V$

To obtain $I_E = 1mA$

$R_E = \frac{-0.7 - (-5)}{1}$

$= 4.3\,K\Omega$



CONT.

To maximize gain while allowing ±1V signal at collector, design for a dc collector voltage of +1V. Thus,

$$R_C = \frac{5-1}{I_C} \simeq \frac{4}{1} = \underline{\underline{4\,K\Omega}} \text{ (for } \alpha=1)$$

For 100°C rise in temperature, V_{BE} decreases by $2 \times 100 = 200mV$ and thus I_E increases by $\frac{0.2V}{R_E}$

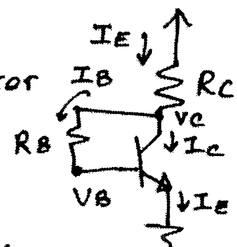
$$= \frac{0.2V}{4.3K\Omega} = 0.047\,mA$$

i.e. an increase of $\underline{\underline{4.7\%}}$

The change in β from 50 to 150 causes α to change from 0.980 to 0.993 which implies an increase in collector current of $\underline{1.3\%}$

Thus the overall increase in I_C is $\underline{\underline{6\%}}$

4.106



To allow a collector voltage swing of ±1V, we design for:

$$V_C = V_B + 1$$
$$= 0.7 + 1 = 1.7V$$

$I_E = 0.5mA$

$$\rightarrow R_C = \frac{5-1.7}{0.5} = \underline{\underline{6.6\,K\Omega}}$$

For $\beta = 100$:

$$I_B = \frac{I_E}{\beta+1} = \frac{0.5}{101} \simeq 5\mu A$$

$I_B \cdot R_B = 1V$

$$R_B = \frac{1V}{5\mu A} = \frac{1}{5} M\Omega = \underline{\underline{200 K\Omega}}$$

Now, if the BJT used has $\beta = 50$, the emitter current resulting can be found from Eq (5.74)

$$I_E = \frac{V_{CC} - V_{BE}}{R_C + \frac{R_B}{\beta+1}}$$

$$= \frac{5-0.7}{6.6 + \frac{200}{51}} = \underline{\underline{0.41mA}}$$

and $I_B = \frac{0.41}{51} \simeq 8\mu A$

Thus the collector will be higher than the base by $8 \times 0.2 = 1.6V$, allowing for a $\underline{\underline{\pm 1.6V}}$ signal swing at the collector.

For $\beta = 150$:

$$I_E = \frac{5-0.7}{6.6 + \frac{200}{151}} = 0.54mA$$

$$I_B = \frac{0.54}{151} = 36\mu A$$

Thus the collector voltage will be higher than that of the base by

$$3.6 \times 0.2 = 0.72V$$

allowing for only $\pm \underline{\underline{0.72V}}$ signal swing.

4.107

$I_B = I_C/\beta = 3mA/90 = 0.033m$

$V_C = R_B \cdot I_B + 0.7$

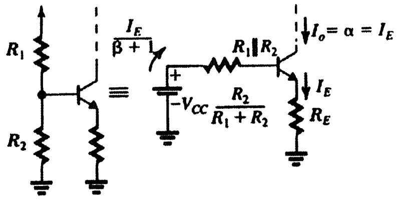
$V_C = 1.5V \rightarrow R_B = \underline{24.2K\Omega}$

$I_E = \frac{I_C}{\alpha} = 3.03mA$

$I = I_C - I_B \equiv I_E$

$I = \underline{3.03mA}$

4.108



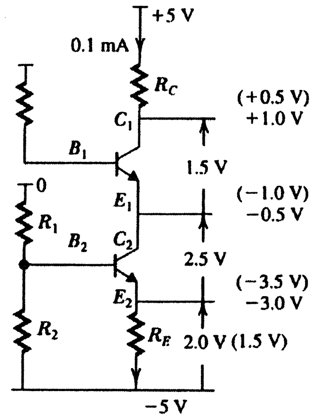
$$V_{CC} \cdot \frac{R_2}{R_1 + R_2} = \frac{I_E}{\beta + 1}(R_1 \| R_2) + V_{BE} + I_E R_E$$

$$\Rightarrow I_E = \frac{V_{CC}\frac{R_2}{R_1 + R_2} - V_{BE}}{R_E + \frac{(R_1 \| R_2)}{\beta + 1}}$$

Thus,

$$I_O = \alpha I_E = \frac{\alpha \cdot \left[\frac{V_{CC}R_2}{R_1 + R_2} - V_{BE}\right]}{R_E + \frac{(R_1 \| R_2)}{\beta + 1}}$$

4.109



The constraints imposed cannot be met

$V_{E1} < -0.7$ V for Q_1 active.

Change V_{RE} to 1.5 V then

$V_{E2} = -3.5$ V

$V_{C2} = V_{E2} + 2.5 = -1.0$ V

$V_{C1} = V_{C2} + 1.5 = +0.5$ V

For $\beta = \infty$

$R_{E2} = 1.5$ V/0.1 mA $= 15$ kΩ

$V_{B2} = -3.5 + 0.7 = -2.8$ V

Then $\frac{V_{R1}}{V_{R2}} = \frac{2.8}{2.2} = \frac{R_1}{R_2}$

$V_{B1} = 0\ (I_{B1} = 0)$

$V_{E1} = -0.7$ V

$V_{C1} = V_{E1} + 1.5 = +0.8$ V

$R_{C2} = \frac{V_{CC}}{0.1\ mA} \cdot 42$ kΩ

For I_{E2} ($\beta = 50$) within 5% I_{E2} ($\beta = \infty$)

For $\beta = 50$

$$I_E = \frac{1.5}{R_E + (R_1 \| R_2)/51}$$

$\beta = \infty$

$$I_E = \frac{1.5}{R_E}$$

Need $\frac{R_1 \parallel R_2}{51} \le \frac{5}{100} R_E$

$\therefore R_1 \parallel R_2 \le 51 R_E / 2 = 38.25\text{ k}\Omega$

$$\frac{R_1 R_2}{R_1 + R_2} = \frac{R_1}{1 + R_1/R_2} = \frac{R_1}{1 + 28/22} < 38.25\text{ k}\Omega$$

$\therefore R_1 < 86.9\text{ k}\Omega$ use $82\text{ k}\Omega$

$R_2 < 68.3\text{ k}\Omega$ use $68\text{ k}\Omega$

$R_1 \parallel R_2 = 37\text{ k}\Omega < 38.25\text{ k}\Omega$

For $\beta = \infty$ and 5% values

$$V_{B2} = \frac{-5 \times R_1}{R_1 + R_2} = -2.73\text{ V}$$

$V_{E2} = 2.27 + 0.7 = -1.57\text{ V}$

$I_{E2} = 1.57/15 = 0.1046\text{ mA}$

For $\beta = 50$ V determine R_B

$$I_{E2} = \frac{2.27 - 0.7}{37/51 + 15} = 0.0998\text{ mA}$$

$I_{C2} = 0.98 \times I_{E2} = 0.098\text{ mA}$

$I_{C1} = 0.98 \times I_{C2} = 0.096\text{ mA}$

$I_{B1} = I_{C1}/50$

$V_{RE} = 0.099 \times 15 = 1.47\text{ V}$

$V_{E2} = -5 + V_{RE} = -3.53\text{ V}$

For $V_{CE2} = 2.5\text{ V}$

$V_{E1} = V_{C2} = V_{E2} + V_{CE2} = -1.03\text{ V}$

$V_{B1} = V_{E1} + 0.7 = -0.33\text{ V}$

$$R_B = V_{B1} \times \frac{\beta}{I_{C1}} = 173.7\text{ k}\Omega \text{ use } 180\text{ k}\Omega$$

For $\beta = 50$

$I_{C1} = 0.096\text{ mA}$

$$V_{B1} = -\frac{0.09}{50} \times 180 = -0.35\text{ V}$$

$V_{E1} = V_{B1} - 0.7 = -1.05\text{ V}$

$V_{C1} = 5 - 0.096 \times 43 = 0.872\text{ V}$

$V_{CE1} = 1.9\text{ V}$

For $\beta = 100$

$$I_{E2} = \frac{1.57}{37/101 + 15} = 0.102\text{ mA}$$

$I_{C1} = 0.99 \times 0.99 \times I_{E2} = 0.10\text{ mA}$

$V_{C1} = 0.7\text{ V}$

$$V_{B1} = -\frac{0.10}{101} \times 180 = -0.878\text{ V}$$

$V_{E1} = V_{B1} - 0.7 = -0.878\text{ V}$

$V_{CE1} = 0.7 + 0.878 = 1.578\text{ V}$

For $\beta = 200$

$$I_{E2} = \frac{1.57}{37/201 - 15} = 0.103\text{ mA}$$

$I_{C1} = 0.995 \times 0.995 \times I_{E2} = 0.102\text{ mA}$

$V_{C1} = 0.615\text{ V}$

$$V_{B1} = -\frac{0.102}{201} \times 180 = -0.091\text{ V}$$

$V_{E1} = V_{B1} - 0.7 = -0.791\text{ V}$

$V_{CE1} = 1.45\text{ V}$

4.110

$$I_O = 2\text{mA} = \alpha \times \frac{5 - 0.7}{R} \simeq \frac{4.3}{R}$$

$\Rightarrow R = 2.15\text{ K}\Omega$

V_C min $= 0\text{V}$ (In actual practice, $V_C$min $\simeq 0.4\text{V}$)

4.111

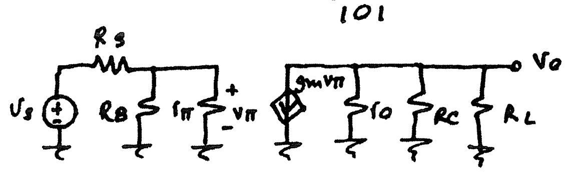
$$I_E = \frac{V_{BB} - V_{BE}}{R_E + R_B/(\beta+1)}$$

where, $V_{BB} = V_{CC} \cdot \frac{R_2}{R_1 + R_2}$

$$= 9 \cdot \frac{15}{27+15} = 3.21\text{V}$$

$R_B = R_1 \parallel R_2 = 15 \parallel 27 = 9.64\text{ K}\Omega$

Thus, $I_E = \frac{3.21 - 0.7}{1.2 + \frac{9.64}{101}} = 1.94\text{mA}$



$$g_m = \frac{I_C}{V_T} = \frac{0.99 \times 1.94}{0.025} = 76.8\,\frac{\text{mA}}{\text{V}}$$

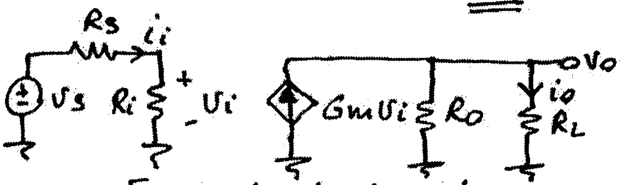
$$r_\pi = \frac{\beta}{g_m} = \frac{100}{76.8} = 1.3\text{ K}\Omega$$

$$r_o = \frac{V_A}{I_C} = \frac{100}{0.99 \times 1.94} = 52.1\text{ K}\Omega$$

$R_i = R_B \parallel r_\pi = 9.64 \parallel 1.3 = 1.15\text{ K}\Omega$

$G_m = -g_m = -76.8\,\frac{\text{mA}}{\text{V}}$

$R_o = R_C \| r_o = 2.2 \| 52.1 = \underline{2.11}\ k\Omega$



Equivalent circuit

$A_v \equiv \frac{v_o}{v_s} = \frac{v_i}{v_s} \cdot \frac{v_o}{v_i}$

$= \frac{R_i}{R_s + R_i} \cdot \frac{G_m (R_o \| R_L) v_i}{v_i}$

$= \frac{-1.15}{10 + 1.15} \times 76.8 \times (2.11 \| 2)$

$= \underline{-8.13}\ V/V$

$A_i = \frac{i_o}{i_i} = \frac{v_o / R_L}{v_s / (R_s + R_i)}$

$\Rightarrow A_i = \frac{v_o}{v_s} \cdot \frac{R_s + R_i}{R_L}$

$= -8.13 \times \frac{(10 + 1.15)}{2}$

$= \underline{-45.3}\ A/A$

4.112

$V_{CC} = 9V \quad V_{BB} = \frac{1}{3} V_{CC} = 3V$

Neglecting the base current,

$R_1 + R_2 = \frac{9}{0.2} = 45\ k\Omega$

$\frac{R_2}{R_1 + R_2} = \frac{1}{3}$

$\Rightarrow R_2 = \underline{15\ k\Omega}, \quad R_1 = \underline{30\ k\Omega}$

$R_B = R_1 \| R_2 = \frac{30 \times 15}{45} = 10\ k\Omega$

$I_E = \frac{V_{BB} - V_{BE}}{R_E + \frac{R_B}{\beta + 1}}$

$2 = \frac{3 - 0.7}{R_E + 10/101} \Rightarrow R_E = \underline{1.05\ k\Omega}$

Use $R_E = \underline{1\ k\Omega}$

The resulting I_E will be

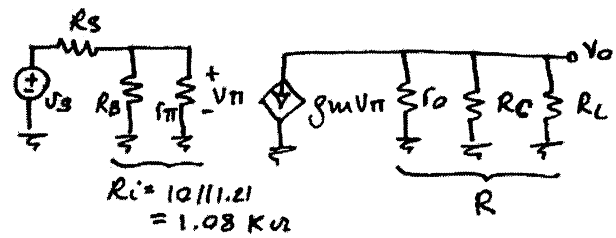
$I_E = \frac{3 - 0.7}{1 + 10/101} = 2.09\ mA$

$I_C = \alpha I_E = 0.99 \times 2.09 = 2.07\ mA$

$g_m = \frac{I_C}{V_T} = \frac{2.07}{0.025} = 82.9\ \frac{mA}{V}$

$r_\pi = \frac{\beta}{g_m} = \frac{100}{82.9} = 1.21\ k\Omega$

$r_o = \frac{V_A}{I_C} = \frac{100}{2.07} = 48.3\ k\Omega.$



$\frac{v_o}{v_s} = \frac{v_\pi}{v_s} \cdot \frac{v_o}{v_\pi} = \frac{R_i}{R_s + R_i} \cdot \frac{-g_m v_\pi R}{v_\pi}$

$= \frac{-1.08}{10 + 1.08} \times 82.9 \times R$

To obtain $\frac{v_o}{v_s} = -8\ \frac{V}{V}$ we use:

$R = \frac{8 \times 11.08}{1.08 \times 82.9} = 0.99\ k\Omega$

Now $R = r_o \| R_C \| R_L$

$0.99 = 48.3 \| R_C \| 2$

$\Rightarrow R_C = 2.04\ k\Omega$

use $R_C = \underline{2\ k\Omega}$

Check: $V_C = 9 - 2.07 \times 2 = 4.86V$ while $V_B \simeq 3V$. Thus in active mode as assumed.

4.113

$V_{BB} = 9 \cdot \frac{47}{82+47} = 3.28\text{ V}$

$R_B = 47 \| 82 = 29.88\text{ K}\Omega$

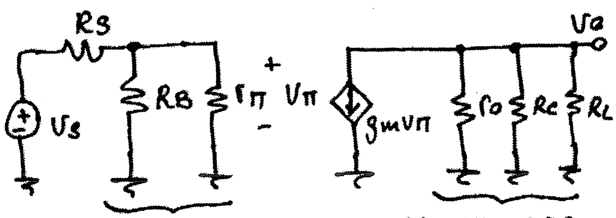
$I_E = \frac{3.28 - 0.7}{3.6 + \frac{29.88}{101}} = 0.66\text{ mA}$

$I_C = 0.99 \times 0.66 = 0.65\text{ mA}$

$g_m = \frac{0.65}{0.025} = 26\ \frac{\text{mA}}{\text{V}}$

$r_\pi = \frac{100}{26} = 3.85\text{ K}\Omega$

$r_o = \frac{100}{0.66} = 151.5\text{ K}\Omega$



$R_i = 29.88 \| 3.85 = 3.41\text{ K}\Omega$

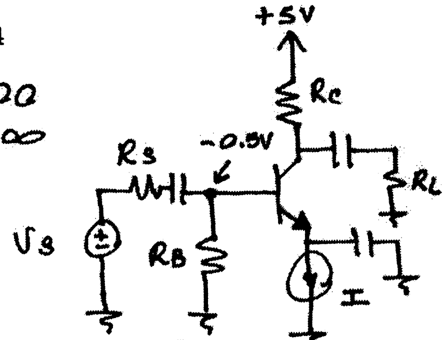
$151.5 \| 6.8 \| 2 = 1.53\text{ K}\Omega$

$A_v = \frac{v_o}{v_s} = \frac{3.41}{10 + 3.41} \times -26 \times 1.53$

$= -10.1\text{ V/V}$ Which is about 25% higher than in the original design. The improvement is not as large as might have been expected because although R_i increases, g_m decreases by about the same factor. Indeed most of the improvement is due to the increase in R_C and hence in the effective load resistance.

4.114

$\beta = 100$
$r_o = \infty$



$R_{in} = 5\text{K}\Omega$, $R_{in} = R_B \| r_\pi$

$\Rightarrow 5K = \frac{R_B \cdot r_\pi}{R_B + r_\pi}$

$5K r_\pi = R_B (r_\pi - 5K)$

but: $r_\pi = \frac{V_T}{I_B}$ and $R_B \cdot I_B = 0.5$

$\rightarrow 5K \cdot \frac{V_T}{I_B} = \frac{0.5}{I_B}(r_\pi - 5K)$

thus, $r_\pi = 5250\Omega$

then $R_B = 105K$

choose $R_B = 100\text{K}\Omega$

and $I_B = 4.76\ \mu A$

$I_E = (\beta+1) I_B = 101 \times 4.76\ \mu A$

$I_E = 0.48\text{ mA}$

$I = I_E \rightarrow I \simeq 0.5\text{ mA}$

To avoid saturation:

$V_C - V_B \geq -0.5$

$V_C = 5V - R_C [I_C + g_m v_{be}]$

$I_C = I \cdot \alpha = 0.5m \times 100/101$

$= 0.49\text{ mA}$

$g_m = \frac{V_T}{I_C} = \frac{25m}{0.49m} \simeq 50\ \frac{\text{mA}}{\text{V}}$

$v_{be} = 0.005V$

$\rightarrow V_C = 5 - R_C [0.49m + 50m \times 5m]$

$= 5 - 0.74 \times 10^{-3} \times R_C$

Then:

$v_C - v_B = (5 - 0.74m R_C) - (-0.5 + v_{be})$

$= 5.495 - 0.74m R_C \geq -0.5$

$R_C \leq \underline{\underline{8.1 K\Omega}}$.

Base-to-Collector open circuit gain:

$\frac{v_c}{v_b} = -g_m R_C = -50m \times 8.1K$

$= -\underline{\underline{405}} V/V$

For $R_s = 10K$, $R_L = 10K$

$\frac{v_o}{v_b} = -g_m (R_C \| R_L)$

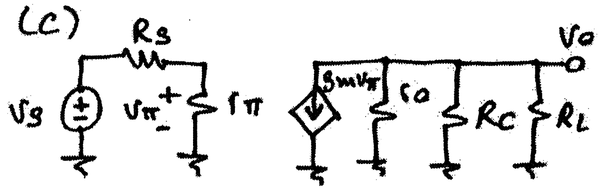
$= -50m \times 4.47K$

$= -223 V/V$

$\frac{v_c}{v_s} = \frac{v_b}{v_s} \cdot \frac{v_o}{v_b} = \frac{5}{5+10} \times -223$

$= -\underline{\underline{74.3}} V/V$

(c)



$R_L = 10K\Omega$, $R_s = 2.5K$

$r_o = 200K\Omega$

$g_m = \frac{I_C}{V_T} \simeq \frac{0.5m}{25m} = 20 \frac{mA}{V}$

$r_\pi = \frac{\beta}{g_m} = \frac{100}{20} = 5K\Omega$

$A_v = \frac{v_o}{v_s} = \frac{v_\pi}{v_s} \times \frac{v_o}{v_\pi}$

$= \frac{r_\pi}{r_\pi + R_s} \times -g_m (r_o \| R_C \| R_L)$

$= -\frac{5}{5+2.5} \times 20 (200 \| 20 \| 10)$

$= -\underline{\underline{86}} V/V$

4.115

$I_E = 0.5mA$

(a) $I_E = \frac{15 - 0.7}{R_E + \frac{R_s}{\beta+1}}$

$0.5 = \frac{14.3}{R_E + \frac{2.5}{100}}$

$\Rightarrow R_E = \underline{\underline{28.57}} K\Omega$.

(b) $V_C = 15 - R_C \cdot I_C$

$5 = 15 - R_C \times 0.99 \times 0.5m$

$\Rightarrow R_C = 20.2K\Omega$

$\simeq \underline{\underline{20K\Omega}}$

4.116

(a) For each transistor

$V_{BB} = 15 \times \frac{47}{100+47} = 4.8V$

$R_B = R_1 \| R_2 = 100 \| 47 = 32K\Omega$

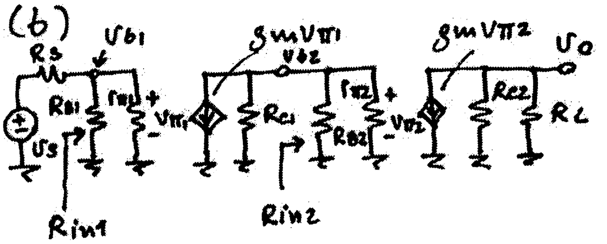
$I_E = \frac{4.8 - 0.7}{3.9 + \frac{32}{101}} = 0.97 mA$

$I_C = 0.99 \times 0.97 = \underline{\underline{0.96 mA}}$

$V_C = V_{CC} - I_C \times R_C$

$= 15 - 0.96 \times 6.8 = \underline{\underline{8.5V}}$

(b)



$R_{B1} = R_{B2} = R_B = 32K\Omega$

$g_{m1} = g_{m2} = \frac{0.96}{0.025} = 38.4 \, mA/V$

$r_{\pi 1} = r_{\pi 2} = \frac{100}{38.4} = 2.6 \, K\Omega$

$R_{C1} = R_{C2} = 6.8 \, K\Omega$

$r_{o1} = r_{o2} = \infty$

(c) $R_{in1} = R_{B1} // r_{\pi 1}$
$= 32 // 2.6 = \underline{\underline{2.4 K\Omega}}$

$\frac{v_{b1}}{v_s} = \frac{R_{in1}}{R_s + R_{in1}}$
$= \frac{2.4}{5 + 2.4} = \underline{\underline{0.32}} \, V/V$

(d) $R_{in2} = R_{B2} // r_{\pi 2}$
$= 32 // 2.6 = \underline{\underline{2.4 K\Omega}}$

$v_{b2} = -g_{m1} v_{\pi 1} \, (R_{C1} // R_{in2})$
$= -38.4 \, v_{b1} \, (6.8 // 2.4)$

$\frac{v_{b2}}{v_{b1}} = \underline{\underline{-68.1}} \, V/V$

(e) $v_o = -g_{m2} v_{\pi 2} \, (R_{C2} // R_L)$
$= -38.4 \, v_{b2} \, (6.8 // 2)$

$\frac{v_o}{v_{b2}} = \underline{\underline{-59.3}} \, V/V$

(f) $\frac{v_o}{v_s} = \frac{v_{b1}}{v_s} \times \frac{v_{b2}}{v_{b1}} \times \frac{v_o}{v_{b2}}$
$= 0.32 \times -68.1 \times -59.3$
$= \underline{\underline{1292}} \, V/V$

4.119

$R_{in} = (\beta + 1)(r_e + 250)$

$\beta = 100 \quad r_e = \frac{V_T}{I_E} = \frac{0.025}{0.1} = 250\Omega$

$R_{in} = 101 \times (250 + 250)$
$= \underline{\underline{50.5 K\Omega}}$

$\frac{v_b}{v_s} = \frac{R_{in}}{R_s + R_{in}} = \frac{50.5}{20 + 50.5}$
$= 0.72 \, V/V$

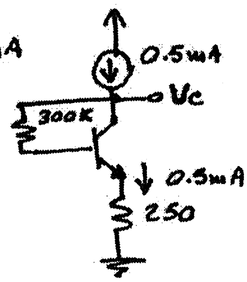
$\frac{v_o}{v_b} = \frac{-\alpha \, (20 // 20)}{(r_e + R_E)}$
$= \frac{-0.99 \times 10}{0.250 + 0.250} = \underline{\underline{-19.8}} \, V/V$

Thus, $\frac{v_o}{v_s} = 0.72 \times -19.8 = \underline{\underline{-14.2}} \, V/V$

For $v_{be} = 5mV$, $v_e = 5mV$ also
(since $R_e = r_e = 250\Omega$)

Thus,
$v_b = 5 + 5 = 10mV$
$v_s = \frac{10m}{0.72} = \underline{\underline{13.88}} \, mV$
$v_o = 13.88 \times 14.2 = \underline{\underline{197.2}} \, mV$

4.120



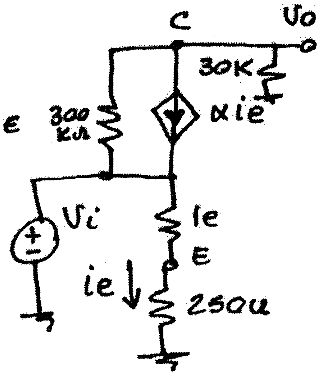
(a) $I_C = 0.99 \times 0.5$ mA

$= 0.495$ mA

$V_C = I_E R_E + V_{BE} + I_B R_B$

$= 0.5 \times 0.175 + 0.7 + 0.005 \times 300$

$= \underline{\underline{2.28 V}}$



(b) $i_e = \dfrac{v_i}{r_e + R_E}$

$r_e = \dfrac{V_T}{I_E} = 50\Omega$

$\rightarrow i_e = \dfrac{v_i}{50 + 250}$

$i_e = \dfrac{v_i}{300}$

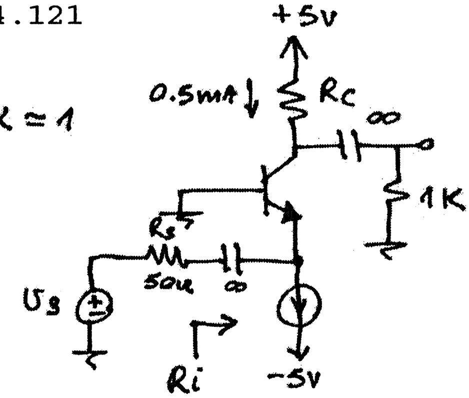
Node equation at c:

$$\frac{v_o - v_i}{300K} + \alpha i_e + \frac{v_o}{30K} = 0$$

$$\frac{v_o - v_i}{300K} + \frac{\alpha v_i}{(250+50)} + \frac{v_o}{30K} = 0$$

$$\Longrightarrow \frac{v_o}{v_i} = \underline{\underline{-90 V/V}}$$

4.121



$\alpha \simeq 1$

$R_i = \dfrac{V_T}{I} = 50\Omega \Rightarrow I = \underline{\underline{0.5mA}}$

$V_C = 5 - 0.5 R_C$

$V_{Cmin} = V_C - 0.01 g_m (R_C \| 1K)$

To prevent saturation $V_{Cmin} = 0$

$\rightarrow 0 = V_C - 0.01 \times 20 (R_C \| 1)$

$= 5 - 0.5 \dfrac{R_C}{R_C + 1}$

$5R_C + 5 - 0.5R_C^2 - 0.5R_C - 0.2R_C = 0$

$0.5R_C^2 - 4.3R_C + 5 = 0$

$R_C = \dfrac{4.3 + \sqrt{4.3^2 + 10}}{1}$

$= 9.64 K\Omega$

Select $R_C = \underline{\underline{9.1 K\Omega}}$

$V_C = 0.45 V$

$\dfrac{v_o}{v_s} = \dfrac{R_i}{R_s + R_i} g_m (R_C \| 1)$

$= \dfrac{50}{50 + 50} \times 20 \times (9.1 \| 1)$

$= \underline{\underline{9 V/V}}$

For $v_{be\,max} = 10\,mV$
$v_{s\,max} = 20\,mV$
$V_{c\,max} = 180\,mV$
Thus the collector voltage swings from
$(0.45 - 0.18)V$ to $(0.45 + 0.18)V$
i.e from 0.27V to 0.63V

4.122

$R_i = r_e = \frac{V_T}{I_E} = \frac{V_T}{0.5} = 50\Omega$

To find the voltage gain v_o/v_s first note that

$$\frac{v_e}{v_s} = \frac{R_i}{R_s + R_i} = \frac{50}{50+50} = 0.5$$

Then,

$$\frac{v_c}{v_e} = \frac{\alpha \times (\text{Total resistance at } c)}{r_e}$$
$$\simeq \frac{1 \times (100K\Omega \,//\, 1K\Omega)}{50\Omega}$$
$$= 19.8\ V/V$$

Thus, $\frac{v_o}{v_s} = 19.8 \times 0.5 = 9.9\ V/V$

4.123

(a) $I_E = \frac{9 - 0.7}{1 + 100/(\beta+1)}$

for $\beta = 40$, $I_E = \frac{8.3}{1 + \frac{100}{41}} = 2.41\,mA$

$V_E = 1 \times 2.41 = 2.41V$
$V_B = 2.41 + 0.7 = 3.11V$

for $\beta = 200$, $I_E = \frac{8.3}{1 + \frac{100}{201}} = 5.54\,mA$

$V_E = +5.54V$

$V_B = +6.24V$

(b) $R_i = 100K\Omega \,//\, (\beta+1)[r_e + (1//1)]$
$= 100 \,//\, (\beta+1)[r_e + 0.5]$

For $\beta = 40$, $I_E = 2.41\,mA$
$\rightarrow r_e = 10.37\,\Omega$
thus $R_i = 100 \,//\, 41 \times (0.01037 + 0.5)$
$= 100 \,//\, 21$
$= 17.30\,\Omega$

For $\beta = 200$, $I_E = 5.54\,mA$
$\rightarrow r_e = 4.51\,\Omega$
thus $R_i = 100 \,//\, 201(0.0045 + 0.5)$
$= 100 \,//\, 101.4$
$= 50.3\,K\Omega$

(c) $\frac{v_o}{v_s} = \frac{v_b}{v_s} \cdot \frac{v_o}{v_b}$
$= \frac{R_i}{R_s + R_i} \cdot \frac{(1//1)}{(1//1) + r_e}$

For $\beta = 40$,
$$\frac{v_o}{v_s} = \frac{17.3}{10 + 17.3} \times \frac{0.5}{0.5 + 0.01037}$$
$$= 0.621\ V/V$$

For $\beta = 200$,
$$\frac{v_o}{v_s} = \frac{50.3}{10 + 50.3} \cdot \frac{0.5}{0.5 + 0.0045}$$
$$= 0.827\ V/V$$

4.124

$$I_E = \frac{5-0.7}{3.3 + \frac{100}{101}} = 1.00\ \text{mA}$$

$$r_e = \frac{25}{1.00} = 25\ \Omega$$

$$R_i = (\beta+1)[r_e + (3.3 \parallel 1)] = 80.0\ \text{k}\Omega$$

$$\frac{v_o}{v_s} = \frac{v_b}{v_s} \cdot \frac{v_o}{v_b} = \frac{R_i}{R_s+R_i} \cdot \frac{(3.3 \parallel 1)}{r_e + (3.3 \parallel 1)}$$

Thus,

$$\frac{v_o}{v_s} = \frac{80}{100+80} \times \frac{(3.3 \parallel 1)}{0.025 + (3.3 \parallel 1)} = 0.430\ \text{V/V}$$

$$\frac{i_o}{i_i} = \frac{v_o/R_L}{v_s/(R_s+R_i)} = \frac{v_o}{v_s} \cdot \frac{(R_s+R_i)}{R_L} = 0.43 \times \frac{(100+80)}{1} = 77.4\ \text{A/A}$$

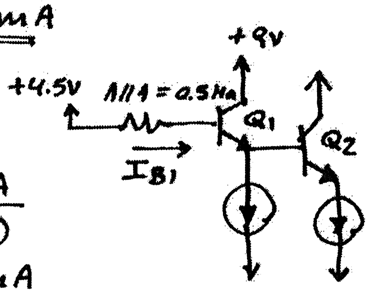
$$R_{out} = 3.3 \parallel \left[r_e + \frac{100}{\beta+1}\right] = 3.3 \parallel \left[0.025 + \frac{100}{101}\right] = 0.776\ \text{k}\Omega$$

4.125

(a) $I_{E2} = 5\ \text{mA}$

$\beta_1 = 50, \quad \beta_2 = 100$

$$I_{E1} = 50\mu + I_{B2} = 50 + \frac{I_{E2}}{\beta_2+1} = 50 + \frac{5000}{101} \simeq 0.1\ \text{mA}$$



$$I_{B1} = \frac{0.1\ \text{mA}}{(50+1)} = 1.96\ \mu\text{A}$$

$$V_{B1} = 4.5 - 0.5 \times 1.96 = 3.52\ \text{V}$$

$$V_{B2} = 3.52 - 0.7 = 2.82\ \text{V}$$

(b) Refer to Fig. P.5.148

$$\frac{v_o}{v_{b2}} = \frac{R_L}{R_L + r_{e2}}$$

$R_L = 1\ \text{k}\Omega \quad r_{e2} = \frac{25}{5} = 5\ \Omega$

$$\frac{v_o}{v_{b2}} = \frac{1}{1 + 0.005} = 0.995\ \text{V/V}$$

$$R_{ib2} = (\beta_2+1)(r_{e2} + R_L) = (101) \times (1.005) = 101.5\ \text{k}\Omega$$

(c) $$\frac{v_{e1}}{v_{b1}} = \frac{R_{ib2}}{R_{ib2} + r_{e1}}$$

$$r_{e1} = \frac{V_T}{100\ \mu\text{A}} = 250\ \Omega$$

$$\rightarrow \frac{v_{e1}}{v_{b1}} = \frac{101.5}{101.5 + 0.25} = 0.997\ \frac{\text{V}}{\text{V}}$$

$$R_i = 1\ \text{M}\Omega \parallel 1\ \text{M}\Omega \parallel (\beta_1+1)(r_{e1} + R_{ib2})$$
$$= 1 \parallel 1 \parallel 51 \times (0.25 + 101.5)\ \text{k}\Omega$$
$$= 1 \parallel 1 \parallel 5.2\ \text{M}\Omega$$
$$= 0.499\ \text{M}\Omega = 499\ \text{k}\Omega$$

(d) $$\frac{v_{b1}}{v_s} = \frac{R_i}{R_s + R_i} = \frac{499}{100+499} = 0.833\ \text{V/V}$$

(e) $\frac{v_o}{v_s} = \frac{v_{b1}}{v_s} \cdot \frac{v_{e1}}{v_{b1}} \cdot \frac{v_o}{v_{e1}}$

$= 0.833 \times 0.997 \times 0.995$

$= \underline{\underline{0.826}}$ V/V

5.1

The capacitance per unit area is: $C_{ox} = \frac{\epsilon_{ox}}{t_{ox}}$

$\epsilon_{ox} = 3.45 \times 10^{-11}$ F/m

$t_{ox} = 5nm \Rightarrow C_{ox} = \frac{3.45 \times 10^{-11}}{5 \times 10^{-9}} = 6.9\ fF/\mu m^2$

$t_{ox} = 40nm \Rightarrow C_{ox} = 0.86\ fF/\mu m^2$

For 1PF capacitance, we require an area A:

$A = \frac{10^{-12}}{6.9 \times 10^{-15}} = 145\ \mu m^2$ for $t_{ox} = 5nm$

$A = \frac{10^{-12}}{0.86 \times 10^{-15}} = 1163\ \mu m^2$ for $t_{ox} = 20nm$

For a square plate capacitor of 10PF:

$A = 10 \times 145 = 1450\ \mu m^2$ or $38 \times 38\ \mu m^2$ square for $t_{ox} = 5^{nm}$

$A = 10 \times 1163 = 11630\ \mu m^2$ or $108 \times 108\ \mu m^2$ square for $t_{ox} = 20^{nm}$

5.2

With V_{DS} small, compared to V_{ov},

$$r_{DS} = \frac{1}{(\mu_n C_{ox})\left(\frac{W}{L}\right)(V_{OV})}$$

(a) V_{ov} is doubled $\rightarrow r_{DS}$ is halved. factor = 0.5

(b) W is doubled $\rightarrow r_{DS}$ is halved. factor = 0.5

(c) W and L are doubled $\rightarrow r_{DS}$ is unchanged. factor = 1.0

(d) If oxide thickness t_{ox} is halved, and

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}},$$

C_{ox} is doubled. If W and L are also halved, r_{DS} is halved, factor = 0.5

5.3

The transistor size will be minimized if W/L is minimized, since W/L appears in the equations that must be satisifyed, we can minimize (W/L). Clearly we want to minimize L by using the smallest feture size.

$L = 0.18\,\mu m$

$$r_{DS} = \frac{1}{k_n'(W/L)(v_{GS} - V_t)}$$

$$r_{DS} = \frac{1}{k_n'(W/L)v_{OV}}$$

Two conditions need to met for v_{OV} and r_{DS}

Condition 1:

$$r_{DS,1} = \frac{1}{400 \times 10^{-6}(W/L)v_{OV,1}}$$

$$= 200 \Rightarrow (W/L)v_{OV,1} = 12.5$$

Condition 2:

$$r_{DS,2} = \frac{1}{400 \times 10^{-6}(W/L)v_{OV,2}}$$

$$= 1000 \Rightarrow (W/L)v_{OV,2} = 2.5$$

If condition 1 is met, condition 2 will be met since the over-voltage can always be reduced to satisfy this requirement. For condition 1, we want to decrease W/L as much as possible (so long as it is greater than or equal to 1), while still meeting all of the other constraints.

This requires our using the largest possible $v_{GS,1}$ voltage. $v_{GS,1} = 1.8$ Volts, so $v_{OV,1} = 1.4$ Volts that

$$W/L = \frac{12.5}{v_{OV,1}} = \frac{12.5}{1.4} \cong 8.93$$

Condition 2 now can be used to find $v_{GS,2}$

$$v_{OV,2} = \frac{12.5}{W/L} = \frac{2.5}{12.5/1.4} = 0.28$$

$$\Rightarrow v_{GS,2} = 0.68\text{Volts} \Rightarrow 0.68 \le v_{GS} \le 1.8$$

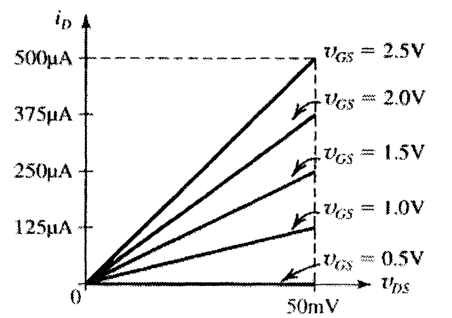
5.4

$k_n = 5\ \text{mA}/\text{V}^2 \quad V_t = 0.5\ \text{V}$

Small v_{DS}

$$i_D = k_n\ (v_{GS} - V_t)v_{DS} = k_n\ v_{OV}\ v_{DS}$$

$$g_{DS} = \frac{1}{r_{DS}} = k_n v_{OV}$$



(V)	(V)	(mS)	(Ω)
V_{GS}	V_{OV}	g_{DS}	r_{DS}
0.5	0	0	∞
1.0	0.5	2.5	400
1.5	1.0	5.0	200
2.0	1.5	7.5	133
2.5	2.0	10	100

5.5

$V_{DS\,sat} = V_{ov}$

$V_{ov} = V_{GS} - V_t = 2.5 - 1 = 1.5$ V

$\Rightarrow V_{DS\,sat} = 1.5$ V

In saturation:

$$i_D = \frac{1}{2} K'_n \left(\frac{W}{L}\right) V_{OV}^2 = \frac{1}{2} K_n V_{OV}^2$$

$$i_D = \frac{1}{2} \times \frac{1\text{mA}}{V^2} \times (1.5\ \text{V})^2$$

$$i_D = (1.125\ \text{mA})$$

5.6

a) $C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}} = \frac{3.45\times10^{-11}}{15\times10^{-9}} = 2.3\ fF/\mu m^2$

$K'_n = \mu_n C_{ox} = 550\times10^{-4}\times2.3\times10^{-3} = 126.5\ \mu A/V^2$

b) $i_O = \frac{1}{2} K'_n \frac{W}{L} (V_{GS} - V_t)^2 \Rightarrow 100 = \frac{1}{2} \times 126.5 \times \frac{16}{0.8} (V_{GS} - 0.7)^2$

$V_{GS} - 0.7 = 0.28 \Rightarrow V_{OV} = 0.28V$

$V_{GS} = 0.98V$

$V_{DS\,min} = V_{GS} - V_t = 0.28V$

c) for small V_{DS}: (triode region) $i_D \simeq K'_n \frac{W}{L} V_{OV} \cdot V_{DS}$

$r_{DS} = \frac{V_{DS}}{i_D} = \frac{1}{K'_n \frac{W}{L} V_{OV}} = \frac{1}{126.5\times10^{-6}\times\frac{16}{0.8} V_{OV}} = 1000$

$\Rightarrow V_{OV} = 0.4V$

$V_{GS} = V_{OV} + V_t = 0.4 + 0.7 = 1.1V$

5.7

p–Channel

$V_{tp} = -0.7$ V.

(a) $|v_{OV}| = 0.5$ V.

$v_{GS} = -1.2$ V. $= v_G$

(b) *for* $v_{GD} = V_{tp}, v_{DS} = v_{GS} - v_{DS}$

$= (-1.2) - (-0.5) = -0.7$ V.

$v_{DS} = v_D \leq -0.7$ V.

(c) $i_D = 1$ mA in saturation mode

$$\therefore k_p = \frac{2i_D}{(v_{GS} - v_{tp})^2} = 8\ \text{mA/V}^2$$

For $v_D = -10$ mV, ohmic mode

$$i_D = k_p\left(v_{GS} - V_{tp} - \frac{1}{2}v_{DS}\right)(v_{DS})$$

$= 39.6\ \mu$A

For $v_D = -2$ V, sat mode, $i_D = 1$ mA

5.8

$$i_D = \frac{1}{2}k'_n \frac{W}{L}(V_{GS} - V_t)^2 \quad k'_n = \mu_n C_{ox}$$

for equal drain currents :

$$\mu_n C_{ox} \frac{W_n}{L} = \mu_p C_{ox} \frac{W_p}{L} = \frac{W_p}{W_n} = \frac{\mu_n}{\mu_p}$$

$$= \frac{1}{0.4} = 2.5$$

5.9

For small $V_{DS} = i_o \simeq k'_n \frac{W}{L_1}(V_{GS} - V_t)\ V_{DS}$

$$r_{DS} = \frac{V_{DS}}{i_D} = \frac{1}{k'_n \frac{W}{L}(V_{GS} - V_t)}$$

$$= \frac{1}{50\times10^{-6}\times20\times(5-0.8)}$$

$r_{DS} = 238\ \Omega \quad V_{DS} = r_{DS}\times i_D = 238$ mV

for the same performance of a p-channel device :

$$\frac{W_p}{W_n} = \frac{\mu_n}{\mu_p} = 2.5 \Rightarrow \frac{W_p}{L} = \frac{W_n}{L}\times2.5 =$$

$$20\times2.5 \Rightarrow \frac{W_p}{L} = 50$$

5.10

$k_n' = \mu_n C_{ox} = \mu_n \frac{\epsilon_{ox}}{t_{ox}} = 650 \times 10^{-4} \times \frac{3.45 \times 10^{-11}}{20 \times 10^{-9}} = 112.1 \mu A/V^2$

a) triode region: $v_{DS} < V_{GS} - V_t$

$i_D = k_n' \frac{W}{L} \left[(v_{GS} - V_t) v_{DS} - \frac{1}{2} v_{DS}^2 \right]$

$i_D = 112.1 \times 10^{-6} \times 10 \times \left[(5 - 0.8) \times 1 - \frac{1}{2} \times 1^2 \right] = 4.15 mA$

b) edge of saturation region: $V_{DS} = V_{GS} - V_t$

$i_D = \frac{1}{2} k_n' \frac{W}{L} (V_{GS} - V_t)^2 = \frac{1}{2} \times 112.1 \times 10^{-6} \times 10 \times (1.2)^2 = 0.8 mA$

c) triode region: $v_{DS} < V_{GS} - V_t$

$i_D = 112.1 \times 10^{-6} \times 10 \left[(5 - 0.8) \times 0.2 - \frac{1}{2} \times 0.2^2 \right] = 0.92 mA$

d) Saturation region: $v_{DS} > V_{GS} - V_t$

$i_D = \frac{1}{2} \times 112.1 \times 10^{-6} \times 10 \times (5 - 0.8)^2 = 9.9 mA$

5.11

L (μm)	0.5	0.25	0.18	0.13
t_{ox} (nm)	10	5	3.6	2.6
$C_{ox}\left(\frac{fF}{\mu m^2}\right)$ $\epsilon_{ox} = 34.5$ pF/m	3.45	6.90	9.58	13.3
$k_n'\left(\frac{\mu A}{V^2}\right)$ $\mu_n = 500$ cm²/VS	173	345	479	664
$k_n\left(\frac{mA}{V^2}\right)$ for $\frac{W}{L} = 10$	1.73	3.45	4.79	6.64
A (μm²) for $\frac{W}{L} = 10$	2.50	0.625	0.324	0.169
V_{DD} (V)	5	2.5	1.8	1.3
V_t (V)	0.7	0.5	0.4	0.4
I_D (mA) for $v_{GS} = v_{DS} = v_{DD}$ $I_D = \frac{1}{2}k_n(v_{DD} - V_t)^2$	16	6.90	4.69	2.69
P (mW) $P = V_{DD} I_D$	80	17.3	8.44	3.50
$\frac{P}{A}\left(\frac{mW}{\mu m^2}\right)$	32	27.7	26.1	20.7
$\frac{\text{devices}}{\text{chip}}$	n	4n	7.72n	14.8n

$$i_D = 191.7 \times 10^{-6} \times 10\left[(5-0.7)\times 0.2 - \frac{1}{2}(0.2)^2\right]$$
$$= 1.61 \text{ mA}$$

(d) saturation region: $V_{DS} > V_{GS} - V_t$

$$i_D = \frac{1}{2} \times 191.7 \times 10^{-6} \times 10 \times (5-0.7)^2$$
$$= 17.7 \text{ mA}$$

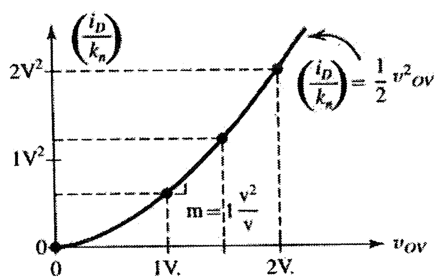
5.12

Sat mode, $\lambda = 0$

$$\left(\frac{i_D}{k_n}\right) = \frac{1}{2}v_{OV}^2$$

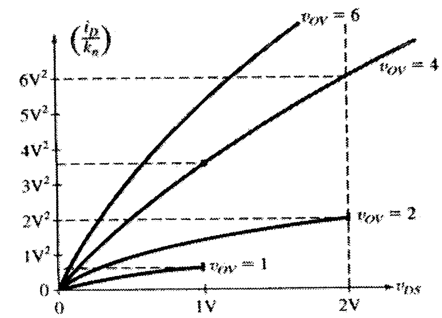
Slope at $v_{OV} = 1$ V.

$$m = 1\frac{\text{V}^2}{\text{V}}$$



Ohmic mode, $\lambda = 0$

$$\left(\frac{i_D}{k_n}\right) = v_{OV}v_{DS} - \frac{1}{2}v_{DS}^2$$



$$\left.\frac{\partial \frac{i_D}{k_n}}{\partial v_{DS}}\right|_{v_{DS}=0} = v_{OV}$$

For pmos, change

$v_{DS} \rightarrow v_{SD}$

$v_{OV} \rightarrow v_{SG} - |V_{tp}|$

5.13

$i_D = \frac{1}{2}k_n' \frac{W}{L}(V_{GS}-V_t)^2 \Rightarrow 0.2\times10^{-3} = \frac{1}{2}\times 0.1\times10^{-3}(V_{GS}-1)^2$

$V_{GS} - 1 = 2 \Rightarrow V_{GS} = 3\text{V}$

$V_{DSmin} = V_{GS} - V_t = 3 - 1 = 2\text{V}$

For $i_D = 0.8$mA: $0.8 = \frac{1}{2}\times 0.1(V_{GS}-1)^2$

$V_{GS} - 1 = 4 \Rightarrow V_{GS} = 5\text{V}$

$V_{DSmin} = V_{GS} - V_t = 5 - 1 = 4\text{V}$

5.14

$V_{GS} = V_{DS}$ indicates operation in saturation mode: $i_D = \frac{1}{2}k_n' \frac{W}{L}(V_{GS}-V_t)^2$

$$\left.\begin{array}{l} 4 = \frac{1}{2}k_n' \frac{W}{L}(5-V_t)^2 \\ 1 = \frac{1}{2}k_n' \frac{W}{L}(3-V_t)^2 \end{array}\right\} \Rightarrow 4 = \frac{(5-V_t)^2}{(3-V_t)^2} \Rightarrow$$

$(5-V_t) = 2(3-V_t) \Rightarrow V_t = 1\text{V}$, $k_n' \frac{W}{L} = 0.5 \text{ mA/V}^2$

5.15

$i_D = \frac{1}{2}k_n' \frac{W}{L}(V_{GS}-V_t)^2 \Rightarrow 0.8 = \frac{1}{2}\times 50\times 10^{-3}\frac{W}{L}(5-1)^2$

$\frac{W}{L} = 2 \Rightarrow W = 2\times 2 = 4\,\mu\text{m}$

5.16

For the channel to remain continous:

$V_{DS} \leq V_{GS} - V_t \Rightarrow V_{DSmax} = 1.5 - 0.8 = 0.7\text{V}$

5.17

$$r_{DS} = \left[k_n' \frac{W}{L} V_{OV}\right]^{-1}$$

$$= \frac{1}{50 \times \frac{100}{5}(V_{GS}-1)} \text{M}\Omega$$

$$r_{DS} = \frac{1}{V_{GS}-1} \text{k}\Omega$$

$V_{GS} = 1.1V \Rightarrow r_{DS} = 10 \text{ k}\Omega$

$V_{GS} = 11V \Rightarrow r_{DS} = 100\ \Omega$

$\Rightarrow 100\Omega \leq r_{DS} \leq 10 \text{ k}\Omega$

a) $r_{DS} \propto \frac{1}{W}$ so if W is halved, r_{DS} is doubled:

$200\ \Omega \le r_{DS} \le 20\ \text{k}\Omega$

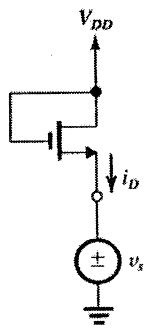
b) $r_{DS} \propto L$ so if L is halved, r_{DS} is also halved:

$50\ \Omega \le r_{DS} \le 5\ \text{k}\Omega$

c) $r_{DS} \propto \frac{L}{W}$ so if both W and L are halved, $\frac{W}{L}$ stays unchanged and so does r_{DS}.

$100\ \Omega \le r_{DS} \le 10\ \text{k}\Omega$

5.18



$v_{GD} = 0 \Rightarrow$ saturation

$i_D = \frac{1}{2}k_n(v_{GS} - V_t)^2$

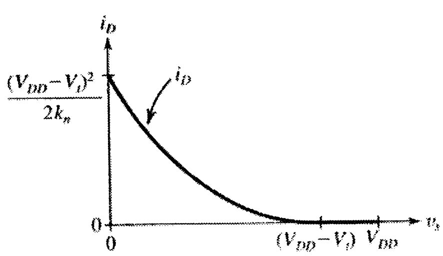
$v_{GS} = V_{DD} - v_S$

$\therefore\ i_D = \frac{1}{2}k_n\ [(V_{DD} - V_t) - v_S]^2$

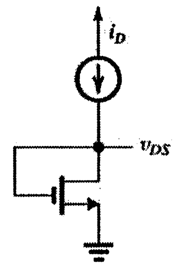
$i_D = \frac{1}{2}k_n[(V_{DD} - V_t)^2 - 2(V_{DD} - V_t)v_S + v_S^2]$

$0 \le v_S \le (V_{DD} - V_t)$

$i_D = 0,\ v_S \ge (V_{DD} - V_t)$

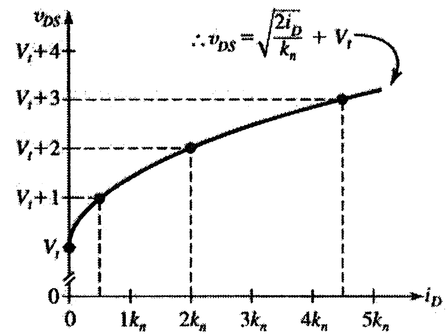


5.19



$v_{DS} = v_{GS}$

$i_D = \frac{1}{2}k_n(v_{DS} - V_t)^2$



5.20

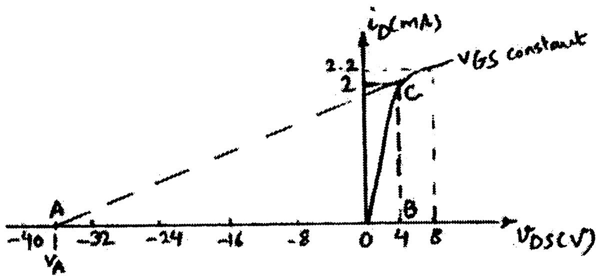
$V_{DS} = V_D - V_S \qquad V_{GS} = V_G - V_S$

$V_{OV} = V_{GS} - V_t = V_{GS} - 1.0$ According to Table 5.1, three regions are possible.

Case	V_S	V_G	V_D	V_{GS}	V_{OV}	V_{DS}	Region of Operation
a	+1.0	+1.0	+2.0	0	−1.0	+1.0	cut-off
b	+1.0	+2.5	+2.0	+1.5	+0.5	+1.0	sat.
c	+1.0	+2.5	+1.5	+1.5	+0.5	+0.5	sat.
d	+1.0	+1.5	0	+0.5	−0.5	−1.0	sat.
e	0	+2.5	1.0	+2.5	+1.5	+1.0	triode.
f	+1.0	+1.0	+1.0	0	−1.0	0	cut-off.
g	−1.0	0	0	+1.0	0	+1.0	sat.
h	−1.5	0	0	+1.5	+0.5	+1.5	sat.
i	−1.0	0	+1.0	+1.0	0	+2.0	sat.
j	+0.5	+2.0	+0.5	+1.5	+0.5	0	triode.

* with V_{DS} negative, drain and source are reversed to show the device is in the saturation region.

5.21



$$r_o = \frac{\Delta V_{DS}}{\Delta i_D}\Bigg|_{V_{GS}\,\text{Const}} = \frac{8-4}{2.2-2} = 20\text{K}\Omega$$

To calculate V_A, consider the ABC triangle:

$V_A + 4 = 2\text{mA} \times r_o = 2 \times 20 = 40\text{V} \Rightarrow V_A = 36\text{V}$

$\lambda = \frac{1}{V_A} = 0.028\ V^{-1}$

5.22

$\lambda = 0.02\ v^{-1} \Rightarrow v_A = 50\text{V}$ for

$L = 1\ \mu\text{m}$

$V_A = v'_A L \Rightarrow v'_A = 50\text{V}$

for $L = 3\ \mu\text{m}$: $V_A = 50 \times 3 = 150\text{V}$

$$r_O = \frac{V_A}{I_D} = \frac{150}{0.08} = 1875\ \text{k}\Omega$$

$$r_O = \frac{\Delta v_{DS}}{\Delta i_D} \Rightarrow \Delta i_D$$

$$= \frac{\Delta v_{DS}}{r_o} = \frac{5-1}{1875} = 2.13\ \mu\text{A}$$

for V_{DS} raised from 1V to 5V, i_D increases from 80 μA to 82.13 μA.

$\frac{\Delta i_D}{i_D} = 2.7\%$ change in i_D.

In order to reduce $\frac{\Delta i_D}{i_D}$ by a factor of 2, Δi_D has to be halved, or equivalently r_o has to be doubled. In order to double r_o, V_A has to be doubled and this can be done by doubling the length. $L = 2 \times 3 = 6\ \mu\text{m}$

5.23

original

$$r_O = \left[\lambda \frac{1}{2} k_n' \frac{W}{L}(V_{GS} - V_t)^2\right]^{-1}$$

$$= \left[\frac{1}{2}\lambda k_n' \frac{W}{L}(V_{ov})^2\right]^{-1}$$

$$\text{new } r_O = \left[\frac{1}{2}\lambda k_n' \frac{4W}{4L}\left(\frac{1}{2}V_{ov}\right)^2\right]^{-1} = 4r_O$$

Note that quadrupling W and L had no effect, but decreasing the overdrive voltage by half increased the output resistance by a factor of 4.

5.24

MOS	1	2	3	4
λ (V^{-1})	0.02	0.01	0.1	0.005
V_A (V)	50	100	10	200
I_D (mA)	5	3.33	0.1	0.2
r_o (kΩ)	10	30	100	1000

$r_o = \frac{V_A}{I_D}$, $\lambda = \frac{1}{V_A}$

5.25

$v_{GS} = -3\text{V}\quad v_{SG} = 3\text{V}\quad V_t = -1\text{V}$

$v_{DS} = -4\text{V}\quad v_{SD} = 4\text{V}\quad V_A = -50\text{V}$

$\lambda = -0.02\text{V}^{-1}$

$$i_D = \frac{1}{2}k_p' \frac{W}{L}(V_{GS} - V_t)^2(1 + \lambda V_{DS})$$

$$3 = \frac{1}{2}k_p' \frac{W}{L}(-3+1)^2(1 + 0.02 \times 4)$$

$$= 2.16 k_p' \frac{W}{L}$$

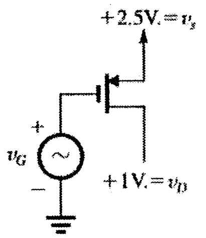
$$k_p' \frac{W}{L} = 1.39\text{mA}/\text{V}^2$$

5.26

	V_s	V_G	V_D	V_{SG}	$\|V_{ov}\|$	V_{SD}	Region of Operation
a	+2	+2	0	OV.	OV.	2V.	cutoff
b	+2	+1	0	+1V.	OV.	2V.	cutoff/sat
c	+2	0	0	+2V.	1V.	2V.	Sat
d	+2	0	+1	+2V.	1V.	1V.	Sat/ohmic
e	+2	0	+1.5	+2V.	1V.	0.5V	ohmic
f	+2	0	+2	+2V.	1V.	OV.	ohmic

pmos $V_{tp} = -1V$,

5.27



pmos

$V_{tp} = -0.5V.$

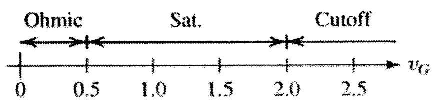
$v_{SD} = 1.5V.$

$v_{GS} \geq V_{tp} \Rightarrow$ Cutoff

$\therefore\ v_G \geq 2.0V. \Rightarrow$ Cutoff

$v_{GD} \leq V_{tp} \Rightarrow$ ohmic

$\therefore v_{GD} \leq\ +0.5V \Rightarrow$ ohmic



5.28

$$\frac{\Delta i_D}{I_D} = \frac{\left.\frac{\partial i_D}{\partial k_n'}\right|_{ID}\frac{dk_n'}{dT}\Delta T + \left.\frac{\partial i_D}{\partial V_t}\right|_{I_D}\frac{dV_t}{dT}\Delta T}{\left.\left[\frac{1}{2}k_n'\frac{W}{L}(v_{ES} - V_t)^2\right]\right|_{I_D}}$$

(a) $\frac{\Delta i_D}{I_D} = \frac{1}{k_n'}\frac{dk_n'}{dT}\Delta T + \frac{-2}{(V_{GS} - V_t)}\frac{dV_t}{dT}\Delta T$

(b)

$$\frac{\left(\frac{\Delta i_D}{I_D}\right)}{\Delta T} = -\frac{0.002}{C^\circ} = \frac{1}{k_n'}\frac{dk_n'}{dT} - \left(\frac{2}{4V}\right)\left(\frac{-2mV}{C^\circ}\right)$$

for $V_t = +1$ V, $V_{GS} = 5$ V, $V_{ov} = 4$ V,

$$\therefore\ \frac{\left(\frac{dk_n'}{dT}\right)}{k_n'} = -0.003 / C^\circ\ (-0.3\% C^\circ)$$

5.29

a) $I_D = \frac{1}{2}k_n'\frac{W}{L}(V_{GS}-V_t)^2 \Rightarrow 2 = \frac{1}{2}k_n'\frac{W}{L}(3-1)^2$

$\Rightarrow k_n'\frac{W}{L} = 1\,mA/V^2$

$V_1 = V_{DS} = 3V$

b) $V_2 = V_S = V_D - V_{DS} = 1-3 = -2^V$

c) $V_3 = V_S = V_D - V_{DS} = 0-(-3) = 3^V$

d) $V_4 = V_D = V_S + V_{DS} = 5+(-3) = 2^V$

In order to calculate R_{max} that can be inserted in series with the drain, V_{DS} has to be equal to $V_{GS}-V_t$, so that the device is operating on the edge of saturation: $|V_{DS}| = 3-1 = 2^V$. Note that since i_D is the same, V_{GS} stays the same.

a) $R_{Dmax} = \frac{3-2}{2^{mA}} = 0.5K\Omega$

b) $V_2 = -2^V \Rightarrow V_D = -2+2 = 0 \Rightarrow R_{Dmax} = \frac{1}{2} = 0.5^{K\Omega}$

Note that V_2 is fixed through $V_{GS} = 3V$.

c) $V_{GS} = -3V \Rightarrow V_S = V_3 = 3^V$. Now for V_{DS} to be $-2V$, V_D has to be 1^V.

$R_{DMax} = \frac{1V}{2^{mA}} = 0.5k\Omega$

d) $V_{GS} = -3V \Rightarrow V_G = V_4 = 2^V$. Adding the resistor between V_4 and drain means that V_D has to be $5-2 = 3^V$ and this leaves 1^V voltage drop on the resistor: $R_{Dmax} = \frac{1}{2} = 0.5k\Omega$

In order to calculate the largest resistor added to the gates, note that since the gate doesn't draw any current, the value of the resistor is immaterial.

Now we calculate R_{Smax}, assuming that the voltage drop across the current source is at least 2^V:

a) $V_1 = 8^V$ then $V_{GS} = 3^V \Rightarrow V_S = 8-3 = 5^V$

$R_{SMax} = \frac{5}{2} = 2.5k\Omega$

b) $V_2 = -9+2 = -7^V$, $V_S = 1-|V_{GS}| = -2^V$

$R_{Smax} = \frac{-2-(-7)}{2} = 2.5k\Omega$

c) $V_3 = 10-2 = 8^V$, $V_S = 0+|V_{GS}| = 3^V$

$R_{Smax} = \frac{8-3}{2} = 2.5k\Omega$

d) $V_4 = -5+2 = -3^V$; $V_S = -3+|V_{GS}| = 0^V$

$R_{Smax} = \frac{5}{2} = 2.5k\Omega$

5.30

$I_D = \frac{V_{DD}-V_D}{R_D} \Rightarrow \frac{5-0}{R_D} = 1^{mA} \Rightarrow R_D = 5k\Omega$

$V_D = V_G \Rightarrow$ saturation

therefore: $i_D = \frac{1}{2}k_n'\frac{W}{L}(V_{GS}-V_t)^2$

$1 = \frac{1}{2}\times 60\times 10^{-3}\times\frac{100}{3}(V_{GS}-1)^2$

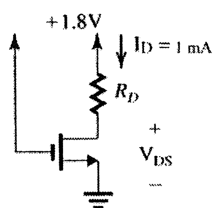
$\Rightarrow V_{GS} = 2V \Rightarrow V_S = -2^V$

$R_S = \frac{-2-(-5)}{1} = 3k\Omega$

5.31

$I_D = 1$ mA, $V_t = 0.5$ V, $V_{DD} = 1.8$ V.

To operate at the edge of saturation, V_{DS} must equal V_{ov}



$V_{GS} = V_G - V_S = 1.8 - 0 = 1.8$ V

$V_{OV} = V_{GS} - V_t = 1.8 - 0.5 = +1.3$ V

with $V_{DS} = V_{OV} = 1.3$ V,

$$R_D = \frac{V_{DD}-V_D}{I_D} = \frac{1.8-1.3}{1\text{ mA}} = 500\ \Omega$$

5.32

$R = \frac{3.5}{0.115} = 3.04\,k\Omega$

$0.115\,mA = \frac{1}{2} \times 60 \times 10^{-3} \times \frac{W}{0.8}(-1.5 - (-0.7))^2 \Rightarrow W = 4.8\,\mu m$

5.33

$V_{GS1} = 1.5V$, $120\,\mu A = \frac{1}{2} \times 120 \times \frac{W_1}{1}(1.5 - 1)^2$

$\Rightarrow W_1 = 8\,\mu m$

$V_{GS2} = 3.5 - 1.5 = 2^V$, $120 = \frac{1}{2} \times 120 \frac{W_2}{1}(2-1)^2$

$\Rightarrow W_2 = 2\,\mu m$

$R = \frac{5 - 3.5}{0.120} = 12.5\,k\Omega$

5.34

$V_{GS1} = 1.5\text{ V}$

$120\ \mu\text{A} = \frac{1}{2} \times 120 \times \frac{W_1}{1}(1.5 - 1)^{\frac{1}{2}}$

$W_1 = 8\ \mu\text{m}$

$V_{GS2} = 2\text{ V}$

$120\ \mu\text{A} = \frac{1}{2} \times \frac{W_2{}^2}{1}(2 - 1)$

$W_2 = 2\ \mu\text{m}$

$V_{GS3} = 1.5\text{V}$

$W_3 = 8\ \mu\text{m}$

5.35

$V_I = V_{GS} = 5V$, $V_O = V_{DS} = 0.05\,V$

$r_{DS} = 50\Omega = \frac{V_{DS}}{I_D} \Rightarrow I_D = \frac{0.05}{50} = 0.001A = 1\,mA$

$R = \frac{V_{DD} - V_O}{I_D} = \frac{5 - 0.05}{1} = 4.95\,k\Omega$

$V_{DS} < V_{GS} - V_t \Rightarrow$ triode region

$I_D = k_n' \frac{W}{L}\left[(V_{GS} - V_t)V_{DS} - \frac{V_{DS}^2}{2}\right]$

$1 = 100 \times 10^{-3} \frac{W}{L}\left[(5-1) \times 0.05 - \frac{0.05^2}{2}\right] \Rightarrow \frac{W}{L} = 50$

5.36

In circuit a: $V_2 = 10 - 4 \times 2 = 2\text{ V}$

assume saturation:

$I_D = 2 = \frac{1}{2} \times 1 \times (V_{GS} - 2)^2$

$= V_{GS} = 4\text{V}$

$\Rightarrow V_1 = -4\text{V}$, $V_{DS} = 6\text{V} > V_{GS} - V_t$

so our assumption was correct.

In circuit b:

$I_D = 1 = \frac{1}{2} \times 1 \times (V_{GS} - 2)^2 \Rightarrow$

$V_{GS} = 3.41\text{ V}$ $V_3 = 3.41\text{ V}$

In circuit c:

$I_D = 2\text{ mA} \Rightarrow V_{GS} = -4\text{ V} \Rightarrow V_S$

$= 4\text{ V} = V_4$

$V_5 = -10 \times 2.5 \times 2 = -5\text{ V}$

In circuit d:

$I_D = 2\text{ mA} \Rightarrow V_{GS} = -4\text{ V} \Rightarrow V_6 = 6\text{ V}$

$\Rightarrow V_7 = V_6 - 4 = 2\text{ V}$

If we replace the current source with a resistor in each of those circuits:

In circuit a:

$R = \frac{-4 - (-10)}{2} \simeq 3.01\text{ k}\Omega$

(by looking at the table for 1% resistors)

now recalculate I_D: $I_D = \frac{1}{2} \times 1 \times (V_{GS} - V_t)^2$

$V_{GS} - V_t = 0 - (-10 + 3.01 I_D) - 2$

$= 8 - 3.01 I_D \Rightarrow$

$2 I_D = (8 - 3.01 I_D)^2 \Rightarrow I_D$

$= 1.99\text{ mA} \Rightarrow V_2 = 2.04\text{ V}$

$V_1 = -4.01\ \text{V}$

In circuit b:

$$R = \frac{10-3.41}{1} = 6.59\ \text{k} \simeq 6.65\ k\Omega$$

then

$$V_{GS} = 10 - 6.65I$$

$$= \frac{1}{2}\times 1(10-6.65I-2)^2 \Rightarrow I = 0.99\ \text{mA}$$

$$V_3 = 10 - 6.65\times 0.99 = 3.41\ \text{V}$$

In circuit c:

$$R = \frac{10-4}{2} \simeq 3.01\ \text{k}\Omega,$$

$$V_{GS} = -(10+3.01I)$$

$$I = \frac{1}{2}\times 1\times(-10+3.01I+2)^2$$

$$I_D = 1.99\ \text{mA}$$

$$V_4 = 10 - 3.01\times 1.99 = 4.01\ \text{V}$$

$$V_5 = -10 + 2.5\ \text{k}\times 1.99 = -5.03\ \text{V}$$

In circuit d:

$R = \frac{2}{2} = 1\text{k}$ so V_7 is still 2 V.

5.37

a) $V_{GS} = -V_1$. $10\mu A = \frac{1}{2}\times 0.4\times 10^3 (V_{GS}-1)^2 \Rightarrow$
$V_{GS} = 1.22^V \Rightarrow V_1 = -1.22^V$

b) $100\mu A = \frac{1}{2}\times 0.4\times 10^3 \times (V_{GS}-1)^2 \Rightarrow V_{GS} = 1.71^V,\ V_2 = -1.71^V$

c) $1 = \frac{1}{2}\times 0.4\times (V_{GS}-1)^2 \Rightarrow V_{GS} = 3.23^V \Rightarrow V_3 = -3.23^V$

d) $10 = \frac{1}{2}\times 0.4\times 10^3 (V_{GS}-1)^2 \Rightarrow V_{GS} = 1.22^V \Rightarrow V_4 = 1.22^V$

e) $1 = \frac{1}{2}\times 0.4 (V_{GS}-V_t)^2 \Rightarrow V_{GS} = 3.24^V \Rightarrow V_5 = 3.24^V$

f) $I = \frac{1}{2}\times 0.4\times (5-100I-1)^2 \Rightarrow I = 0.045\text{mA},\ 0.036\text{mA}$
$V_6 = 5 - 100\times 0.036 = 1.4^V$

g) $I = \frac{1}{2}\times 0.4\times (5-1\times I-1)^2 \Rightarrow I = 1.38\ \text{mA}$
$V_7 = 5 - 1.38\times 1 = 3.62^V$

h) $I = \frac{1}{2}\times 0.4\times (5-100-I)^2 \Rightarrow I = \cancel{0.045^{mA}},\ 0.036^{mA}$
$V_8 = -5 + 100\times 0.036 = -1.4^V$

Note that $I = 0.045^{mA}$ in circuits h and f is not acceptable, because it results in $V_{GS} < V_t$ that is not physically possible.

5.38

a) $V_{GS2} = -V_2$, $I = \frac{V_2-(-5)}{1K} = \frac{1}{2}\times 2\times(-V_2-1)^2$
$\Rightarrow V_2 + 5 = V_2^2 + 2V_2 + 1 \Rightarrow V_2^2 + V_2 - 4 = 0 \Rightarrow$ ✗ $V_2 = 1.55^V$
$V_2 = -2.56^V$

$V_2 = 1.55^V$ is not acceptable because it results in $V_{GS} < 0$ that is not possible for an NMOS. Therefore $V_2 = -2.56\text{V}$

$i_{D1} = i_{D2} \Rightarrow \frac{V_2-(-5)}{1K} = \frac{1}{2}\times 2(5-V_1-1)^2 \Rightarrow$
$2.44 = (4-V_1)^2 \Rightarrow 4 - V_1 = \pm 1.56^V \Rightarrow V_1 = 2.44^V$
$V_1 = 5.56^V$ ✗

The second answer results in $V_{GS} = 5 - 5.56 < 0$ which is not acceptable. Therefore $V_1 = 2.44^V$

b) $\frac{10-V_3}{1K} = \frac{V_5}{1K} = i_D \Rightarrow 10 - V_3 = V_5$ ①
$i_{D1} = \frac{V_5}{1K} = \frac{1}{2}\times 2\times(V_3-V_4-1)^2 \Rightarrow V_5 = (V_3-V_4-1)^2$ ②
$i_{D2} = V_5/_{1K} = \frac{1}{2}\times 2\times(V_4-V_5-1)^2 \Rightarrow V_5 = (V_4-V_5-1)^2$ ③
②,③ $\Rightarrow V_3 - V_4 - 1 = V_4 - V_5 - 1 \Rightarrow V_5 = 2V_4 - V_3$ ④
①,④ $\Rightarrow 2V_4 - V_3 = 10 - V_3 \Rightarrow V_4 = 5^V$
③ $\Rightarrow V_5 = (4-V_5)^2 \Rightarrow V_5^2 - 9V_5 + 16 = 0 \Rightarrow V_5 = 6.55^V$ ✗
$V_5 = 2.45^V$

$V_5 = 6.55$ results in $i_D = 6.55\text{mA}$, $V_3 = 4.45^V$ and thisisnot physicly possible. So $V_5 = 2.45^V$
$V_3 = 10 - 2.45 = 7.55\ \text{V}$

5.39

The PMOS transistor operates in saturation region if

$V_{SD} \geq V_{SG} - |V_t|$

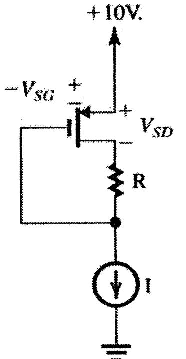
or

$V_{SD} \geq V_{SG} - 1$

Also, $V_{SD} + IR = V_{SG} \Rightarrow V_{SD}$

$= V_{SG} - IR$

$\Rightarrow IR \leq |V_t|$ for PMOS to be in saturation.



a) $R = 0 \Rightarrow IR = 0 < |V_t|$

saturation:

$I = 100 = \frac{1}{2} \times 8 \times 2.5$

$\times (V_{SG} - |V_t|^2)$

$V_{SG} - 1 = \pm 1 \Rightarrow V_{SG} = 2\ \text{V}$

$= V_{SD}$

b)

$R = 10\ \text{k}\Omega = IR = 10 \times 0.1 = 1\ \text{V} \Rightarrow$ saturation

$V_{SG} = 2\ \text{V} \Rightarrow V_{SD} = 2 - 1 = 1\ \text{V}$

c) $R = 30\ \text{k}\Omega \Rightarrow \text{IR} = 30 \times 0.1$

$= 3\ \text{V} \Rightarrow$ triode region

$100 = 8 \times 25$

$\left[(V_{SG} - |V_t|)V_{SD} - \frac{1}{2}V_{SD}^2\right]$

$0.5 = \left[(V_{SG} - 1)(V_{SG} - 3) - \frac{1}{2}(V_{SG} - 3)^2\right]$

$0.5 = 0.5V_{SG}^2 - V_{SG} - 1.5$

$\Rightarrow V_{SG}^2 - 2V_{SG} - 4 = 0$

$V_{SG} = 3.24\text{V}, -1.2\ \text{V}$ X

$V_{SD} = 3.24 - 3 = 0.24\ \text{V}$

$100\ \text{k}\Omega \Rightarrow \text{IR} = 100 \times 0.1$

$= 10\ \text{V} \Rightarrow$ triode region

$100 = 8 \times 25$ X

5.40

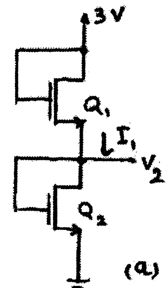
a) Q_2, Q_1 operating in Saturation: $i_{D1} = i_{D2}$

$\Rightarrow V_{GS1} = V_{GS2}$

$3V = V_{GS1} + V_{GS2} \Rightarrow V_{GS1} = V_{GS2} = 1.5^V$

$V_2 = 1.5^V$

$I_1 = \frac{1}{2} \times 20 \times \frac{30}{10}(1.5 - 1)^2 = 7.5\mu A$



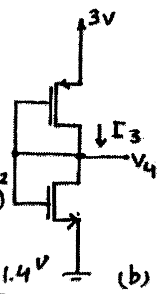
b) Both transistors have $V_D = V_G$ and therefore they are operating in Saturation: $i_{D_1} = i_{D2}$

$\frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_4 - 1)^2 = \frac{1}{2}\mu_p C_{ox}\frac{W}{L}(3 - V_4 - 1)^2$

$2.5(V_4 - 1)^2 = (2 - V_4)^2$

$1.58(V_4 - 1) = (2 - V_4) \Rightarrow V_4 = 1.39\ V \simeq 1.4^V$

$I_3 = \frac{1}{2} \times 20 \times \frac{30}{10}(1.39 - 1)^2 = 4.6\mu A$



c) $\frac{W_1}{L_1} = \frac{75}{10} = 7.5$

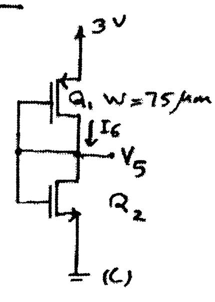
$\frac{W_2}{L_2} = \frac{30}{10} = 3$ $\quad \frac{W_1/L_1}{W_2/L_2} = 2.5$

$i_{D1} = i_{D2}$

Since $\mu_n C_{ox}\frac{W_2}{L_2} = \mu_p C_{ox}\frac{W_1}{L_1}$

$\Rightarrow V_{GS_1} = V_{GS2} = \frac{3}{2} = 1.5V = V_5$

$I_6 = \frac{1}{2} \times 20 \times \frac{30}{10}(1.5 - 1)^2 = 7.5\mu A$



5.41

Since $V_{G1} = V_{D1}$ then Q_1 is in saturation. We assume that Q_2 is also in saturation, then because $I_{D1} = I_{D2}$, V_{GS} would be equal to V_{GS2}.

$$V_{GS1} = V_{GS2} = \frac{5}{2} = 2.5 \text{ V}$$

$$I_1 = \frac{1}{2} \times 50 \times \frac{10}{1}(2.5 - 1)^2 = 562.5 \ \mu\text{A}$$

$V_{GS3} = V_{GS1} = 2.5$ V. Since Q_3 and Q_4 have the same drain current, then

$V_{GS3} = V_{GS4} = 2.5$ V. This is based on the assumption that Q_3 & Q_4 are saturated:

$$V_{GS3} = V_{GS1} \Rightarrow I_2 = I_{GS3} = I_{GS1}$$

$$= 562.5 \ \mu\text{A}$$

$$V_2 = 5 - 2.5 = 2.5 \text{ V}$$

Now if Q_3 and Q_4 have $W = 100$ μm then:

$$I_2 = \frac{1}{2} \times 50 \times \frac{100}{1}(2.5 - 1)^2 = 5.625 \text{ mA}$$

or

$$\frac{I_{Q3}}{I_{Q1}} = \frac{W_3}{W_1} = \frac{100}{10} \Rightarrow I_{Q3} =$$

$$10 \times 562.5 \ \mu\text{A} = 5.625 \text{ mA}$$

5.42

Part a

Find the R_D corresponding to point B, which is the saturation-triode boundary with $V_{DS,B} = 0.5$ Volts

Also on the boundary

$$i_{D,B} = \frac{k'_n \frac{W}{L} v^2_{DS,B}}{2}$$

$$5 \ \frac{(0.25x10^{-3})(40)(0.5)^2}{2} \ 5 \ 1.25 \text{ mA}$$

$$R_D \ 5 \ \frac{2.5 - 0.5}{1.25x10^{-3}} \ 5 \ 1600 \ \Omega$$

Part b

Find v_{GS} corresponding to point B.

$V_{DS,B} = 0.5 \Rightarrow V_{GS,B} = V_{GD,S} + V_{DS,B} = V_t + V_{DS,B} =$
$0.5 + 0.5 = 1.0$ Volts

Part c

Find $V_{DS,c}$ corresponding to point C, where $v_{GS,C} = 2.5$ Volts and the transistor is in the triode region$v$

$$V_{DS,C} + R_D\left[k'_n\frac{W}{L}\left((v_{GS,C} - V_t)v_{DS,C} - \frac{v^2_{DS,C}}{2}\right)\right]$$

$$= V_{DD} \Rightarrow V_{DS,C} + 1600$$

$$\left[(0.25 \ x \ 10^{-3})40((2.5 - 0.5) - 0.5v_{DS,C} - \frac{v^2_{DS,C}}{2}\right]$$

The roots of this equation are 0.07720 & 4.04778

Clearly the $v_{ds} \approx 0.07720$ is the choice because the other one is above V_{DD}.

The current, i_D, corresponding to point C, $i_{D\,C}$, is

$$i_{D,C} = \frac{V_{DD} - V_{DS,C}}{R_D}$$

$$= \frac{2.5 - 0.07720}{1600} = 1.514\text{mA}$$

An equivalent resistor value can now be calculated at point C

$$R_{\text{equivalent}} = \frac{V_{DS,C}}{i_{DS,C}} = \frac{0.07720}{1.514 \times 10^{-3}} = 50.98\,\Omega$$

This can be compared to the value of r_{DS}, which is really derived for $v_{DS} = 0$.

$$r_{DS} = \frac{1}{k'_n\frac{W}{L}(V_{GS} - V_t)}$$

$$= \frac{1}{(0.25 \times 10^{-3})(40)(2.5 - 0.5)} = 50$$

The value is close to the equivalent resistor value, but they are not exactly equal.

Part d

$V_{GS} = 0.8$, so the transistor is in saturation.

Find V_{DS}.

$$V_{DS} + R_D\left[k'_n\frac{W}{L}\frac{(v_{GS} - V_t)^2}{2}\right] = V_{DD} \Rightarrow V_{DS}$$

$$+ \ 1600\left[\frac{(0.25x\,10^{-3})40(0.8 - 0.5)^2}{2}\right]$$

$$= 2.5$$

$$V_{DS} \approx 1.78\text{Volts}$$

The voltage gain is

$$A_v = -k'_n\frac{W}{L}(V_{GS} - V_t)R_D = -(0.25x\,10^{-3})$$

$$(40)(0.8 - 0.5)(1600) = -4.8$$

5.43

a)

Point A : $V_{IA} = V_t = 1^V$, $V_{OA} = V_{DD} = 5^V$

For $V_i < V_t$, the transistor is not on, $V_{GS} < V_t$.
Point A is when $V_{GS} = V_t$ and the transistor turns on. As V_i increases, the i_D increases and v_o decreases. v_o decreases to the point that it is below V_I by V_t volts. At this point, B, the MOSFET enters the triode region: $V_{OB} = V_{IB} - V_t$ or $V_{DS} = V_{GS} - V_t$. So at point B: $I = \frac{V_{DD} - V_{OB}}{R}$

$$I = \frac{V_{DD} - (V_{GS} - V_t)}{R} = \frac{1}{2} \times k_n' \frac{W}{L} (V_{GS} - V_t)^2$$

$$\frac{5 - V_{GS} + 1}{24} = \frac{1}{2} \times 1 \times (V_{GS} - 1)^2 \Rightarrow 12V_{GS}^2 - 23V_{GS} + 6 = 0$$

$V_{GS} = 1.61^V \Rightarrow V_I = 1.61^V$ $\quad V_O = 1.61 - 1 = 0.61^V$

Point B: $\underline{V_{OB} = 0.61^V}$ $\quad \underline{V_{IB} = 1.61^V}$

b) $I_Q = \frac{1}{2} \times 1 \times 0.5^2 = \underline{0.125\,mA}$

$V_{OQ} = 5 - 24 \times 0.125 = \underline{2^V}$

$V_{IQ} = V_{GS} = V_{OV} + V_t = 0.5 + 1 = \underline{1.5^V}$

Now to calculate the incremental gain A_v at this bias point, From equation 4.41, we have: $A_v = -2\,V_{RD}/V_{OV} = \frac{-2(V_{DD} - V_{OQ})}{V_{OV}}$

$$A_v = -\frac{2(5-2)}{0.5} = -12\,V/V$$

c) $V_{IQ} = 1.5^V$, $V_t = 1^V$, $V_{IB} = 1.61^V$. Thus the largest amplitude of a sine wave that can be applied to the input while the transistor remain in saturation is: $\quad 1.61 - 1.5 = 0.11V$

The amplitude of the output voltage signal that results is approximately equal to $V_{OQ} - V_{OB} = 2 - 0.61 = 1.39^V$. The gain implied by this amplitudes is:

$$\text{gain} = -\frac{1.39}{0.11} = 12.64\,V/V$$

This gain is 5.3% different from the incremental gain calculated in part (b). This difference is due to the fact that the segment of the voltage transfer curve considered here is not perfectly linear.

5.44

$R_D = 20\,k\Omega$, $V_{RD} = 2^V \Rightarrow I_D = 0.1\,mA$

$A_v = -\frac{2V_{RD}}{V_{OV}} \Rightarrow -10 = -\frac{2 \times 2}{V_{OV}} \Rightarrow V_{OV} = 0.4^V$

$V_{GS} = 1.2V \Rightarrow V_t = 1.2 - 0.4 = \underline{0.8V}$

$I_D = \frac{1}{2} k_n' \frac{W}{L} V_{OV}^2 \Rightarrow 0.1 = \frac{1}{2} \times 50 \times 10^{-3} \frac{W}{L}\, 0.4^2$

$\Rightarrow \underline{\frac{W}{L} = 25}$

5.55 the maximum gain achievable is:

$$|A_{v\max}| = \frac{V_{DD}}{(V_{OV}/2)} = \frac{5}{(0.2/2)} = 50\ V/V$$

the gain is maximum when V_{ov} is minimum ($= 0.2$ V) and when the drop across R_D ($= I_D R_D$) is largest possible, which occurs when we operate closest to point B

At B: $V_{DS}| = V_{GS}|_B - V_t = V_{OV}$

$V_{DS}|_B = 0.2$

to allow for ± 0.5 V swing

$V_{DS} = 0.2 + 0.5 = 0.7$ V

$$\rightarrow |A_v| = \frac{(5 - 0.7)}{0.2/2} = 43\ V/V$$

$\Delta V_i \times 43 = \Delta V_O$

$$\Delta V_i = \frac{\pm 0.5}{43} = \pm 11.6\ mV$$

c) If $I_D = 100\ \mu A$,

$k_n' = 100\ \mu A/V^2 \Rightarrow \frac{W}{L} = ?$

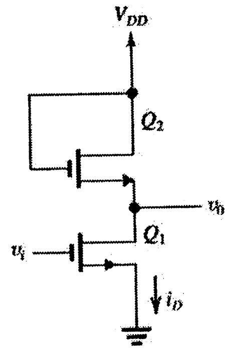
In saturation: $I_D = \frac{1}{2} k_n' \frac{W}{L} V_{OV}^2 \Rightarrow \frac{W}{L} = \frac{2I_D}{k_n' V_{OV}^2}$

$$\frac{W}{L} = \frac{2 \times 100\ \mu}{100\ \mu \times (0.2)^2} = 50$$

(d) $V_{DD} - I_D R_D = 0.7$

$5 - 100\ \mu \cdot R_D = 0.7 \Rightarrow R_D = 43\ k\Omega$

5.46



given $V_{t1} = V_{t2} = V_t$

for Q_2 $i_D = \frac{1}{2}k_n'\left(\frac{W}{L}\right)_2 [V_{DD} - V_o - V_t]^2$

for Q_1 $i_D = \frac{1}{2}k_n'\left(\frac{W}{L}\right)_1 [V_i - V_t]^2$

for $V_t \le V_i \le V_o + V_t$

equate i_{D1} and i_{D2}

$$\left(\frac{W}{L}\right)_2 [V_{DD} - V_o - V_t]^2 = \left(\frac{W}{L}\right)_1 [V_i - V_t]^2$$

$$[V_{DD} - V_o - V_t] = \sqrt{\frac{(W/L)_1}{(W/L)_2}} \cdot [V_i - V_t]$$

$$V_o = V_{DD} - V_t + V_t\sqrt{\frac{(W/L)_1}{(W/L)_2}}$$

$$-V_i\sqrt{\frac{(W/L)_1}{(W/L)_2}}$$

for $\sqrt{\frac{(W/L)_1}{(W/L)_2}} = \sqrt{\frac{\left(\frac{50}{0.5}\right)}{\left(\frac{5}{0.5}\right)}} = \sqrt{10}$

$A_v = -\sqrt{10} = -3.16$

5.47

$I_D = \frac{1}{2} k_n' \frac{W}{L} V_{ov}^2 \Rightarrow I_D = \frac{1}{2} \times 2 \times 1^2 = 1\,mA$

$i_D = \frac{1}{2} \times 2 \times (1+0.1)^2 = 1.21\,mA \quad (v_{gs} = 0.1V)$

$i_D = 1.21 - 1 = 0.21\,mA$

If $v_{gs} = -0.1^V \Rightarrow i_D = \frac{1}{2} \times 2(1-0.1)^2 = 0.81\,mA$

$i_D = 0.81 - 1 = -0.19\,mA$

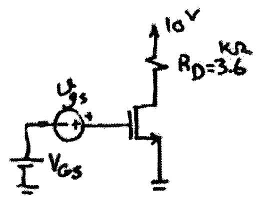
For positive increment : $g_m = \frac{\Delta i_D}{\Delta v_{gs}} = \frac{0.21}{0.1} = 2.1\,mA/V$

For negative increment : $g_m = \frac{0.19}{0.1} = 1.9\,mA/V$

An estimate of $g_m = \frac{2.1+1.9}{2} = 2\,mA/V$

$g_m = k_n' \frac{W}{L} V_{ov} = 2 \times 1 = 2\,mA/V$. same as estimate.

5.48



a) $I_D = \frac{1}{2} \times 1 \times (4-2)^2 = 2\,mA$

$V_D = V_{DD} - R_D I_D = 10 - 2 \times 3.6$

$V_D = 2.8V$

b) $g_m = k_n' \frac{W}{L} V_{ov} = 1 \times (4-2) = 2\,mA/V$

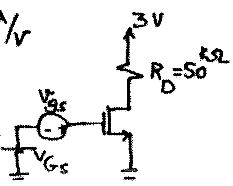
c) $A_v = \frac{v_d}{v_{gs}} = -g_m R_D = -2 \times 3.6 = -7.2\,V/V$

d) $r_o \simeq \frac{1}{\lambda I_D} = \frac{1}{0.01 \times 2} = 50\,k\Omega$

$A_v = \frac{v_d}{v_{gs}} = -g_m (R_D \| r_o) = -2(3.6 \| 50) = -6.7\,V/V$

5.49

$g_m R_D = 5 \Rightarrow g_m = \frac{5}{50} = 0.1\,mA/V$



For 0.5V output signal and a gain of $5V/V$, $v_{gs} = \frac{0.5}{5} = 0.1^V$

So we can write $V_{DS} - 0.5 \geq V_{GS} + 0.1 - V_t$

or $V_{DS} \geq V_{GS} + 0.6 - 0.8 \Rightarrow V_{DS} \geq V_{GS} - 0.2$

Also, from the other side: $V_{DS} + 0.5 \leq V_{DD}$

or $V_{DS} \leq 3 - 0.5 \Rightarrow V_{DS} \leq 2.5V$

We design the circuit for lowest possible V_{DS} that guarantees the device operation in saturation: $V_{DS} = V_{GS} - 0.2$

$V_{DS} = V_{DD} - R_D I_D \Rightarrow V_{GS} - 0.2 = 3 - 50 \times I_D$

$\Rightarrow I_D = \frac{3.2 - V_{GS}}{50}$

Also, from eq. 4.71: $g_m = \frac{2 I_D}{V_{GS} - V_t} = 0.1$

$0.1 = \frac{2}{V_{GS} - 0.8} \times \frac{3.2 - V_{GS}}{50}$

$\Rightarrow V_{GS} = 1.49^V$, $\underline{I_D = 0.034\,mA}$

$V_{DS} = 1.49 - 0.2 = 1.29^V$ $\quad$ $V_{OV} = 1.49 - 0.8 = \underline{0.69^V}$

$\frac{W}{L} = \frac{I_D}{\frac{1}{2} k'_n V_{OV}^2} = \frac{0.034 \times 10^3}{\frac{1}{2} \times 100 \times 0.69^2} = 1.43$

$\underline{\frac{W}{L} = 1.43}$

5.50

$A_v = -g_m R_D$, $\quad g_m = \frac{2 I_D}{V_{OV}}$ eq. 4.71 $\Big\} \Rightarrow A_v = -\frac{2 R_D I_D}{V_{OV}} = -\frac{2(V_{DD} - V_D)}{V_{OV}}$ ①

Minimum v_{DS} for edge of saturation:

$v_{DS} \geq v_{GS} - V_t$ or $v_{DSmin} = v_{GSmax} - V_t$

$V_{DS} - |A_v| \hat{v}_i = V_{GS} + \hat{v}_i - V_t$

If we replace A_v with ①:

$V_D - \frac{2(V_{DD} - V_D)}{V_{OV}} \hat{v}_i = V_{OV} + \hat{v}_i$

$\Rightarrow V_D \left(1 + \frac{2 \hat{v}_i}{V_{OV}}\right) = V_{OV} + \hat{v}_i + \frac{2 V_{DD} \hat{v}_i}{V_{OV}}$

$$V_D = \frac{V_{OV} + \hat{v}_i + 2 V_{DD} (\hat{v}_i / V_{OV})}{1 + 2(\hat{v}_i / V_{OV})}$$

$V_{DD} = 3V$, $\hat{v}_i = 20\,mV$ $\quad m = 10 = \frac{V_{OV}}{\hat{v}_i} \Rightarrow \underline{V_{OV} = 0.2^V}$

$V_D = \frac{0.2 + 0.02 + 2 \times 3 \times 10^{-1}}{1 + 2 \times 0.1} = \underline{0.68^V}$

$A_v = -\frac{2(3 - 0.68)}{0.2} = \underline{-23.2\ V/V}$

If $I_D = 100\,\mu A = 0.1\,mA$:

$A_v = -\frac{2 R_D I_D}{V_{OV}} \Rightarrow 23.2 = \frac{2 \times R_D \times 0.1}{0.2} \Rightarrow$

$\underline{R_D = 23.2\,k\Omega}$

$I_D = \frac{1}{2} k'_n \frac{W}{L} V_{OV}^2 \Rightarrow 0.1 = \frac{1}{2} \times 100 \times 10^{-3} \frac{W}{L} 0.2^2$

$\Rightarrow \underline{\frac{W}{L} = 50}$

5.51

Given $u_n = 500\text{cm}^2/Vs$

$$\mu_p = 250\text{cm}^2/Vs \quad C_{ox} = 0.4\frac{\text{fF}}{\mu\text{m}^2}$$

$$k'_n = \mu_n C_{ox} = 20\mu\text{A}/\text{V}^2$$

$$k'_p = 10\mu\text{A}/\text{V}^2$$

Use equations

$$(5.55)\, g_m = k'\frac{W}{L}V_{ov}$$

$$(5.56)\, g_m = \sqrt{2k'\frac{W}{L}I_D}$$

$$(5.57)\, g_m = \frac{2I_D}{V_{ov}}$$

case type	I_D (mA)	$\lvert V_{GS}\rvert$	$\lvert V_t\rvert$	$\lvert V_{ov}\rvert$	W (µm)	L (µm)	$\frac{W}{L}$	$k'\frac{W}{L}$ (mA/V²)	gm(ms)
a(N)	1	3	2	1	100	1	100	2	2
b(N)	1	1.2	0.7	0.5	50	0.125	400	8	4
c(N)	10	–	–	2	250	1	250	5	10
d(N)	0.5	–	–	0.5	–	–	200	4	2
e(N)	0.1	–	–	1.41	10	2	5	0.1	0.141
f(N)	0.1	1.8	0.8	1	40	4	10	0.2	0.2
g(P)	1	–	–	2	–	–	25	*	* See comment
h(P)	1	3	1	2	–	–	50	0.5^{+}	1
i(P)	10	–	–	1	4000	2	2000	20	20
j(P)	10	–	–	4	–	–	125	1.25	5
k(P)	0.05	–	–	1	30	3	10	0.1	0.1
l(P)	0.1	–	–	5	–	–	0.8	$0.008^{\times}$	0.04

Note - the circled entries are the givens.

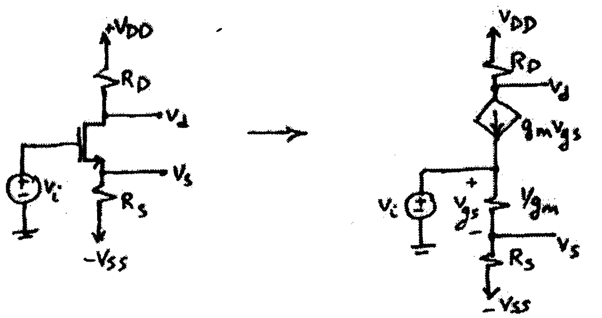
5.52

$$g_m = \sqrt{2K_n'\frac{W}{L}I_D} \Rightarrow \frac{W}{L} = \frac{g_m^2}{2K_n I_D}$$

$$\frac{W}{1} = \frac{1}{2\times 50\times 10^{-3}\times 0.5} \Rightarrow W = 20\,\mu m$$

$$g_m = \frac{2I_D}{V_{ov}} \Rightarrow V_{ov} = \frac{2\times 0.5}{1} = 1 \Rightarrow V_{GS} = 1 + V_t = 1.7^V$$

5.53



$$\frac{v_s}{v_i} = \frac{R_s}{R_s + \frac{1}{g_m}} = \frac{R_s g_m}{R_s g_m + 1}$$

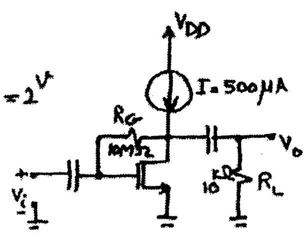
$$\frac{v_d}{v_i} = \frac{-g_m V_{gs} R_D}{v_i} = -g_m R_D \frac{1/g_m}{1/g_m + R_s} = \frac{-g_m R_D}{1 + g_m R_s}$$

5.54

$$r_o \simeq \frac{V_A}{I_D} = \frac{50}{0.5} = 100\,K\Omega$$

$$g_m = \frac{2I_D}{V_{GS} - V_t}, \quad V_{GS} = V_{DS} = 2^V$$

$$g_m = \frac{2\times 0.5}{2 - 0.9} = 0.91\,mA/V$$



$$\frac{v_o}{v_i} = -g_m(r_o \| R_L) = -0.91(100^k \| 10^k) = -8.3\,V/V$$

For I=1mA or twice the current:

$$\frac{I_{D1}}{I_{D2}} = \frac{(V_{GS} - V_t)^2}{(V_{GS_2} - V_t)^2} \Rightarrow V_{GS2} = V_t + \sqrt{2}\,(V_{GS_1} - V_t)$$

5.55

NMOS: $g_m = \sqrt{2K_n'\frac{W}{L}I_D} = \sqrt{2\times 90\times 10^{-3}\times \frac{20}{2}\times 0.1} = 0.42\frac{mA}{V}$

$$r_o = \frac{|V_A|}{I_D} = \frac{8\times 2}{0.1} = 160\,k\Omega$$

$$\chi = \frac{\gamma}{2\sqrt{2\phi_f + |V_{SB}|}} = \frac{0.5}{2\sqrt{2\times 0.34 + 1}} = 0.2$$

$$g_{mb} = \chi g_m = 0.2\times 0.42 = 0.084\,mA/V$$

$$g_m = \frac{2I_D}{V_{ov}} \Rightarrow V_{ov} = \frac{2\times 0.1}{0.42} = 0.48V$$

PMOS: $g_m = \sqrt{2\times 30\times 10^{-3}\times \frac{20}{2}\times 0.1} = 0.24\,mA/V$

$$r_o = \frac{|V_A|}{I_D} = \frac{12\times 2}{0.1} = 240\,K\Omega$$

$$\chi = 0.2 \Rightarrow g_{mb} = 0.2\times 0.24 = 0.048\,mA/V$$

$$V_{ov} = \frac{2\times 0.1}{0.24} = 0.83V$$

5.56

$$V_t = 1V,\ k_n' \frac{W}{L} = 2\ mA/V^2$$

(a) dc analysis $V_G = \frac{5}{15}\,15\ V = 5V$, assume

$I_D = 1$ mA

$V_S = 3$ V, $V_{GS} = 2$ V, $V_{OV} = 1$V.

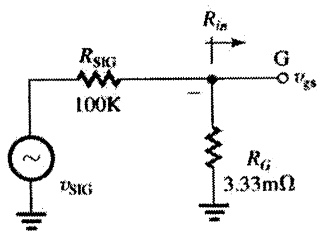
$I_D = \frac{1}{2}k' V_{OV}^2 = 1$ mA (check)

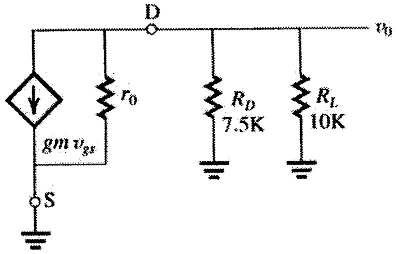
$V_D = V_{DD} - I_D R_D = 7.5$V.

(b) $r_o = \frac{V_A}{I_D} = \frac{100\ V.}{1\ mA} = 100\ k\Omega$

$g_m = \sqrt{2k_n I_D} = 2$ mS

(c)





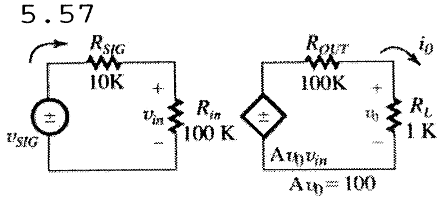
(d) $R_{in} = R_G = 3.33\ \text{M}\Omega$

$$\frac{v_{gs}}{v_{sig}} = \frac{R_{in}}{R_{sig} + R_{in}} = 0.97$$

$$\frac{v_0}{v_{gs}} = -g_m(r_0 \| R_D \| R_L) = -8.2$$

$$\frac{v_0}{v_{sig}} = -8.0$$

5.57

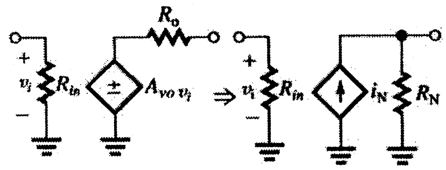


$$G_v = \frac{v_0}{v_{SIG}} = \frac{R_{in}}{R_{SIG} + R_{in}} A_{v_0} \frac{R_L}{R_{out} + R_L}$$

$= 82.6$

$$A_i = \frac{i_0}{i_i} = \frac{v_0}{v_{SIG}} \quad \frac{R_{SIG} + R_{in}}{R_L} = 9090$$

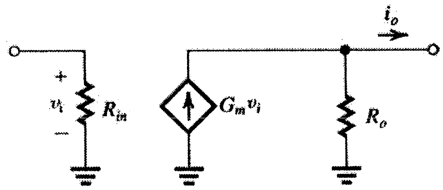
5.58



Where i_N = Norton's current source = $\dfrac{A_{VO}V_i}{R_O}$

and $R_N = R_o$ this is equivalent to Fig. P5.82

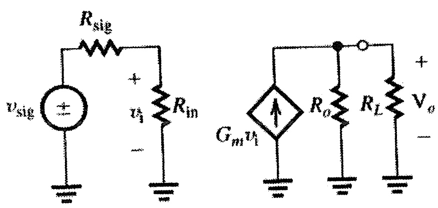
where $G_m = \dfrac{A_{VO}}{R_O}$



If the output is shorted, $i_o = G_m V_i$ or

$G_m = \left.\dfrac{i_O}{V_i}\right|_{R_L = O}$ with a signal source and

load connected,



by voltage division, $V_i = \dfrac{V_{sig}R_{in}}{R_{in} + R_{sig}}$

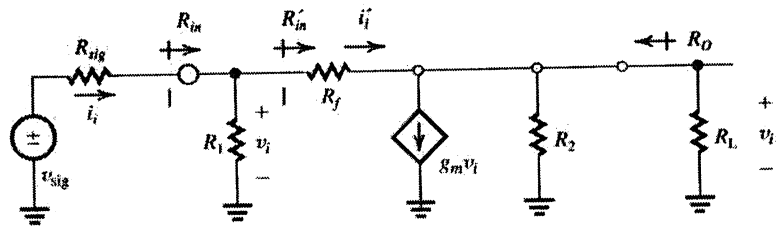
since $V_o = G_m V_i\,(R_O \ \|\ R_L)$, substitution for V_i

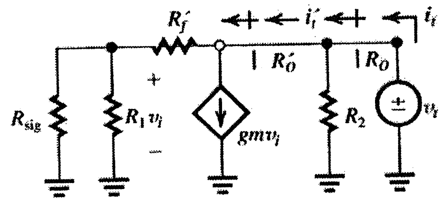
yields

$V_O = \dfrac{V_{sig}R_{in}}{R_{in} + R_{sig}} \cdot G_m(R_O \ \|\ R_L)$, so that

$$G_V = \frac{V_O}{V_{sig}} = \frac{R_{in}}{R_{in} + R_{sig}} G_m(R_O \ \|\ R_L)$$

5.59





note $R_{in} = R_1 \parallel R_{in}'$

$R_L' \equiv R_2 \parallel R_L$

$$v_O = v_i = \frac{R_L'}{R_f + R_L'} - g_m v_i (R_f \parallel R_L')$$

$$= v_i\left[\frac{R_L' - g_m R_f R_L'}{R_f + R_L'}\right] = v_i \frac{R_L'(1 - g_m R_f)}{R_L' + R_f}$$

$$A_{VO} = \left.\frac{v_O}{v_i}\right| = \frac{R_2(1 - g_m R_f)}{R_2 + R_f} = -g_m R_2 \frac{\left(1 - \frac{1}{g_m R_f}\right)}{1 + \frac{R_2}{R_f}}$$

$$i_i' = \frac{v_i - v_o}{R_f} = \frac{v_i}{R_f}\left[1 - \frac{R_L'(1 - g_m R_f)}{R_L' + R_f}\right]$$

$R_L' = R_2 \ (R_L \to \infty)$

$$\frac{i_i'}{v_i} = \frac{1}{R_{in}'} = \frac{1}{R_f}\frac{R_f + R_L' - R_L' + g_m R_f R_L'}{R_f + R_L'}$$

$$= \frac{1 + g_m R_L'}{R_f + R_L'}$$

$$R_{in}' = \frac{R_f + R_L'}{1 + g_m R_L'} = \frac{R_f + R_2 \parallel R_L}{1 + g_m(R_2 \parallel R_L)}$$

$$R_{in} = R_1 \parallel R_{in}' = R_1 \parallel \frac{R_f + R_2 \parallel R_L}{1 + g_m(R_2 \parallel R_L)}$$

Output resistance assuming $R_{sig} = 0$

$R_O = R_2 \parallel R_f \cong R_2$

Output resistance including R_{sig}

$R_1' \equiv R_1 \parallel R_{sig}$

$$i_t' = \frac{v_t}{R_1' + R_f} + g_m v_t \frac{R_1'}{R_1' + R_f}$$

$$v_t = \frac{1 + g_m R_L'}{R_1' + R_f}$$

$$R_O' = \frac{R_1' + R_f}{1 + g_m R_L'}$$

$$R_O = R_2 \parallel R_O' = R_2 \parallel \frac{(R_1 \parallel R_{sig}) + R_f}{1 + g_m(R_1 \parallel R_{sig})}$$

Evaluate for

$R_1 = 100\ \text{k}\Omega$, $R_f = 1\ \text{m}\Omega$, $g_m = 100\ \text{mA/V}$

$R_2 = 100\ \Omega$, $R_L = 1\ \text{k}\Omega$ (R_{sig} assumed ϕ)

$R_{in} = 49.8\ \text{k}\Omega$

$A_{vo} = -10.0$

$R_O = 100\ \Omega$

R_{in} is out in half by R_f.

Given $R_{sig} = 100\ \text{k}\Omega$, $R_f \to \infty$, and

$R_f = 1\ \text{m}\Omega$

$$G_V = \frac{v_O}{v_{sig}} = \frac{R_{in}}{R_{in} + R_{sig}}\left(-g_m R_L' \frac{1 - \frac{1}{g_m R_f}}{1 + \frac{R_L'}{R_f}}\right)$$

$G_V = -4.55 (R_f \to \infty)$, $G_V = -3.02$

$(R_f = 1\ \text{m}\Omega)$

5.60

R_{in} = depends on biasing

$A_{vo} = -g_m(r_o \parallel R_D)$

$= -0.4\frac{mA}{v}(50\ k\Omega \parallel 6\ k\Omega)$

$= -2.14V/V$

$r_o = \frac{V_A}{I_D} = \frac{10v}{0.2\ mA} = 50\ k\Omega$

$g_m = \sqrt{2\mu_n C_{ox}\frac{W}{L}I_D} = 0.4\ mA/V$

$R_o = r_o||R_D = 50\ k\Omega||6\ k\Omega = 5.36\ k\Omega$

WITH R_L=10 kΩ and assuming losses due to source impedance are negligible

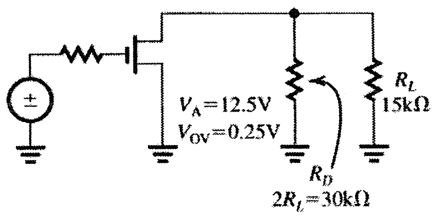
$G_v = A_v = -g_m(r_o\|R_D\|R_L)$

$= -0.4\frac{mA}{v}(5.36\ k\Omega||10\ k\Omega) = -1.40V/V$

For a 0.2V peak output, the input must be

$\frac{0.2V}{1.4} = 0.143V$ peak

5.61



a) $g_m r_o$ = ? $g_m = \frac{2.I_D}{V_{ov}}$ and $r_o = \frac{V_A}{I_D}$

$g_m r_o =$

$\frac{2.I_D}{V_{ov}} \cdot \frac{V_A}{I_D} = \frac{2V_A}{V_{ov}} = \frac{2 \cdot 12.5}{0.25} = 100$

b) If $Gv = -10V/V$,

$R_L = 15\ k\Omega, R_D = 2R_L = 30\ k\Omega$.

what is gm?

$Gv = Av = -g_m(R_D\|\infty\|R_L)$

$-10V/V = -g_m(30K \parallel 15K) = -g_m \cdot 10K$

$gm = \frac{1\ mA}{V}$

therefore:

$I_D = \frac{Vov}{2} \cdot g_m = \frac{0.25V\ 1mA/V}{2} = 0.125\ mA.$

c) If $R_D = R_L$

$\Rightarrow Gv = -g_m \cdot \frac{R_L}{2} = \frac{-1\ mA}{V} \cdot 7.5\ K$

$Gv = -7.5V/V$

5.62

$Gv = Av = -g_m(R_D \| R_L \| r_O)$

If $R_D \| R_L = \infty \Rightarrow Gv = -g_m r_O$

since $g_m = \frac{2I_D}{V_{ov}}$ and $r_O = \frac{V_A}{I_D}$

$Gv = \frac{-2I_D}{V_{ov}} \cdot \frac{V_A}{I_D} = \frac{-2V_A}{V_{ov}}$

5.63

$g_m = 5\ mS$

$i_d = g_m v_{gs} = \frac{g_m}{1 + g_m R_s} v_g$

$\frac{g_m}{1 + g_m R_s} = 1\ mS$

$\therefore R_s = \frac{4}{g_m} = 800\Omega$

5.64

$Rs = 1\ k\Omega$

$\frac{-g_m R'_L}{1 + g_m R_s} = -15$

$-g_m R'_L = -30$

$\therefore g_m = \frac{1}{R_s} = 1ms$

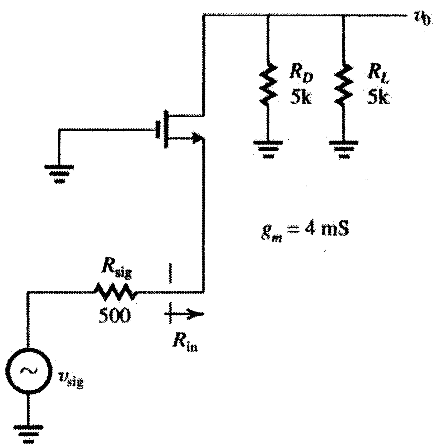
for $A_v = -10$, let $R_s = \frac{2}{g_m} = 2\ k\Omega$

5.65

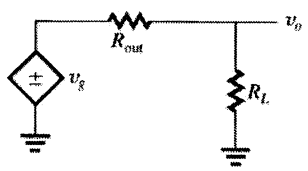
$$R_{in} = \frac{1}{g_m} = 250\Omega$$

$$Gv = \frac{v_o}{v_{sig}} = \frac{R_{in}}{R_{sig} + R_{in}} g_m(R_D \| R_L) = +3.3$$

$g_m = \sqrt{2k_n I_D}$, so for $\frac{1}{g_m} = R_{sig}, g_m$ must decrease to 1/2, and I_D must decrease to 1/4



5.66



$1\text{K} < R_L < 3\text{K}$

$R_{L,\,nom} = 2\text{K}$

for $R_{L,\,min}$

$$\frac{R_{L,\,min}}{R_{L,\,min} + R_{out}} \geq (0.80)\frac{R_{L,\,nom}}{R_{L,\,nom} + R_{out}}$$

$$\frac{1\text{K}}{1\text{K} + R_{out}} \geq \frac{1.6\text{K}}{2\text{K} + R_{out}}$$

$2\text{K}^2 + 1\text{K}R_{out} \geq 1.6\text{K}^2 + 1.6\text{K}R_{out}$

$400 \geq 0.6R_{out}$

$R_{out} \leq 667\Omega$

for $R_{L,\,max}$

$$\frac{R_{L,\,max}}{R_{L,\,max} + R_{out}} \leq (1.20)\frac{R_{L,\,nom}}{R_{L,\,nom} + R_{out}}$$

$$\frac{3\text{K}}{3\text{K} + R_{out}} \leq \frac{2.4\text{K}}{2\text{K} + R_{out}}$$

$R_{out} \leq 2\ \text{k}\Omega$

Therefore $R_{L,\,min}$ is the ruling case and

$R_{out} \leq 667\Omega$

$$g_m = \sqrt{2k_n I_D} \geq \frac{1}{667\Omega}$$

$k_n = 16\text{mA}/\text{V}^2$

$\therefore I_D \geq 70\ \mu\text{A}$

$$V_{ov} = \frac{2I_D}{g_m} = 0.093\text{V}.$$

5.67

Source Follower

$|v_{gs}| \leq 50\text{mV}$

$|v_o| \leq 0.5\text{V}$

$R_L = 2\text{k}\Omega$

$$v_o = g_m v_{gs} R_L \Rightarrow g_m \geq \frac{500\text{mV}}{50\text{mV}}\frac{1}{2\text{k}\Omega} = 5\text{mS}$$

For low distortion, keep

$|v_{gs}| < 0.2V_{ov} \Rightarrow V_{ov} = 0.25\text{V}.$

$$\therefore I_D \geq \frac{g_m V_{ov}}{2} = 0.625\text{mA}$$

$$i_{D,\,peak} = \frac{500\text{mVpK}}{2\text{k}\Omega} = 250\mu\text{Apk}$$

$i_{D,\,max} = 0.625\ \text{mA} + 250\ \mu\text{A} = 0.875\ \text{mA}$

$i_{D,\,min} = 0.625\ \text{mA} - 250\ \mu\text{A} = 0.375\ \text{mA}$

$v_{sig} = v_{gs} + v_o = 550\text{mVpk}$

5.68

$I_D = 2\,mA = \frac{1}{2} \times 80\times10^{-3} \times \frac{240}{6} \times (V_{GS}-1.2)^2 \Rightarrow$

$V_{GS} = 2.32V$

$R_D I_D = \frac{15}{3} = 5^V \Rightarrow R_D = \frac{5}{2^{mA}} = 2.5^{K\Omega}$

$R_S I_D = 5^V \Rightarrow R_S = \frac{5}{2} = 2.5K\Omega$

$V_G = 5 + V_{GS} = 7.32V$

$\frac{15}{R_{G1}+R_{G2}} \times R_{G2} = 7.32 \qquad R_{G1} = 22M\Omega \Rightarrow R_{G2} = 20.97\,M\Omega$

$V_{DS} = 5^V$

at the edge of saturation $V_{DS} = V_{GS} - V_t$ or $V_{DS} = 2.32 - 1.2 = 1.12^V$. So V_{DS} is $5 - 1.12 = 3.88^V$ away from the edge of saturation.

5.69

$I_D = 2mA = \frac{1}{2} k_n' \frac{W}{L} V_{ov}^2 \Rightarrow 2 = \frac{1}{2} \times 50\times10^{-3} \times \frac{200}{4} V_{ov}^2$

$V_{ov} = 1.26\,V$

$V_{DS} = V_{ov}$ edge of triode

Midway of cutoff ($V_{DS} = V_{DD}$) and beginning of triode operation ($V_{DS} = V_{ov}$) is when $V_{DS} = \frac{30+1.26}{2}$

$V_{DS} = 15.63V$

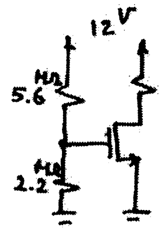
$V_{GS} = 2.32\,V \Rightarrow V_S = -2.32V \Rightarrow R_S = \frac{-2.32+15}{2}$

$\underline{R_S = 6.34K\Omega}$

$V_D = V_S + V_{DS} = -2.32 + 15.63 = 13.31\,V \Rightarrow R_D = \frac{15-13.31}{2}$

$\underline{R_D = 0.85K\Omega}$

5.70



$V_G = 12 \times \frac{2.2}{2.2+5.6} = 3.4^V$

$k_n' \frac{W}{L} = 220$ to $380\ \mu A/V^2$

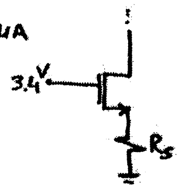
$V_t = 1.3$ to $2.4V$

$I_D = \frac{1}{2} \times k_n' \frac{W}{L}\,(3.4 - V_t)^2$

$I_{Dmin} = \frac{1}{2} \times 220\,(3.4-2.4)^2 = 110\,\mu A$

$I_{Dmax} = \frac{1}{2} \times 380 (3.4-1.3)^2 = 838\,\mu A$

to limit I_{Dmax} to $150\mu A$:



$150 = \frac{1}{2} \times 380 (3.4 - 0.15R_S - 1.3)^2$

$R_S = 8.1K\Omega$

Select $\underline{R_S = 8.2K\Omega}$

$I_{Dmax} = \frac{1}{2} \times 380 \times (3.4 - I_{Dmax} \times 8.2 - 1.3)^2$

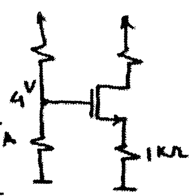
$I_{Dmax} = 0.15\,mA$ or ~~0.34~~ mA

The second answer results in negative V_{GS} and therefore it is not acceptable.

$I_{Dmin} = \frac{1}{2} \times 0.22 \times (3.4 - 8.2 I_{Dmin} - 2.4)^2$

$\underline{I_{Dmin} = 0.04\,mA}$

5.71



$V_t = 2^V$, $k_n' \frac{W}{L} = 2\,mA/V^2$

$I_D = \frac{1}{2} \times 2 \times (4 - I_D \times 1 - 2)^2$

$I_D = 4 + I_D^2 - 4I_D \Rightarrow \underline{I_D = 1^{mA}}$, ~~$4^{mA}$~~

$I_D = 4^{mA}$ results in $V_{GS} = 0$ which is not acceptable, therefore $I_D = 1mA$.

For $k' \frac{W}{L}$ 50% larger, i.e. $k' \frac{W}{L} = 3mA/V^2$

$I_D = \frac{1}{2} \times 3\,(4 - I_D - 2)^2 \Rightarrow \underline{I_D = 1.13\,mA}$

I_D increases by 13%.

5.72

$V_{GS} = 5-2 = 3V$, $I_D = \frac{V_S}{R_S} = \frac{2}{1} = 2mA$

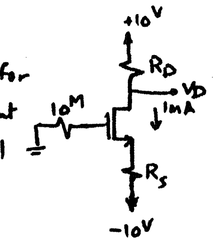
$I_D = 2 = \frac{1}{2} \times 2 \times (3-V_t)^2 \Rightarrow \pm 1.41 = 3-V_t \Rightarrow V_t = 1.59^V$

For a device with $V_t = 1.59 - 0.5 = 1.09^V$:

$I_D = \frac{1}{2} \times 2 \times (5 - I_D \times 1 - 1.09)^2 \Rightarrow I_D = 2.37^{mA}$

$V_S = 2.37V$

5.73



To maximize gain, we design for the lowest possible V_D consistant with allowing a 2V p-p signal swing. $V_{Dmin} = V_D - 1$

$V_{Dmin} = V_G - V_t = 0 - 2$

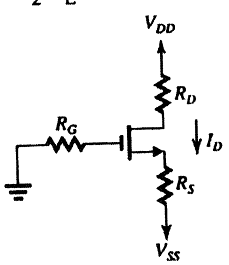
$V_D - 1 = -2 \Rightarrow V_D = -1V \Rightarrow R_D = \frac{10-(-1)}{1mA} = 11K\Omega$

$I_D = \frac{1}{2} \times 2 [0 - (-10 + 1 \times R_S) - 2]^2 = 1 \Rightarrow 1 = (8 - R_S)^2$

$R_S = 7K\Omega$

5.74

$$k = \frac{1}{2}k'\frac{W}{L}$$



a) $I_D = \frac{1}{2}k'_n\frac{W}{L}(V_{GS} - V_t)^2$

$$I_D = K(0 + V_{SS} - R_S I_D - V_t)^2$$

$$\frac{\partial I_D}{\partial K} = (V_{SS} - R_S I_D - V_t)^2 +$$

$$+ 2K(V_{SS} - R_S I_D - V_t)(-R_S)\frac{\partial I_D}{\partial k}$$

$$\frac{\partial I_D}{\partial K} = \frac{I_D}{K} - 2R_S\sqrt{\frac{I_D}{K}}K\frac{\partial I_D}{\partial k}$$

$$\frac{\partial I_D}{\partial K}(1 + 2\sqrt{KI_D}\,R_S) = \frac{I_D}{K} \Rightarrow$$

$$S_K^{I_D} = \frac{\partial I_D K}{\partial K I_D} = \frac{1}{1 + 2\sqrt{KI_D}\,R_S}$$

b) $K = 100\mu A/V^2, \frac{\Delta K}{K} = \pm 10\%,$

$V_t = 1$ V, $I_D = 100$ μA

$$\frac{\Delta I_D}{I_D} = \pm 1\ \%$$

$$S_K^{I_D} \simeq \frac{\partial I_D / I_D}{\frac{\partial k}{k}} = \frac{1}{10} = 0.1$$

$$= \frac{1}{1 + 2\sqrt{100 \times 10^{-3} \times 100 \times 10^{-3} R_S}}$$

$\Rightarrow R_S = 45$ kΩ

Now find V_{GS} and V_{SS} when $I_D = 100$ μA and $K = 100$ μA/V^2 $100 = 100(V_{GS} - 1)^2$

$\Rightarrow V_{GS} = 2$ V

Also $V_{GS} = V_{SS} - I_D R_S$

$2 = V_{SS} - 100 \times 10^{-6} \times 45 \times 10^3$

$\Rightarrow V_{SS} = 6.5$ V

C. For $V_{SS} = 5$ V

$$R_S = \frac{-V_{GS} + V_{SS}}{I_D}$$

$$= \frac{-2+5}{100 \times 10^{-6}} = 30\ k\Omega$$

$$S_k^{I_D} = \frac{1}{1 + 2\sqrt{100 \times 10^{-6} \times 100 \times 10^{-6} \times R_S}}$$

$= 0.14$

$\therefore$ For $\frac{\Delta K}{K} = \pm 10\%$, $\frac{\Delta I_D}{I_D} = \pm 1.4\%$

5.75

Both cases are in saturation region, because $V_{DG} \geq V_t$.

$V_D = 10 - 5 \times 1 = 5\text{V}$

a) $1 = \frac{1}{2} \times 0.5 \times (V_{GS} - 1)^2 \Rightarrow V_{GS} = 3\text{V}$, $V_S = -3\text{V}$

$V_{DS} = 8\text{V}$

b) $1 = \frac{1}{2} \times 1.25 \times (V_{GS} - 2)^2 \Rightarrow V_{GS} = 3.3\text{V}$, $V_S = -3.3\text{V}$

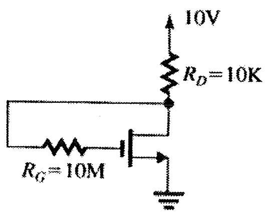
$V_{DS} = 8.3\text{V}$

5.76

$$V_D = V_G = V_{GS}$$

$$V_{DS} \geq V_{GS} - V_t \Rightarrow V_{DG} \geq -V_t$$

$$V_{DG} = 0$$



a) $\frac{10 - V_D}{10} = \frac{1}{2} \times 0.5 \times (V_D - 1)^2$

$\Rightarrow V_D = 2.7\ \text{V}$

$V_G = 2.7\ \text{V}$

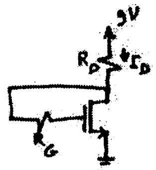
b) $\frac{10 - V_D}{10} = \frac{1}{2} \times 1.25 \times (V_D - 2)^2$

$\Rightarrow V_D = 3.05\ \text{V}$

$V_G = 3.05\ \text{V}$

5.77

For $I_D = 0.2\text{mA}$:



$0.2 = \frac{1}{2} \times 0.4 \times (V_{GS} - 1)^2$

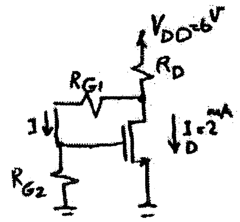
$V_{GS} = 2\text{V}$, $V_D = V_G = V_{GS} = 2\text{V}$

$R_D = \frac{9-2}{0.2} = 35\text{K}\Omega$

Select $R_D = 36\ \text{K}\Omega \Rightarrow \frac{9 - V_D}{R_D} = \frac{1}{2} \times 0.4 (V_D - 1)^2$

$\frac{9 - V_D}{36} = 0.2 (V_D - 1)^2 \Rightarrow V_D = 2\text{V}$, $I_D = 0.21\ \text{mA}$.

5.78



$I_D = 2 = \frac{1}{2} \times 3.2 \times (V_{GS} - 1.2)^2$

$V_{GS} - 1.2 = 1.12 \Rightarrow V_{GS} = 2.32\text{V}$

$V_G = 2.32\text{V}$

$V_{DSmin} = V_{GS} - V_t = 1.12\text{V}$

$V_{DS} = V_{DSmin} + 2 = 3.12\text{V}$

$R_{G_2} = 22\text{M}\Omega \Rightarrow I = \frac{2.32}{22} = 0.11\mu\text{A}$

$R_{G_1} = \frac{3.12 - 2.32}{0.11} = 7.58\text{M}\Omega$

$R_D = \frac{6 - 3.12}{2 + 0.11 \times 10^{-3}} = 1.44\text{K}\Omega$

5.79

a)

$$A_{vo} = -2\frac{(V_{DD} - V_D)}{V_{OV}} = -\frac{2(10 - 2.5)}{1}$$

$$= -15\ \text{V/V}$$

b) if V_{OV} is halved ($V_{OV} = 0.5$) then I_D is divided by 4, i.e. $I_D = \frac{0.5}{4} = 0.125\ \text{mA}$

Since V_D is kept unchanged at 2.5 V then:

$$R_D = \frac{10 - 2.5}{0.25} = 60\ \text{k}\Omega,$$

$$g_m = \frac{2I_D}{V_{OV}} = 0.5\ \text{m}\frac{\text{A}}{\text{V}}$$

$$r_o = \frac{V_A}{I_D} \Rightarrow r_o = 4 \times r_{o1} = 4 \times \frac{75}{0.5} = 600\ \text{k}\Omega$$

$$A_{vo} = -15 \times 2 = -30\text{V/V (without } r_o)$$

c) If we take r_o into account :

$$A_{vo} = -g_m(r_o \| R_D) = -0.5\left(600^k \| 60^k\right)$$
$$= -27.3 V/V$$

$$R_{out} = R_D \| r_o = 600^k \| 60^k = 54.5\ k\Omega$$

d) $R_{in} = R_G = 4.7\ M\Omega$

$R_o = 54.5\ k\Omega$

$$G_v = \frac{R_{in}}{R_{in} + R_{sig}} A_{vo} \frac{R_L}{R_L + R_o}$$

$$= \frac{4.7}{4.7 + 0.1} \times 27.3 \times \frac{15}{15 + 54.5}$$

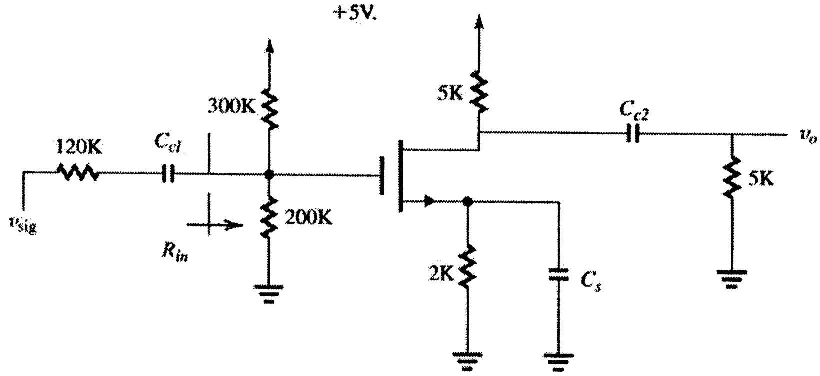
$$G_v = 5.77\ V/V$$

e) As we can see by reducing V_{OV} to half of its value or equivalently multiplying drain current by 4, A_{vo} is almost doubled, while R_{out} is multiplied by 4.

As a result G_v which is proportional to both A_{vo} and $\frac{1}{R_{out}}$ is only shightly reduced.

(G_v was -7 V/V before and it is 5.8 V/V now)

5.80



$V_t = 0.7$ V.

$V_A = 50$ V.

a) with $I_D = 0.5$ mA

$V_G = +2V \quad V_S + 1V. \quad V_{GS} = +1V.$

$V_{OV} = 0.3V$

$0.5 \text{ mA} = \frac{1}{2}k_n V_{oV}^2 \Rightarrow k_n = 11.1\frac{\text{mA}}{\text{V}^2}$

$V_D = 5 - (5 \text{ K})(0.5 \text{ mA}) = +2.5V.$

$V_{GD} = -0.5V < V_t \therefore$ Saturation

b) $R_{in} = 200 \text{ K} \| 300 \text{ K} = 120 \text{ k}\Omega$

$G_V = \frac{v_o}{v_{sig}} = -\frac{R_{in}}{120 \text{ K} - R_{in}} g_m$

$(5 \text{ K} \| r_O \| 5 \text{ K})$

$g_m = \frac{2I_D}{V_{ov}} = 3.33\text{mS}$

$r_O = \frac{V_A}{I_D} = 100 \text{ k}\Omega$

$G_V = -4.1$

c) $v_{sig} = \hat{v}_{sig} \sin\omega t$

$g_m(5\text{K}\|5\text{K}\|100\text{K}) = 8.12$

$V_{gd} + V_{GD} = \hat{v}_o + \frac{\hat{v}0}{8.12} - 0.5$

$\leq V_t = 0.7\text{V}.$

$\hat{v}_o$, max $= 1.07 \text{ V}_{\text{pk}}$

$\therefore \hat{v}_g$, max $= \frac{\hat{v}_o \text{ max}}{8.12} = 132 \text{ mv}_{\text{pk}}$

$\hat{v}_{sig}$, max $= \frac{\hat{v}_o \text{ max}}{4.1} = 261 \text{ mv}_{\text{pk}}$

d) Add $R_S = \frac{1}{g_m} = 300 \ \Omega,$

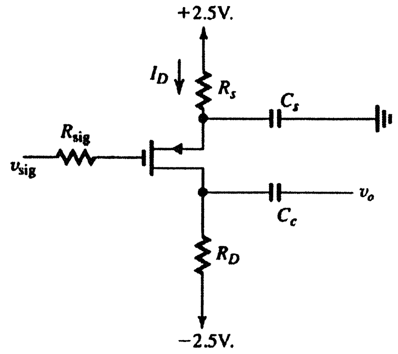
then $v_{gs} = \frac{v_g}{1 + g_m R_s} = \frac{v_g}{2}$

$\frac{g_m R_L'}{1 + g_m R_S} = \left|\frac{v_O}{v_g}\right| = 4.06$

$\hat{v}_O + \frac{\hat{v}}{4.06} - 0.5 \leq 0.7 \text{ V}.$

$\Rightarrow \hat{v}_o$, max $= 0.96$ V.

5.81



$V_{tp} = -0.7\text{V}, \quad V_A \to \infty$

a) for $I_D = 0.3$ mA, $|V_{OV}| = 0.3$ V.

$V_{SG} = 1.0$ V., $V_G = 0$

$V_S = 2.5 - I_D R_S = 1.0$ V.

$\therefore R_S = 5.0\ \text{k}\Omega$

b) $g_m = \dfrac{2I_D}{V_{OV}} = 2$ mS

$G_V = \dfrac{v_0}{v_{sig}} = -g_m R_D = -10$

$\therefore R_D = 5.0\ \text{k}\Omega$

c) $v_{gd} + V_{GD} \ge V_{tp} = -0.7$

$-\left|\hat{v}_o + \dfrac{\hat{v}_o}{10}\right| + 1\text{V} \ge -0.7$

$\hat{v}_O \le 1.55\text{V}_{pk}$

$\hat{v}_{sig} \le \dfrac{\hat{v}_{o}, \max}{10} = 0.155\text{V}_{pk}$

d) for $\hat{v}_{sig} = 50$ mV, changed R_D

$-\left|\hat{v}_O + \dfrac{\hat{v}_O}{g_m R_D}\right| + (2.5 - I_D R_D) \ge -0.7$

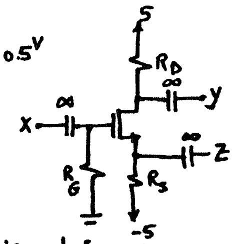
for $g_m = 2$ mS, $I_D = 0.3$ mA

$-\left|\dfrac{1 + g_m R_D}{g_m R_D}\right| g_m R_D \hat{v}_{sig} + 2.5 - I_D R_D \ge -0.7$

$R_D \le 7.88\ \text{k}\Omega \qquad (\hat{v}_{sig} = 50\ \text{mV})$

$G_V = -g_m R_D = -15.8$

5.82



a) $I_D = 0.1 = \frac{1}{2} \times 0.8 \times V_{ov}^2 \Rightarrow V_{ov} = 0.5^V$

$\Rightarrow V_{GS} = 0.5 + 1 = 1.5^V$

$V_G = 0 \Rightarrow V_S = -1.5V$

$R_S = \dfrac{-1.5-(-5)}{0.1} = \underline{35k\Omega}$

$V_{DS} = 5 - R_D \times 0.1$

Largest possible R_D is achieved for V_{DSmin}

$V_{DS} \geq V_{GS} - V_t \Rightarrow V_{DSmin} = V_{ov} \Rightarrow V_{DS} - 1 = V_{ov}$

$\Rightarrow V_{DS} = 1 + 0.5 = 1.5V \Rightarrow R_D = \dfrac{5-1.5}{0.1} = \underline{35k\Omega}$

$\underline{R_G = 10M\Omega}$.

b) $g_m = \dfrac{2I_D}{V_{ov}} = \dfrac{2\times0.1}{0.5} = 0.4\ mA/V$

$r_o = \dfrac{V_A}{I_D} = \dfrac{40}{0.1} = 400k\Omega$

c) If z is grounded then the circuit becomes a common-source configuration. The voltage gain according to Eq. 4.82:

$G_v = -\dfrac{R_G}{R_G + R_{sig}} g_m (r_o \| R_D \| R_L)$

$G_v = \dfrac{10M}{10M + 1M} \times 0.4 \times (400^k \| 35^k \| 40^k) = 6.5\ V/V$

$\underline{G_v = 6.5\ V/V}$

d) If y is grounded, then the circuit becomes a source follower configuration.

Eq. 4.103 : $A_{v_o} = \dfrac{r_o}{r_o + \frac{1}{g_m}} = \dfrac{400}{400 + \frac{1}{0.4}} = 0.99V/V$

$R_{out} = \frac{1}{g_m} \| r_o = \frac{1}{0.4} \| 400$

$\underline{R_{out} = 2.48k\Omega}$

e) If x is grounded, the circuit becomes a common-gate configuration.

$R_{in} = \frac{1}{g_m} \| R_s = 35k \| \frac{1}{0.4} = 2.33k\Omega$

Eq. 4.98 : $i_i = i_{sig} \dfrac{R_{sig}}{R_{sig} + R_{in}} \Rightarrow$

$i_i = 10\mu A \dfrac{100^k}{100^k + 2.33^k} = 9.77\mu A$

$v_y = R_D \times i_i = 35 \times 9.77\mu A = \underline{0.34V}$

5.83

a) is a source Follower:

$$A_{vo} = \frac{r_o}{r_o + \frac{1}{g_m}},$$

$$r_o \gg \frac{1}{g_m} \Rightarrow A_{vo} \simeq 1\text{V / V}$$

$$R_{out} = \frac{1}{g_m} = \frac{1}{5} = 0.2 \text{ k}\Omega$$

b) is a common - gate configuration:

$$R_{in} = \frac{1}{g_m} = \frac{1}{5} = 0.2 \text{ k}\Omega$$

$$A_v = g_m(R_D||R_L) = 5(5\text{K}||2\text{K}) = 7.1 \text{ V/V}$$

c) If we connect both stages together, then: for the first stage: $A_v = A_{vo}\frac{R_L}{R_L + R_{out}}$

where R_L is fact R_{in} of the second stage.

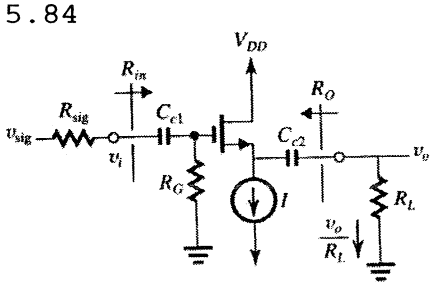
Therefore: $A_{v_1} = 1 \times \frac{0.2\text{K}}{0.2 + 0.2} = 0.5\text{V / V}$

For the second stage : $A_{v_2} = 7.1 \text{ V / V}$

overall gain

$$A_v = A_{v_1}A_{v_2} = 7.1 \times 0.5 = 3.55\text{V / V}$$

5.84



5.85

$V_{to} = 1.0$ V,

$r = 0.5$ V. $^{1/2}$

$2\phi_f = 0.6$ V.

$0 < V_{SB} < 4$V.

$$V_t = V_{to} + r[\sqrt{2\phi_f + V_{SB}} - \sqrt{2\phi_f}]$$

for $0 < V_{SB} < 4$V., $V_{to} < V_t < V_{to} + 0.685$V.

so 1V. $< V_t < 1.68$ V.

Since $r = \sqrt{\frac{2qN_A\epsilon_S}{C_{ox}}}$, an increase of 4x in t_{ox} makes C_{ox} 4x lower. and V_t becomes

$1\text{V} < V_t < 3.74$ V.

5.86

The test for region of operation for a depletion mode MOSFET is the same as for a enhancement mode MOSFET. The threshold voltage is negative; however.

$v_t = -3$ Volts, $v_G = 0$, $v_S = 0$ P$v_{GS} = 0$

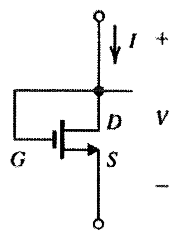
a) $v_D = 0.1$ Volts P$v_{DS} = 0.1$ and $v_{GS} - v_t = 3$ P$v_{DS} - v_t$, so transistor is in the triode region

b) $v_D = 1$ Volts P$v_{DS} = 1$ and $v_{GS} - v_t = 3$ P$v_{DS} < v_{GS} - v_t$, so transistor is in the triode region.

c) $v_D = 3$ Volts P$v_{DS} = 3$ and $v_{GS} - v_t = 3$ P$v_{DS} = v_{GS} - v_t$, so transistor is at triode-saturation boundary.

d) $v_D = 5$ Volts P$v_{DS} = 51$ and $v_{GS} - v_t = 3$ P$v_{DS} > v_{GS} - v_t$, so transistor is in the saturation region.

5.87



$V_{GS} = V_{DS} = V$ $\quad V_t$ is negative so

$V_{DS} < V_{GS} - V_t$ (always)

First, when $V = V_{GS} > V_t$

- From TABLE 5.1, this is triode region

$$i_D = k_n'\left(\frac{W}{L}\right)[(V_{GS} - V_t)V_{DS} - \frac{1}{2}V_{DS}^2]$$

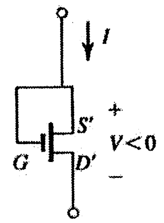
$$= k_n'\left(\frac{W}{L}\right)[(V - V_t)V - \frac{1}{2}V^2]$$

$$= k_n'\left(\frac{W}{L}\right)\left[\frac{1}{2}V^2 - V_tV\right]$$

$$= \frac{1}{2}k_n'\left(\frac{W}{L}\right)[V^2 - 2V_tV]$$

Note that when $V < 0$, $I = i_D$ is negative.

- When $V = V_{GS} < V_t$ AND assuming the device can operate symmetrically with D acting as the source and S acting as the drain, the circuit can be modeled as below. In this configuration



$V_{GS'} = 0 > V_t$ $V_{D'S'} = -V$

($V_{D'S'}$ is therefore positive)

Since $V_{GD'} = V < V_t$ this is saturation region (see Table 5.1)

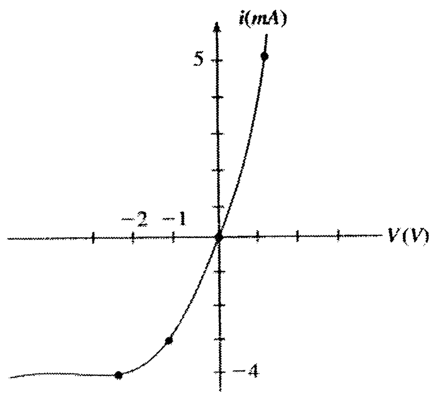
so

$$I = -i_D = -\frac{1}{2}k_n'\left(\frac{W}{L}\right)(V_{G'S'} - V_t)^2$$

$$= -\frac{1}{2}k_n'\left(\frac{W}{L}\right)(o - V_t)^2$$

$$= -\frac{1}{2}k_n'\left(\frac{W}{L}\right)V_t^2$$

$$V_t = -2\text{ V},\ k_n'\left(\frac{W}{L}\right) = 2\text{ m}\frac{\text{A}}{\text{V}^2}$$



$$i = -\frac{1}{2}k_n'\left(\frac{W}{L}\right)(V^2 - 2V_t v) \qquad V \geq V_t$$

$$= 1(\text{mA}/\text{V}^2)(V^2 + 4\text{V}/V)$$

$$i = -\frac{1}{2}k_n'\left(\frac{W}{L}\right)V_t^2 \qquad V \leq V_t$$

$$= -\frac{1}{2}(2\text{ mA}/\text{V}^2)(-2\text{ V})^2$$

$$= -4\text{ mA}$$

6.1

For I = 10 μA:

$$g_m = \frac{I}{V_T} = \frac{10\ \mu A}{25\ mV} = 0.4\ mA/V$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{0.4\ mA/V} = 250\ k\Omega$$

$$r_o = \frac{V_A}{I} = \frac{10\ V}{10\ \mu A} = 1\ M\Omega$$

$$A_o = g_m r_o = \frac{V_A}{V_T} = \frac{10\ V}{0.025\ V} = 400\ V/V$$

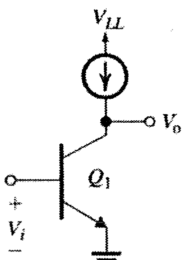
For I = 100 μA:

$$g_m = \frac{100\ \mu A}{25\ mV} = 4\ mA/V$$

$$r_\pi = \frac{100}{4\ mA/V} = 25\ k\Omega$$

$$r_o = \frac{10\ V}{100\ \mu A} = 100\ k\Omega$$

$$A_o = 4mA/V(100\ k\Omega) = 400$$



For I = 1 mA:

$$g_m = \frac{1\ mA}{25\ mV} = 40\ mA/V$$

$$r_\pi = \frac{100}{40\ mA/V} = 2.5\ k\Omega$$

$$r_o = \frac{10\ V}{1\ mA} = 10\ k\Omega$$

$$A_o = 40\ mA/V\ (10\ K) = 400$$

I	g_m	r_π	r_o	A_o
10 μA	0.4 mA/V	250 kΩ	1 MΩ	400
100 μA	4.0 mA/V	25 kΩ	100 kΩ	400
1 mA	40 mA/V	2.5 kΩ	10 kΩ	400

6.2

$$g_m = \frac{I_D}{\frac{V_{OV}}{2}}, \text{ so}$$

$$I_D = \frac{g_m V_{OV}}{2} = \frac{2\ mA/V(0.25\ V)}{2} = 0.25\ mA$$

From chapt. 5, $k_n' = \mu_n C_{OX}$

since $g_m = \sqrt{2\mu_n C_{ox}(W/L)}\sqrt{I_D}$,

$$2\ mA/V = \sqrt{2(200\ \mu A/V^2)(W/L)(250\ \mu A)}$$

yielding

$W/L = 40$

so that

$W = 40(0.5\ \mu m) = 20\ \mu m$

6.3

Assuming that the MOSFET is operating above V_t,

$$A_o = \frac{V_A'\sqrt{2(\mu_n C_{ox})(WL)}}{\sqrt{I_D}}$$

If I_D is decreased to 25 μA,

A_o is increased by $\frac{1}{\sqrt{1/4}} = 2$

$g_m = \sqrt{2(\mu_n C_{ox})(W/L)} \cdot \sqrt{I_D}$

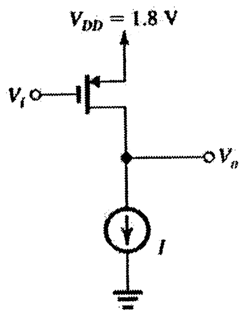
so, g_m is decreased by

$\sqrt{1/4} = 1/2$

If I_D is increased to 400 μA,

A_o is decreased by $\frac{1}{\sqrt{4}} = \frac{1}{2}$

g_m increases by $\sqrt{4} = 2$



The edge of the Saturation region is defined as when $|V_{DS}| = |V_{GS}| - |V_t| = |V_{OV}|$

$\therefore$ The highest instantaneous output voltage is

$V_{DD} - |V_{OV}| = 1.8 - 0.3 = 1.5$ V

6.4

$$L = 2\,(0.18\ \mu\text{m}) = 0.36\ \mu\text{m}$$

a) $$g_m = \frac{I_D}{\frac{V_{OV}}{2}} = \frac{10\ \mu\text{A}}{\frac{0.25}{2}} = 80\ \frac{\mu\text{A}}{\text{V}}$$

$$V_A' = 5\ \text{V}/\mu\text{m}$$

so,

$$r_o = \frac{V_A' L}{I_D} = \frac{5\ \text{V}/\mu\text{m}\ (2)\ (0.18\ \mu\text{m})}{10\ \mu\text{A}} = 180\ \text{k}\Omega$$

$$A_o = \frac{2V_A' L}{V_{OV}} = \frac{2\ (5\ \text{V}/\mu\text{m})\ (0.36\ \mu\text{m})}{0.25\ \text{V}}$$

$$= 14.4\ \text{V/V}$$

b) with $I_D = 10\ \mu\text{A}$

$$k_n = \frac{2I_D}{V_{OV}^2} = \frac{2(10\ \mu\text{A})}{(0.25\ \text{V})^2} = 320\ \mu\text{V/A}^2$$

Solving for V_{OV} with $I_D = 100\ \mu\text{A}$:

$$V_{OV} = \frac{2I_D}{k_n} \rightarrow$$

$$V_{OV} = \sqrt{\frac{2(100\ \mu\text{A})}{320\ \mu\text{A/V}^2}} = 0.79\ \text{Volts}$$

$$g_m = \frac{I_D}{V_{OV}/2} = \frac{100\ \mu\text{A}}{0.79\ \text{V}/2} = 253\ \mu\text{A/V}$$

$$r_o = \frac{V_A' L}{I_D} = \frac{5\text{V}/\mu\text{m}\ (0.36\mu\text{m})}{100\ \mu\text{A}} = 18\ \text{k}\Omega$$

$$A_o = g_m r_o = 253\ \mu\text{A/V}(18\ \text{k}\Omega) = 4.56\ \text{V/V}$$

c) Now, with a new W and $V_{OV} = 0.25$ V,

$I_D = 100\ \mu\text{A}$,

$$g_m = \frac{I_D}{V_{OV}/2} = \frac{100\ \mu\text{A}}{0.25\ \text{V}/2} = 800\ \mu\text{A/V}$$

$$r_o = \frac{V_A' L}{I_D} = \frac{5\ \text{V}/\mu\text{m}(0.36\ \mu\text{m})}{100\ \mu\text{A}} = 18\ \text{k}\Omega$$

$$A_o = \frac{2V_A' L}{V_{OV}} = \frac{(2)(5\text{V}/\mu\text{m})(0.36\ \mu\text{m})}{0.25\ \text{V}}$$

$$= 14.4\ \text{V/V}$$

d) I_D is now $10\ \mu\text{A}$, first, find k_n:

$$k_n = \frac{2I_D}{V_{OV}^2} = \frac{2(100\ \mu\text{A})}{(0.25\ \text{V})^2} = 3\ \text{mA/V}^2$$

so, now with $I_D = 10\ \mu\text{A}$,

$$V_{OV} = \sqrt{\frac{2I_D}{k_n}} = \sqrt{\frac{2(10\ \mu\text{A})}{3.2\ \text{mA/V}}} = 0.079\ \text{V}$$

$$g_m = \frac{I_D}{V_{OV}/2} = \frac{10\ \mu\text{A}}{0.079/2\text{V}} = 253\ \mu\text{A/V}$$

$$r_o = \frac{V_A' L}{I_D} = \frac{(5\text{V}/\mu\text{m})(0.36\ \mu\text{m})}{10\ \mu\text{A}} = 180\ \text{k}\Omega$$

$$A_o = \frac{2V_A' L}{V_{OV}} = \frac{2(5\text{V}/\mu\text{m})(0.36\ \mu\text{m})}{0.079\ \text{V}}$$

$$= 45.6\ \text{V/V}$$

e) The lowest A_O is 4.56 V/V

when $V_{OV} = 0.79$ V, $I_D = 100\ \mu\text{A}$,

$L = 0.36\ \mu\text{m}$

The highest A_O is 45.6 V/V

with $I_D = 10\ \mu\text{A}$, $V_{OV} = 0.079$ V

If W/L is held constant, and L is increased 10 times,

since $A_o = \dfrac{2V_A' L}{V_{OV}}$ (or since g_m remains constant, and r_O is increased by L)

Each gain is increased by a factor of 10:

Low $A_O = 45.6$ V/V

High $A_O = 456$ V/V

6.5

$$I_D = \frac{1}{2}k_n'\left(\frac{W}{L}\right)V_{OV}^2$$

$$\frac{W}{L} = \frac{2I_D}{k_n' V_{OV}^2} = \frac{2(100\ \mu A)}{200\ \mu A/V^2 (0.25\ V)^2} = 16$$

so, $W = 16(0.4\ \mu m) = 6.4\ \mu m$

$$g_m = \frac{I_D}{V_{OV}/2} = \frac{100\ \mu A}{\left(\frac{0.25\ V}{2}\right)} = 800\ \mu A/V$$

$$r_o = \frac{V_A' L}{I_D} = \frac{20\ V/\mu m(0.4\ \mu m)}{100\ \mu A} = 80\ k\Omega$$

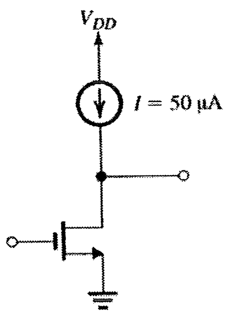
If $L = 0.8\ \mu m$,

$W = 0.8\ \mu m(16) = 12.8\ \mu m$

$$g_m = \frac{100\ \mu A}{\left(\frac{0.25\ V}{2}\right)} = 800\ \mu A/V$$

$$r_o = \frac{V_A' L}{I_D} = \frac{20\ V/\mu m(0.8\ \mu m)}{100\ \mu A} = 160\ k\Omega$$

6.6



Since $A_0 = \frac{2V_A' L}{V_{OV}}$, and the current source is ideal,

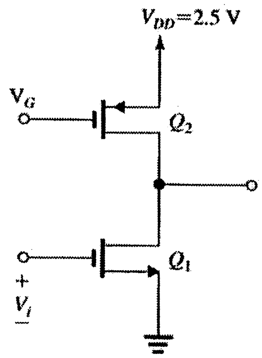
$$L = \frac{A_0 V_{OV}}{2V_A'} = \frac{100(0.2\ V)}{2(20\ V/\mu m)} = 0.5\ \mu m$$

Since $I_D = \frac{1}{2}(\mu_n C_{ox})\left(\frac{W}{L}\right)V_{OV}^2$,

$$\frac{W}{L} = \frac{2I_D}{(\mu_n C_{ox})V_{OV}^2}$$

$$= \frac{2\ (50\ \mu A)}{(200\ \mu A/V^2)(0.2\ V)^2} = 12.5$$

6.7



$V_G = V_{DD} - V_{SD2}$

$= V_{DD} - |V_{tp}| - |V_{ov}|$

$= 2.5 - 0.5 - 0.3 = 1.7\ V$

Since $I_{D_1} = \frac{1}{2}(\mu_n C_{ox})\left(\frac{W}{L}\right)_1 V_{OV}^2$

$$\left(\frac{W}{L}\right)_1 = \frac{2I_{D1}}{(\mu_n C_{ox})V_{OV}^2}$$

$$= \frac{2(100\ \mu A)}{(200\ \mu A/V^2)(0.3V)^2} = 11.1$$

for Q_2, $\left(\frac{W}{L}\right)_2 = \frac{2I_{D2}}{(\mu_p C_{ox})|V_{ov}|^2}$

$$= \frac{2\ (100\ \mu A)}{(100\ \mu A/V^2)(0.3)^2} = 22.2$$

Since $V_{An}' = |V_{Ap}'| = 20\ V/\mu m$

$$r_{o1} = r_{o2} = r_o = \frac{V_A' L}{I} = \frac{20\ V/\mu m\ (0.5\ \mu m)}{100\ \mu A}$$

$$g_m = \frac{I_{D1}}{\frac{V_{OV}}{2}} = \frac{100\ \mu A}{0.3/2\ V} = 667\ \mu A/V$$

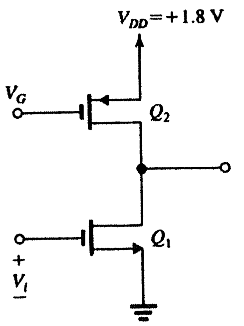
$r_o = 100\ k\Omega$

so,

$$A_V = \frac{1}{2}g_m r_o = -\frac{1}{2}(667\ \mu A/v)(100\ k\Omega)$$

$$= -33.3\ V/V$$

6.8



$$V_G = V_{DD} - |V_{tp}| - |V_{OV}|$$
$$= 1.8 - 0.5 - 0.2 = 1.1 \text{ V}$$

$$g_{m1} = \frac{I_D}{V_{OV}/2} = \frac{100\ \mu\text{A}}{0.2\text{V}/2} = 1 \text{ mA/V}$$

$A_v = -g_{m1}(r_{o1} \| r_{o2})$ so we must find r_{o1} and r_{o2}

$$r_{o1} \| r_{o2} = \frac{A_v}{-g_{m1}} = \frac{-40}{-1 \text{ mA/V}} = 40 \text{ k}\Omega$$

since $r_{o1} = \dfrac{V'_{An}}{I_D}$ and $r_{o2} = \dfrac{|V'_{Ap}|\text{L}}{I_D}$

$$r_{o1} = \frac{5\text{ V}/\mu\text{m}}{100\ \mu\text{A}} \cdot L = \frac{50\text{ K}}{\mu\text{m}} \cdot \text{L}$$

$$r_{o2} = \frac{6\text{ V}/\mu\text{m}}{100\ \mu\text{A}} \cdot L = \frac{60\text{ K}}{\mu\text{m}} \cdot \text{L}$$

so,

$$40\text{ k}\Omega = \frac{50\text{ k}\Omega/\mu\text{m}(60\text{ k}\Omega/\mu\text{m}).\text{L}^2}{50\text{ k}\Omega/\mu\text{m}\cdot\text{L} + 60\text{ k}\Omega/\mu\text{m}\cdot\text{L}}$$

or $L = 1.47\ \mu\text{m}$

since $I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right) V_{OV}^2$,

$$\left(\frac{W}{L}\right)_1 = \frac{2I_{D1}}{\mu_n C_{ox} V_{OV}^2}$$

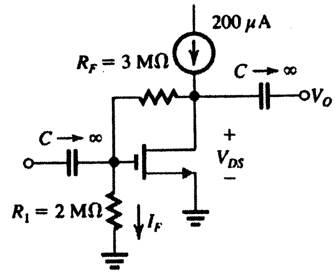
$$= \frac{2(100\ \mu\text{A})}{387\ \mu\text{A}/\text{V}^2(0.2\text{ V})^2} = 12.9$$

similarly,

$$\left(\frac{W}{L}\right)_2 = \frac{2I_{D2}}{\mu_p C_{ox}|V_{OV}|^2}$$

$$= \frac{2(100\ \mu\text{A})}{86\ \mu\text{A}/\text{V}^2\ (0.2\text{ V})^2} = 58.1$$

6.9



(a) If we neglect the current through R_F,

$$I_D = 200\ \mu\text{A} = \frac{1}{2}k_n'(W/L)V_{OV}^2$$

$$V_{OV} = \sqrt{\frac{2I_D}{k_n'(W/L)}} = \sqrt{\frac{2(200\ \mu\text{A})}{2\text{ mA}/\text{V}^2}} = 0.45\text{ V}$$

$$V_{GS} = V_t + V_{OV} = 0.5 + 0.45 = 0.95\text{ V}$$

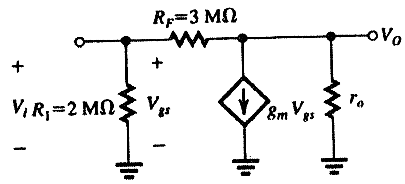
The current through the feedback network is

$$I_F = \frac{V_G}{R_1} = \frac{0.95\text{ V}}{2\text{ M}\Omega} = 0.475\ \mu\text{A}$$

This is << 200 μA, so this assumption is justified.

$V_{DS} \simeq$
$I_F(R_F + R_1) = 0.475\ \mu\text{A}(3\text{M}\Omega + 2\text{M}\Omega)$
$= 2.38\text{ V} \simeq 2.4\text{ V}$

(b) small-signal model:



KCL at the output node yields

$$\frac{V_O}{r_O} + g_m V_{gs} + \frac{V_O - V_i}{R_F} = 0$$

since $V_{gs} = V_i$

$$\frac{V_O}{r_O} + g_m V_i + \frac{v_O}{R_F} - \frac{V_i}{R_F} = 0 \text{ or}$$

$$\frac{V_O}{V_i} = \frac{\left(\frac{1}{R_F} - g_m\right)}{\left(\frac{1}{r_O} + \frac{1}{R_F}\right)}$$

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2(200\ \mu A)}{0.45\ V} = 0.89\ mA / V$$

$$r_o = \frac{V_A}{I_D} = \frac{20\ V}{200\ \mu A} = 100\ k\Omega$$

so,

$$\frac{V_D}{V_i} = \frac{\frac{1}{3000\ K} - 0.89\ mA / V}{\frac{1}{100\ K} + \frac{1}{3000\ K}} = -86.1\ V / V$$

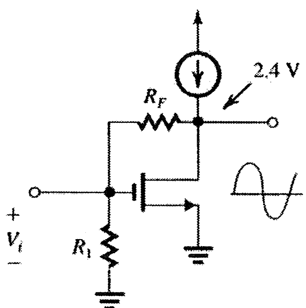
To find the peak of the maximum sinewave output possible, we note that the current source is assumed to be ideal. Therefore, sinewave amplitude will be limited by the negative excursion. Since this happens when

$V_{DS} = V_{OV} = 0.45$ V,

the maximum peak amplitude will be

$2.4 - 0.45 = 1.95$ V

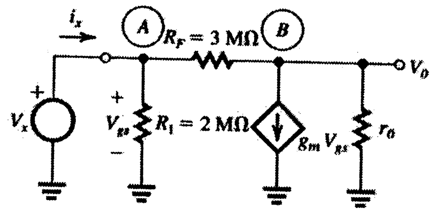
(That is, the output will vary between 0.45V and $2.4 + 1.95 = 4.35$ V.)



The corresponding input voltage is

$$V_{i_{peak}} = \frac{V_{o_{peak}}}{|A_V|} = \frac{1.95\ V}{86.1\ V / V} = 23\ mV_{peak}$$

(c) To find R_{in}, we apply a test voltage V_X to the input



KCL at node A:

$$i_x = \frac{V_x}{R_1} + \frac{V_x - V_o}{R_F}$$

KCL at node B:

$$\frac{V_x - V_o}{R_F} = \frac{V_o}{r_o} + g_m V_x$$

$$\Rightarrow V_o = \frac{V_x\left(\frac{1}{R_F} - g_m\right)}{\frac{1}{r_o} + \frac{1}{R_F}}$$

Substituting into the first equation, we get

$$i_x = \frac{V_x}{R_1} + \frac{V_x}{R_F} - \frac{V_x}{R_F}\frac{\left(\frac{1}{R_F} - g_m\right)}{\left(\frac{1}{r_o} + \frac{1}{R_F}\right)}$$

so that

$$R_{in} = \frac{V_X}{i_x}$$

$$= \frac{1}{\frac{1}{R_1} + \frac{1}{R_F} - \frac{\frac{1}{R_F{}^2}}{1 / r_o + 1 / R_F} + \frac{g_m / R_F}{\frac{1}{r_o} + \frac{1}{R_F}}}$$

$$R_{in} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_F} - \frac{1}{\frac{(R_F)^2}{r_o} + R_F} + \frac{g_m}{\frac{R_F}{r_o} + 1}}$$

$$R_{in} = \frac{1}{\frac{1}{2\ m\Omega} + \frac{1}{3\ m\Omega} - \frac{1}{\frac{(3\ m\Omega)^2}{0.1\ m\Omega} + 3\ m\Omega} + \frac{0.89\ mA / V}{\frac{3\ m\Omega}{0.1\ m\Omega} + 1}}$$

$$R_{in} = 33.9\ k\Omega$$

6.10 the transfer characteristic of the amplifier over the region labeled as segment III, is quite linear.

$V_{OA} = V_{DD} - V_{OV3} = 5 - 0.53 = 4.47$ V

Now to Find the linear equation for segment III, we can write $i_{D1} = i_{D2}$:

$$\frac{1}{2}k_n'\left(\frac{W}{L}\right)_1(v_I - v_{tn})^2\left(1 + \frac{v_O}{V_{An}}\right)$$

$$= \frac{1}{2}k_n'\left(\frac{W}{L}\right)_2(v - |v_{tp}|)^2\left(1 + \frac{V_{DD} - v_O}{V_{AP}}\right)$$

$$\Rightarrow 200(V_I - 0.6)^2\left(1 + \frac{v_O}{20}\right)$$

$$= 65 \times 0.53^2 \times \left(1 + \frac{V_{DD} - V_O}{10}\right)$$

$$(V_{56} - |V_{tp}|)^2\left(1 + \frac{V_{DD} - v_O}{V_{A_P}}\right)$$

$$\frac{200}{65 \times 0.53^2}(V_I - 0.6)^2 = \frac{1.5 - v_O/10}{1 + \frac{v_O}{20}}$$

$$7.3(v_I - 0.6)^2 = \frac{1 - v_O/15}{1 + \frac{v_O}{20}}$$

$$= \frac{1 - 0.067\, v_O}{1 + 0.05\, v_O} \simeq 1 - 0.117\, v_O$$

$$\Rightarrow v_O = 8.57 - 62.57(v_I - 0.6)^2$$

If we substitute for $v_{OA} = 4.47$ V, then

$V_{IA} = 0.86$ V

To determine coordinates of B, note that

$V_{IB} - V_{tn} = V_{OB}$ or $V_{IB} - 0.6 = V_{OB}$

Substitute in 1:

$V_{OB} = 8.57 - 62.57 v_{OB}^2 \Rightarrow V_{OB} = 0.36$ V

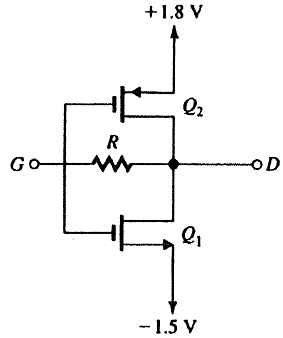
$V_{IB} = 0.6 + 0.36 = 0.96$ V

Therefore the linear region is :

$0.86 \text{ V} \le V_I \le 0.96 \text{ V}$ or

$0.36 \text{ V} \le V_O \le 4.47 \text{ V}$

6.11



(a) If G and D are open, and no current flows to either gate,

$V_D = V_G$ and $I_{D1} = I_{D2}$

$$I_{D1} = \frac{1}{2}k_n'(W/L)_1(V_G - V_S - V_t)^2$$

$$= I_{D2} = \frac{1}{2}k_p'(W/L)_2(V_{DD} - V_G - |V_t|)^2$$

or, $(V_G - (-1.5V) - 0.5V)^2 =$

$(1.5V - V_G - 0.2\text{ V})^2$

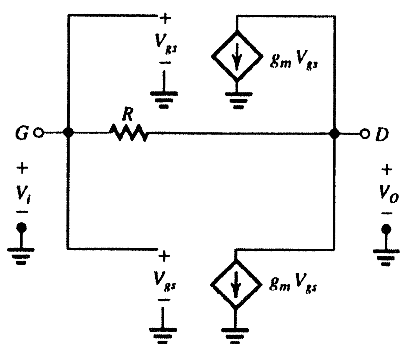
$$(V_G + 1)^2 = (1 - V_G)^2 \Rightarrow V_G = 0$$

so,

$$I_{D2} = I_{D1} = \frac{1}{2}(1 \text{ mA/V}^2)(0 + 1)^2$$

$$= 0.5 \text{ mA}$$

(b) For $r_O = \infty$, the small-signal model becomes:



$$V_O = V_i - 2(g_m V_{gs})R$$

$$V_{gs} = V_i, \text{ so}$$

$$V_O = V_i - 2\, g_m R v_i$$

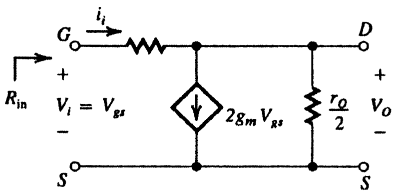
$$Av = \frac{V_O}{V_i} = 1 - 2\, g_m R$$

substituting values,

$$Av = 1 - 2(1\ \text{mA/V})(1\ \text{M}\Omega) = -1999\ \text{V/V}$$

(c) $r_O = \dfrac{|V_A|}{|I_D|} = \dfrac{20\ \text{V}}{0.5\ \text{mA}} = 40\ \text{k}\Omega$

Adding r_{O_1} and r_{O_2} to the model, we get



KCL at D yields,

$$\frac{V_i - V_O}{R} = 2\,g_m V_{gs} + \frac{V_O}{r_{O/2}} \quad \text{and since}$$

$$V_{gs} = V_i,$$

$$\frac{V_i}{R} - 2\,g_m V_i = \frac{V_O}{R} + \frac{2V_O}{r_O} \quad \text{so that}$$

$$Av = \frac{v_O}{v_i} = \frac{\frac{1}{R} - 2g_m}{\frac{1}{R} + \frac{2}{r_O}} = \frac{1 - 2g_m R}{1 + \frac{2R}{r_O}}$$

Substituting numbers, we get:

$$Av = \frac{1 - 2(1\ \text{mA/V})(1000\ \text{k}\Omega)}{1 + \dfrac{2000\ \text{k}\Omega}{40\ \text{k}\Omega}}$$

$$= -39.2\ \text{V/V}$$

To find R_{in}, note that

$$R_{in} = \frac{V_i}{i_i}$$

$$i_i = \frac{V_i - V_O}{R} \quad \text{since} \quad V_O = V_i\left(\frac{1 - 2\,g_m R}{1 + \frac{2R}{r_O}}\right),$$

$$i_O = \frac{V_i\left[1 - \left(\dfrac{1 - 2\,g_m R}{1 + \frac{2R}{r_O}}\right)\right]}{R}$$

so that,

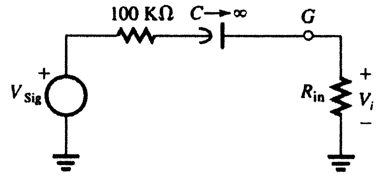
$$R_{in} = \frac{V_i}{i_i} = \frac{R}{1 - \left(\dfrac{1 - 2\,g_m R}{1 + \frac{2R}{r_O}}\right)}$$

Substituting in numerical values,

$$R_{in} = \frac{1\ \text{M}\Omega}{1 - \left[\dfrac{1 - 2(1\ \text{mA/V})(1\ \text{M}\Omega)}{1 + (2)(1\ \text{mA/V})(40\ \text{k}\Omega)}\right]} = 24.9\ \text{K}$$

$$\simeq 25\ \text{k}\Omega$$

(d) If the gate is driven as shown:



$$\frac{V_D}{V_{sig}} = \frac{R_{in}}{100\ \text{k}\Omega + R_{in}} \cdot A_v$$

$$= \frac{25\ \text{k}\Omega}{100\ \text{k}\Omega + 25\ \text{k}\Omega} \cdot (-39.2\ \text{V/V}) = -7.84\ \text{V/V}$$

(e) $|v_{DS}|$ must be $\geq |V_{OV}|$

with $V_G = 0, V_{GS1} = 1.5\ \text{V}, V_{SG2} = 1.5\ \text{V}$

$\therefore |V_{OV}| = 1.5 - 0.5 = 1.0\ \text{V}$

Given the $\pm$ 1.5 V supplies,

$-0.5\ \text{V} \leq v_O \leq 0.5\ \text{V}$

6.12

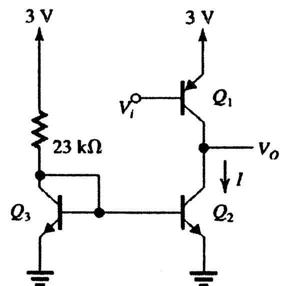
a) $I_{REF} = I_{C3} = \dfrac{3 - V_{BE3}}{23\ \text{k}\Omega}$

$$I_{REF} = \frac{3 - 0.7}{23}$$

$$I_{REF} = 0.1\ \text{mA}$$

$$\Rightarrow I_{C2} = 5 I_{C3}$$

$$I_{C2} = I = 0.5\ \text{mA} \Rightarrow I = 0.5\ \text{mA}$$



b)

$$|V_A| = 50V \Rightarrow r_{O1} = \frac{|V_A|}{I} = \frac{50}{0.5} = 100\ \text{k}\Omega$$

$$r_{O2} = \frac{50}{0.5} = 100\ \text{k}\Omega$$

Total resistance at the collector of Q_1 is equal to $r_{O1} \parallel r_{O2}$, thus:

$r_{tot} = 100\ k\Omega \parallel 100\ k\Omega = 50\ k\Omega$

$r_{tot} = 50\ k\Omega$

c) $g_{m1} = \frac{I_{C1}}{V_T} = \frac{0.5}{0.025} = 20\ mA/V$

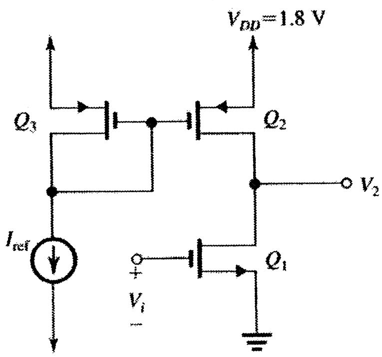
$r_{\pi 1} = \frac{\beta}{g_m} = \frac{50}{20} = 2.5\ k\Omega$

d) $R_{in} = r_{\pi 1} = 2.5\ k\Omega$

$R_O = r_{O1} \parallel r_{O2} = 100\ k\Omega \parallel 100\ k\Omega = 50\ k\Omega$

$A_V = -g_{m1}R_O = -20 \times 50 = -1000\ V/V$

6.13



For an output of 1.6 V,

$V_{SD_{2min}} = |V_{OV}| = 1.8 - 1.6 = 0.2\ V,$

$V_{SD_{1min}} = 0.2\ V$

Since $I_{D2} = I_{D3} = I_{D1} = 50\ \mu A$,

and $I_D = \frac{1}{2}(\mu_p C_{ox})(W/L)V_{OV}^2$

$$\left(\frac{W}{L}\right)_2 = \left(\frac{W}{L}\right)_3 = \frac{2I_{D2}}{(\mu_p C_{Ox})(V_{OV})^2}$$

$$= \frac{2(50\ \mu A)}{(86\ \mu A/V^2)(0.2\ V)^2} = 29.1$$

For Q_1,

$$\left(\frac{W}{L}\right)_1 = \frac{2(50\ \mu A)}{(387\ \mu A/V^2)(0.2\ V)^2} = 6.46$$

A_V must be at least $-10\ V/V$,

and $A_V = -g_{m}(r_{O1} \parallel r_{O2})$

If we want to make r_{O1} and r_{O2} equal,

$$A_V = -\frac{1}{2}g_{m1}r_O$$

$$\text{so, } r_O = \frac{A_V}{-1/2\ g_{m1}}$$

$$g_m = \frac{I_{D1}}{V_{OV/2}} = \frac{50\ \mu A(2)}{0.2\ V} = 0.5\ mA/V$$

$$r_O = \frac{-10\ V/V}{-(1/2)\ (0.5\ mA/V)} = 40\ k\Omega$$

$$r_O = \frac{|V_A'|L}{|I_D|} \text{ so,}$$

for Q_1, $L_1 = \frac{40\ k\Omega(0.05\ mA)}{5\ V/\mu m} = 0.4\ \mu m$

for Q_2 and Q_3,

$$L_2 = L_3 = \frac{40\ k\Omega(0.05\ \mu A)}{6V/\mu m} = 0.33\ \mu m$$

Since we want $L_1 = L_2 = L_3$ and L be an integer multiple of $0.18\ \mu m$, we choose

$L = 3(0.18\ \mu m) = 0.54\ \mu m$

(Note: Choosing $0.36\ \mu m$ results in slightly less than $-10\ V/V$.)

checking,

$$r_{O1} = \frac{V'_{An}L}{I_D} = \frac{5V/\mu m\ (0.54\ \mu m)}{0.05\ mA} = 54\ k\Omega$$

$$r_{O2} = r_{O3} = \frac{6V/\mu m\ (0.54\ \mu m)}{0.05\ mA}$$

$$= 64.8\ k\Omega$$

$$A_V = -g_{m1}(r_{O1} \parallel r_{O2})$$

$$= -0.5\ mA/V(54\ K \| 64.8\ K)$$

$$= -14.7\ V/V$$

If the gain is to be doubled, and the $\frac{W}{L}$ ratios kept the same, $r_{O1} \parallel r_{O2}$ must double.

If r_{O1} and r_{O2} had been equal, this would have meant doubling L and W, making the area 4 times greater.

For a gain of $-20\ V/V$,

$L_1 = 0.8\ \mu m$

$L_2 = 0.67\ \mu m$

The closest integer multiple that satisfies our requirement is $(0.18\ \mu m)(5) = 0.9\ \mu m$.

so, with $L_1 = L_2 = L_3$,

$$r_{O1} = \frac{5\ V/\mu m\ (0.9\ \mu m)}{0.05\ mA} = 90\ k\Omega$$

$$r_{O2} = \frac{6\ V/\mu m\ (0.9\ \mu m)}{0.05\ mA} = 133\ k\Omega$$

This results in a gain of

$A_V = -(0.5\ mA/V)(90\ k\Omega \parallel 133\ k\Omega)$

$A_V = -26.8\ V/V$

This represents an increase in area of

$$\left(\frac{0.9}{0.54}\right)^2 = 2.78 \text{ (instead of 4)}$$

6.14

$$K = 40 = g_{m2}r_{O2} = \frac{V_A}{V_{OV/2}}$$

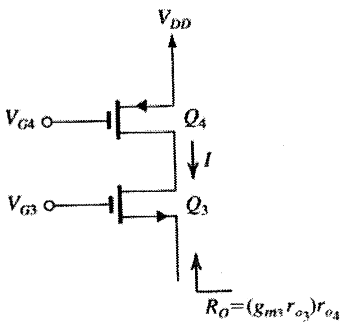
so that

$$V_A = \frac{KV_{OV}}{2} = \frac{40(0.2V)}{2} = 4\text{ V}$$

If $V_A' = 5\text{ V}/\mu\text{m}$,

$$L = \frac{V_A}{V_A'} = \frac{4\text{ V}}{5\text{ V}/\mu\text{m}} = 0.8\ \mu\text{m}$$

6.15



$$V_{OV_3} = V_{OV_4} = V_{OV}$$

$$L_4 = L_3 = L$$

$$V_{A3}' = V_{A4}' = V_A'$$

$$g_{m3}r_{O3} = A_{O3} = \frac{|V_A'|L}{|V_{OV/2}|}$$

$$V_{O4} = \frac{|V_A'|L}{I}$$

so that

$$R_O \simeq \frac{2|V_A'|L}{|V_{OV}|} \cdot \frac{|V_A'|L}{I}$$

Finally, $IR_O = \dfrac{2|V_A'|^2L^2}{|V_{OV}|}$

Now, $|V_A'| = 5\text{ V}/\mu\text{m}$, $|V_{OV}| = 0.2\text{ V}$

For $L = 0.18\ \mu\text{m}$:

$$IR_O = \frac{2(5\text{ V}/\mu\text{m})^2(0.18\ \mu\text{m})^2}{(0.2\text{ V})} = 8.1\text{ V}$$

$$R_O = \frac{IR_O}{I} = \frac{8.1\ V}{0.01\text{ mA}} = 810\text{ k}\Omega$$

$$g_m = \frac{|I|}{|V_{OV}|/2} = \frac{0.01\text{ mA}}{(0.2\text{ V}/2)} = 0.1\text{ mA/V}$$

$$2WL = 2(0.18\ \mu\text{m})(0.18\ \mu\text{m})n = 0.065\text{ n}$$

Assuming that the driving NMOS transistors have similar g_m and R_O,

$$A_V = -\frac{1}{2}g_m R_O$$

$$A_V = -\frac{1}{2}(0.1\text{ mA/V})(810\text{ K}) = -40.5\text{ V/V}$$

For $L = 0.36\ \mu\text{m}$:

$$IR_O = \frac{2(5\text{ V}/\mu\text{m})^2(0.36\ \mu\text{m})^2}{(0.2\text{ V})} = 32.4\text{ V}$$

$$R_O = \frac{32.4\text{ V}}{0.01\text{ mA}} = 3.240\text{ k}\Omega$$

g_m remains unchanged

$$A_V = -\frac{1}{2}(0.1\text{ mA/V})(3.240\text{ K}) = -162\text{ V/V}$$

Area $= 2\text{ LW} = (0.36\ \mu\text{m})^2 n(2) = 0.26\ n\mu\text{m}^2$

For $L = 0.54\ \mu\text{m}$:

$$IR_O = \frac{2(5\text{ V}/\mu\text{m})^2(0.54\ \mu\text{m})^2}{(0.2\text{ V})} = 72.9\text{ V}$$

$$R_O = \frac{72.9\text{ V}}{0.01\text{ mA}} = 7.290\text{ k}\Omega$$

$$A_v = -\frac{1}{2}(0.1\text{ mA/V})(7.290\text{ k}\Omega)$$

$$= -364.5\text{ V/V}$$

Area $= 2(0.54\text{ n})(0.54) = 0.58\text{n}\ \mu\text{m}^2$

Now, use $I = 0.1$ mA:

$L = 0.18\ \mu\text{m}$

Since $I_D = \frac{1}{2}k_p'(W/L)V_{ov}^2$,

W/L will be ten times larger (10n)

$$g_m = \frac{(0.1\text{ mA})(2)}{(0.2\text{ v})} = 1\text{ mA/V}$$

$$R_O = \frac{IR_O}{I} = \frac{8.1\text{ V}}{0.1\text{ mA}} = 81\text{ k}\Omega$$

$$A_v = -\frac{1}{2}(1\text{ mA/v})(81\text{ K}) = -40.5\text{ V/V}$$

Area $= 2\ WL = 2(10\text{ n})(0.18\ \mu\text{m})^2$

$= 0.65\ n\mu\text{m}^2$

For $L = 0.36\ \mu\text{m}$:

$$R_O = \frac{32.4\text{ V}}{0.1\text{ mA}} = 324\text{ k}\Omega$$

$$A_v = \frac{1}{2}(1\text{ mA/V})(324\text{ K}) = -162\text{ V/V}$$

Area $= 2\ WL = 2(10\text{ n})(0.36\ \mu\text{m})^2$

$= 2.59\ n\mu\text{m}^2$

For $L = 0.54\ \mu\text{m}$:

$$R_O = \frac{72.9\text{ V}}{0.1\text{ mA}} = 729\text{ k}\Omega$$

	$L = L_{min} = 0.18\ \mu m$ $IR_O = 8.1$ V				$L = 2L_{min} = 0.36\ \mu m$ $IR_O = 32.4$ V				$L = 3L_{min} = 0.54\ \mu m$ $IR_O = 72.9$ V			
	g_m	R_O	A_{vo}	2WL	g_m	R_O	A_{vo}	2WL	g_m	R_O	A_{vo}	2WL
	mA/V	kΩ	V/V	μm^2	mA/V	kΩ	V/V	μm^2	mA/V	kΩ	V/V	μm^2
I=0.01 mA W/L = n	0.1	810	−40.5	0.065 n	0.1	3,240	−162	0.26 n	0.1	7,290	−364, 5	0.58 n
I=0.01 mA W/L = 10 n	1.0	81	−40.5	0.65 n	1.0	324	−162	2.6 n	1.0	729	−364, 5	5.8 n
I=0.01 mA W/L = 100 n	10.0	8.1	−40.5	6.5 n	10.0	32.4	−162	26 n	10.0	72.9	−364, 5	58 n

$$A_v = -\frac{1}{2}(1\ \text{mA/V})(729\ \text{K}) = -364.5\ \text{V/V}$$

$$\text{Area} = 2\ WL = (2)(10\ \text{n})(0.54\ \mu\text{m})^2$$
$$= 5.8\ \text{n}\ \mu\text{m}^2$$

Now, for $I = 1.0$ mA,

For $L = 0.18\ \mu$m:

$$\frac{W}{L} = 100\ \text{n}$$

$$g_m = \frac{1\ \text{mA}(2)}{(0.2\ \text{V})} = 10\ \text{mA/V}$$

$$R_O = \frac{8.1\ \text{V}}{1\ \text{mA}} = 8.1\ \text{k}\Omega$$

$$A_V = -\frac{1}{2}(10\ \text{mA/V})(8.1\ \text{k})$$
$$= -40.5\ \text{V/V}$$

$$\text{Area} = 2\ \text{WL} = 2(100\ \text{n})(0.18\ \mu\text{m})^2$$
$$= 6.5\ \text{n}\ \mu\text{m}^2$$

For $L = 0.36\ \mu$m:

$$R_O = \frac{32.4\ \text{V}}{1\ \text{mA}} = 32.4\ \text{k}\Omega$$

$$A_V = -\frac{1}{2}(10\ \text{mA/V})(32.4\ \text{K}) = -162\ \text{V/V}$$

$$\text{Area} = 2\ \text{WL} = 2(100\ \text{n})(0.36\ \mu\text{m})^2$$
$$= 26\ \text{n}\ \mu\text{m}^2$$

For $L = 0.54\ \mu$m:

$$R_O = \frac{72.9\ \text{V}}{1\ \text{mA}} = 72.9\ \text{k}\Omega$$

$$A_V = -\frac{1}{2}(10\ \text{mA/V})(72.9\ \text{K})$$
$$= -364.5\ \text{V/V}$$

$$\text{Area} = 2\ \text{WL} = 2(100\ \text{n})(0.5\ \mu\text{m})^2$$
$$= 58\ \text{n}\ \mu\text{m}^2$$

The table summarizes the calculations.

Comments:

(a) R_O and A_V are increased but at the cost of larger device area. As L increases by a factor of X, A_V and R_O increase by a factor of X^2. The device area increases at this same rate.

(b) g_m increases with $|I|$, but R_O decreases with $\frac{1}{|I|}$.

The device area increases with $|I|$.

(c) Smallest area = 0.065 n μm^2

Largest area = 58 n μm^2 Gain and g_m have been increased, but at the expense of increased device area.

6.16

$g_{m1} = \frac{2I_D}{V_{OV}}$, so,

$I_D = \frac{g_{m1}V_{OV}}{2} = \frac{1\text{ mA/V}(0.2\text{ V})}{2} = 100\ \mu\text{A}$

$R_O = (g_{m2}r_{O2})r_{O1}$

However, if we make $g_{m1} = g_{m2} = g_m$

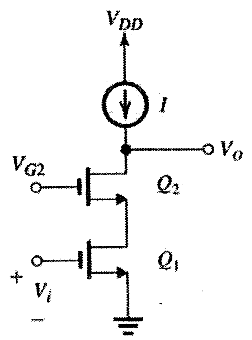
and $r_{O1} = r_{O2} = r_O$, we can say

that $400\text{ k}\Omega = 1\text{ mA/V}\cdot r_O^2$

$r_O^2 = \frac{400\text{ k}\Omega}{1\text{ mA/V}} \Rightarrow r_O = 20\text{ k}\Omega$

since $r_O = \frac{V_A'L}{I_D}$,

$L = \frac{I_D r_O}{V_A'} = \frac{100\ \mu\text{A}(20\text{ k}\Omega)}{5\text{ V}/\mu\text{m}} = 0.4\ \mu\text{m}$



$g_m = \sqrt{2\,\mu_n C_{ox}(W/L)}\cdot\sqrt{I_D}$ so that

$\frac{W}{L} = \frac{g_m^2}{2\,\mu_n C_{ox} I_D}$

$= \frac{(1000\ \mu\text{A/V})^2}{2(400\ \mu\text{A/V}^2)(100\ \mu\text{A})} = 12.5$

For maximum negative excursion at the output, we want the MOSFETs to be biased so that each transistor can reach $V_{DS} = V_{OV} = 0.2$ V.

$\therefore$ Set $V_{G2} = V_{tn} + V_{OV} + V_{OV}$

$= 0.5 + 0.2 + 0.2 = 0.9$ V

minimum output voltage will be

$2\,V_{OV} = 0.4$ V

6.17

$g_{m1} = \frac{I_{D_1}}{\frac{V_{OV}}{2}} = \frac{100\ \mu\text{A}}{(0.25\text{ V})/2} = 800\ \mu\text{A/V}$

Since all devices have the same V_A and I_D,

$r_{O1} = r_{O2} = r_{O3} = r_{O4}$

$= \frac{|V_A|}{I_D} = \frac{4\text{ V}}{0.1\text{ mA}} = 40\text{ k}\Omega$

$R_{on} = g_m r_O^2 = (0.8\text{ mA/V})(40\text{ k}\Omega)^2$

$= 1.28\text{ M}\Omega$

$R_{op} = g_m r_O^2 = 1.28\text{ M}\Omega$

$R_O = R_{on} \| R_{op} = 640\text{ k}\Omega$

$A_V = -g_{m1}R_O$

$= -800\ \mu\text{A/V}\ (640\text{ k}\Omega)$

$= -512$ V/V

6.18

Since $A_V = -g_{m1}R_O$

$R_O = \frac{A_V}{-g_{m1}} = \frac{-200}{-2\text{ mA/V}} = 100\text{ k}\Omega$

If all have the same I_D and V_A', and

since $R_O = R_{on} \| R_{op}$, and

$g_{m1} = g_{m2} = g_{m3} = g_{m4} = g_m$,

$R_O = (g_m r_O^2) \| (g_m r_O^2) = \frac{1}{2} g_m r_O^2$

solving for r_O, we get

$r_O = \sqrt{\frac{2R_O}{g_m}} = \sqrt{\frac{2(100\text{ k}\Omega)}{2\text{ mA/V}}} = 10\text{ k}\Omega$

$I = \frac{g_m|V_{OV}|}{2} = \frac{2\text{ mA/V}(0.2\text{ V})}{2}$

$= 0.2\text{ mA} = 200\ \mu\text{A}$

Since $r_O = \frac{|V_A'|L}{I_D}$,

$L = \frac{r_O I}{|V_A'|} = \frac{10\text{ k}\Omega(0.2\text{ mA})}{5\text{ V}/\mu\text{m}} = 0.4\ \mu\text{m}$

Since $g_m = \sqrt{(2\mu_n C_{ox})(W/L)} \cdot \sqrt{I_D}$

$$\left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = \frac{g_m^2}{2\mu_n C_{ox} I_D}$$

$$= \frac{(2\text{ mA/V})^2}{2(400\ \mu\text{A/V}^2)(200\ \mu\text{A})} = 25$$

Similarly,

$$\left(\frac{W}{L}\right)_3 = \left(\frac{W}{L}\right)_4$$

$$= \frac{(2\text{ mA/V})^2}{2(100\ \mu\text{A/V}^2)(200\ \mu\text{A})} = 100$$

6.19

a) $I = \frac{1}{2} k_n' \frac{W}{L} V_{OV}^2$

$$\Rightarrow \text{ For same } I\text{: } \frac{V_{OVb}^2}{V_{OVa}^2} = \frac{\left(\frac{W}{L}\right)_a}{\left(\frac{W}{L}\right)_b}$$

For same I, if $\frac{W}{L}$ is divided by 4,

then V_{OV}^2 is multiplied by 4, or equivalently

V_{OV} is doubled $g_m = \mu_n C_{ox} \frac{W}{L} V_{OV}$ Thus g_m for circuit (b) is half of the one for circuit(a).

$A_O = g_m r_O = \frac{2I_D}{V_{OV}} \times \frac{V_A}{I_D} = \frac{2V_A' L}{V_{OV}}$. Thus, if L is multiplied by 4, and V_{OV} is halved, then A_O is doubled for circuit(b).

In summary, for circuit (b), V_{OV} is doubled, gm is halved, A_O is doubled.

(b) Each transistor is circuit (c) has the same V_{OV} as the one in circuit (a).

$A_{VO} = -A_O^2 = -(g_m r_O)^2$

$G_m \approx g_{m1} = g_m$ (same as circuit (a))

Note that for the transistor in (c), the g_m and r_O are the same as those in circuit (a). In summary, for circuit(b), V_{ov} is doubled, g_m is halved A_o is doubled.

(b) Each transistor in circuit (c) has the same V_{ov} as the one in circuit (a).

$A_{VO} = -A_o^2 = -(g_m r_o)^2$

$G_m = g_{m1} = g_m$ (same as circuit (a))

Note that for the transistor in (c), the g_m and r_o are the same as those is circuit (a).

Thus, the intrinsic gain for circuit (c), $A_{vo} = -A_o^2$ where A_o is the intrinsic gain for circuit (a).

In general, circuit (c) has a higher output resistance, and for the same V_{OV} of transistors it has lower output swing. The output swing is limited to 2 V_{OV} on the low side for circuits (b) and (c), but limited to only V_{OV} in circuit (a)

6.20

For Q_1,

$V_{OV} = V_i - V_{tn} = 0.8 - 0.5 = 0.3$ V

Since all transistors are identical, and

$k_{n1} = k_{n2} = k_{p3} = k_{p4}$

with $I_{D1} = I_{D2} = I_{D3} = I_{D4}$.

$|V_{OV}| = 0.3$ V (since $I_D = \frac{1}{2} k|V_{OV}|^2$.)

with V_{G2} and V_{G3} fixed,

$V_{S2} = V_{G2} - V_{GS2}$

$= 1.2 - 0.5 - 0.3 = 0.4$ V

$V_{S3} = V_{G3} + V_{GS3}$

$= 1.3 + 0.5 + 0.3 = 2.1$ V

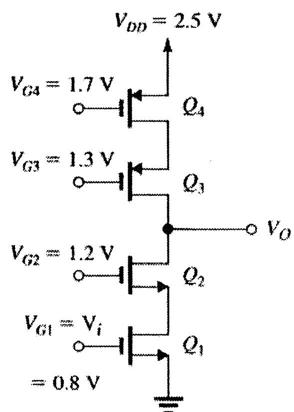
The lowest V_O is

$V_{S2} + V_{OV2} = 0.4 + 0.3 = 0.7$ V

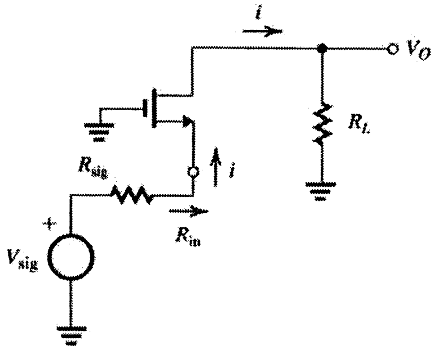
The highest V_O is

$V_{S3} - V_{OV3} = 2.1 - 0.3 = 1.8$ V

so the output range is 0.7 V to 1.8 V



6.21



a)

$$R_{in} = \frac{R_L + r_O}{1 + g_m r_O} = \frac{R_L}{g_m r_O} + \frac{1}{g_m}$$

(b) $V_O = i\,R_L$ and

$$i = \frac{V_{sig}}{R_{sig} + R_{in}} = \frac{V_{sig}}{R_{sig} + \dfrac{R_L + r_O}{1 + g_m r_O}}$$

multiplying and dividing by V_{sig}, we get

$$\frac{V_O}{V_{sig}} = \frac{R_L}{R_{sig} + \dfrac{R_L + r_O}{1 + g_m r_O}} \simeq \frac{R_L}{R_{sig} + \dfrac{R_L}{g_m r_O} + \dfrac{1}{g_m}}$$

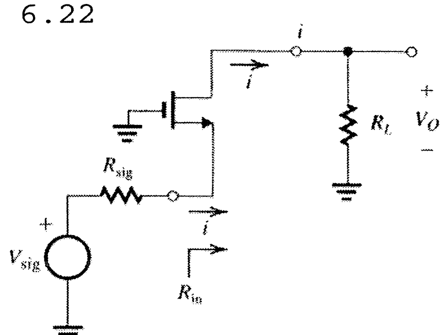
c) If $R_L = r_O = 10\text{ k}\Omega$, $A_O = 20$,

$$R_{sig} = 1\text{K},\ g_m = \frac{A_O}{r_O} = \frac{20}{10\text{k}\Omega} = 2\text{mA/V}$$

$$R_{in} \simeq \frac{10\text{ k}\Omega}{20} + \frac{1}{2\text{ mA/V}} = 1\text{ k}\Omega$$

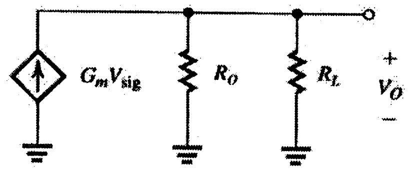
$$\frac{V_O}{V_{sig}} = \frac{R_L}{R_{sig} + R_{in}} = \frac{10\text{ k}\Omega}{1\text{ k}\Omega + 1\text{ k}\Omega} = 5\text{ V/V}$$

6.22



a) If d is shorted to ground, the current flowing through the short is

$$i = \frac{V_{sig}}{R_{sig} + R_{in}} = \frac{V_{sig}}{R_{sig} + \dfrac{1}{g_m}}$$



$$G_m = \frac{i}{V_{sig}} = \frac{1}{R_{sig} + \dfrac{1}{g_m}}$$

From Fig. 713,

$$R_O = r_O + R_{sig} + (g_m r_O)R_{sig}$$

b) If $r_O + 10\text{ k}\Omega$, and

$$g_m = \frac{A_O}{r_O} = \frac{20}{10\text{ k}\Omega} = 2\text{ mA/V},$$

$$G_m = \frac{1}{R_{sig} + \dfrac{1}{g_m}} = \frac{1}{1\text{ k}\Omega + \dfrac{1\text{ V}}{2\text{ mA}}}$$

$$= 0.67\text{ mA/V}$$

$R_O + 10\text{ k}\Omega + 1\text{ k}\Omega + (20)(1\text{ k}\Omega) = 31\text{ k}\Omega$

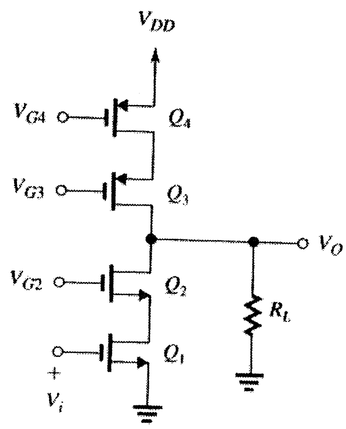
Using the new model,

$$V_O = G_m V_{sig}(R_O \parallel R_L)$$

$$\frac{V_O}{V_{sig}} = G_m(R_O \parallel R_L)$$

$= 0.67\text{ mA/V}\ (31\text{ k}\Omega \parallel 10\text{ k}\Omega) = 5.04\text{ V/V}$

6.23



If $|V'_{Ap}| = V'_{An}$,

$$r_{O1} = r_{O2} = r_{O3} = r_O = \frac{V'_A L}{I_D}$$

$$= \frac{(5\text{ V}/\mu\text{m})(0.36\ \mu\text{m})}{200\ \mu\text{A}}$$

$r_O = 9\text{ k}\Omega$

$$I_D = \frac{1}{2}k_n'\left(\frac{W}{L}\right)V_{OV1}^2$$

$$V_{OV1} = \sqrt{\frac{2I_D}{k_n'(W/L)}}$$

$$= \sqrt{\frac{2(200\ \mu A)}{400\ \mu A/V^2\left(\frac{5.4}{0.36}\right)}} = 0.26\ V$$

$$g_{m1} = \frac{I_{D1}}{\frac{V_{OV}}{2}} = \frac{200\ \mu A}{0.26/2} = 1.54\ mA/V$$

$$V_{OV3} = V_{OV4} = \sqrt{\frac{2(200\ \mu A)}{100\ \mu A/V^2\left(\frac{5.4}{0.36}\right)}}$$

$$= 0.52\ V$$

$$g_{m3} = g_{m4} = \frac{200\ \mu A}{(0.52\ V)/2} = 769\ \mu A/V$$

$$R_{on} \simeq (g_{m3}r_{O3})r_{O1}$$

$$= (1.54\ mA/V)(9\ k\Omega)^2 = 125\ k\Omega$$

$$R_{OP} = (g_{m3}r_{O3})r_{O4}$$

$$= (0.769\ mA/V)(9\ k\Omega)^2 = 62.3\ k\Omega$$

$$R_o = R_{op} \| R_{on}$$

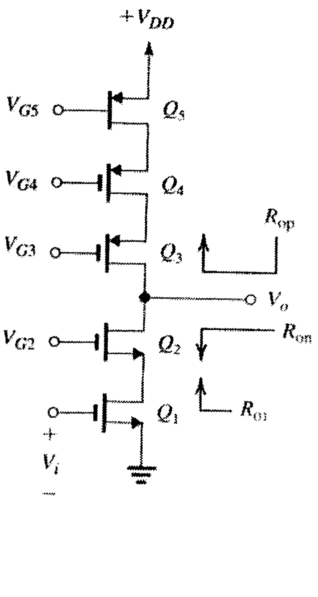
$$= 62.3\ k\Omega \| 125\ k\Omega = 41.6\ k\Omega$$

The value of $R_o \| R_L$ needed is

$$R_O \| R_L = \frac{A_V}{-g_{m1}} = \frac{-100}{1.54\ mA/V} = 64.9\ k\Omega$$

This is greater than R_O!

This can't be done with the present design. One thing we could do is double cascode the current source to raise R_{op}:



This raises R_{op} to

$$R_{op} = (g_{m3}r_{O3})(g_{m4}r_{O4})r_{O5}$$

$$= (0.769\ mA/V)^2(9\ k\Omega)^3 = 431\ k\Omega$$

We can now find an R_L that will allow a gain of -100 V/V: Since

$$431\ k\Omega \| 125\ k\Omega = 96.9\ k\Omega$$

Setting $\frac{R_L(96.9)}{R_L + (96.9)} = 65\ k\Omega$, we get

$$R_L = 197\ k\Omega$$

To find the gain of the CS amplifier, we calculate R_{o1}:

$$R_{O1} = (R_{op} \| R_L)(g_{m3}r_{O3}) \| r_{O1}$$

$$R_{O1} = [(431\ k\Omega \| 197\ k\Omega)(1.5\ mA/V)(9\ k\Omega)] \| (9\ k\Omega)$$

$$R_{O1} = 9\ k\Omega$$

so, $A_{V1} = -g_{m1}R_{O1} = (1.54\ mA/V)(9\ k\Omega)$

$$= -13.9\ V/V$$

6.24

$$R_1 = r_O$$

$$R_2 = g_m r_O^2$$

$$R_3 \simeq \frac{1}{g_m} + \frac{g_m r_O^2}{g_m r_O}$$

$$R_3 \simeq \frac{1}{g_m} + r_O$$

b) $i_1 = V_i g_m$

By current division,

$$i_2 = \frac{i_1 R_3}{r_O + R_3} = \frac{V_i g_m\left(\frac{1}{g_m} + r_O\right)}{r_O + \frac{1}{g_m} + r_O}$$

$$= \frac{V_i(1 + g_m r_O)}{\frac{1}{g_m} + 2r_O}$$

also,

$$i_3 = \frac{i_1 r_O}{r_O + R_3} = \frac{V_i g_m r_O}{r_O + \frac{1}{g_m} + r_O} = \frac{V_i g_m r_O}{\frac{1}{g_m} + 2r_O}$$

$$i_4 = i_5 = i_7 = i_3 = \frac{V_i g_m r_O}{\frac{1}{g_m} + 2r_O}$$

c)

$$V_1 = -V_i g_m (r_O \| R_3)$$

$$= \frac{-V_i g_m (r_O)\left(\frac{1}{g_m} + r_O\right)}{r_O + \frac{1}{g_m} + r_O}$$

$$V_1 = \frac{-V_i g_m r_O}{1 + \frac{r_O}{\frac{1}{g_m} + r_O}}$$

$$= \frac{-V_i g_m r_O}{1 + \frac{1}{1 + \frac{1}{g_m r_O}}} \simeq \frac{-1}{2} V_i g_m r_O$$

$$V_2 = V_i g_m [(g_m r_O^2) \| (g_m \| r_O^2)]$$

$$= \frac{1}{2}(g_m r_O)^2$$

$$V_3 = \frac{-V_i g_m r_O}{\frac{1}{g_m} + 2r_O} r_O = \frac{-V_i g_m r_O}{\frac{1}{g_m r_O} + 2}$$

$$\simeq -\frac{1}{2} V_i g_m r_O$$

d) $V_1(t) \simeq \frac{-1}{2} V_i g_m r_O$

with $V_{i\text{peak}} = 5$ mV,

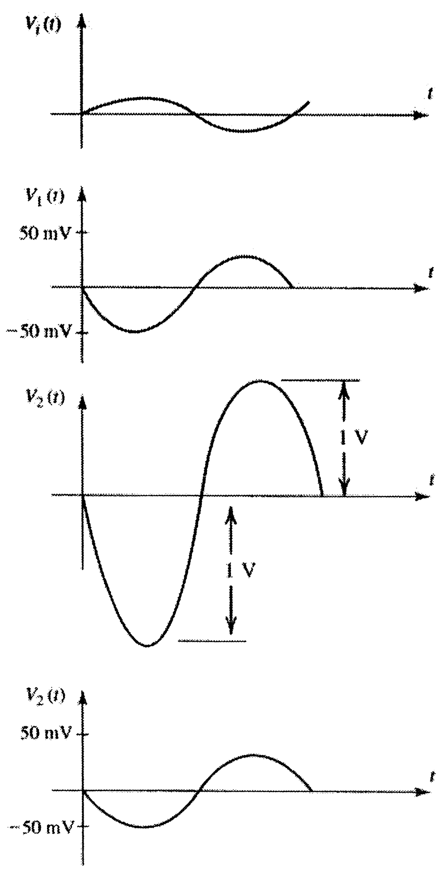
$$V_{1\text{peak}} = -\frac{1}{2}(5 \text{ mV})(20) = -50 \text{ mV}_{\text{peak}}$$

$$V_2(t) = -\frac{1}{2} V_i (g_m r_O)^2$$

$$V_{2\text{peak}} = -\frac{1}{2}(5 \text{ mV})(20)^2 = -1 \text{ V}_{\text{peak}}$$

$$V_3(t) \simeq -\frac{1}{2} V_i (g_m r_O)$$

$$V_{3\text{peak}} = -\frac{1}{2}(5 \text{ mV})(20) = -50 \text{ mV}_{\text{peak}}$$



6.25

Since $I_D = \frac{1}{2}(\mu_p C_{ox})\left(\frac{W}{L}\right)|V_{OV}|^2$,

$$\frac{W}{L} = \frac{2I_D}{(\mu_p C_{ox})|V_{OV}|^2}$$

$$= \frac{2(100\ \mu\text{A})}{(100\ \mu\text{A}/\text{V}^2)(0.2\ \text{V})^2}$$

$$\frac{W}{L} = 50$$

(for all transistors)

$$r_O = \frac{|V_A|L}{I_D} = \frac{(6\ \text{V}/\mu\text{m})(0.18\ \mu\text{m})}{100\ \mu\text{A}}$$

$= 10.8$ kΩ

To permit the maximum swing, each $V_{DS_{\min}}$ should equal $|V_{OV}|$. So,

$V_{G1} = V_{DD} - |V_{tp}| - |V_{OV}|$
$= 1.8 - 0.5 - 0.2 = 1.1\text{ V}$
$V_{G2} = V_{D1_{max}} - |V_{tp}| - |V_{OV}|$
$= (1.8 - 0.2) - 0.5 - 0.2 = 0.9\text{ V}$
$V_{G3} = V_{D2_{max}} - |V_{tp}| - |V_{OV}|$
$= (1.8 - 0.2 - 0.2) - 0.5 - 0.2 = 0.7\text{ V}$
$R_O \simeq r_{O1}(g_{m2}r_{O2})(g_{m3}r_{O3})$
$\simeq$
$g_m^2 r_O^3 = (1\text{ mA/V})^2(10.8\text{ k}\Omega)^3 = 1.26\text{ M}\Omega$

6.26

a) Assuming that all transistors have the same g_m and r_o,
$R_{o1} = r_o$
$R_{o1} = r_o$
$R_{o3} = (g_{m3}\, r_{o3})$.

$$R_{O1} \| R_{O2} = g_m r_O\left(\frac{1}{2}r_O\right) = \frac{1}{2}g_m r_O^2$$

$R_{o4} \simeq (g_{m4}\, r_{o4})\ r_{o5} = g_m r_o^2$

$$R_{in3} = \frac{1}{g_m} + \frac{R_{O4}}{g_m r_o} = \frac{1}{g_m} + \frac{g_m r_o^2}{g_m r_o} = \frac{1}{g_m} + r_o$$

b) $R_o = R_{o3} \| R_{o4} = \left(\frac{1}{2}g_m r_o^2\right) \| (g_m r_o^2)$

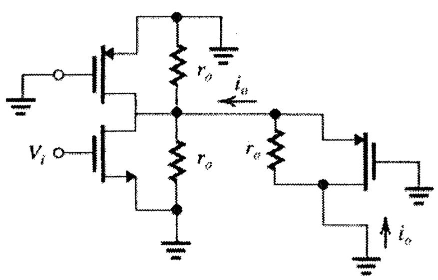
$$R_O = \frac{\frac{1}{2}g_m^2 r_O^4}{\frac{1}{2}g_m r_O^2 + g_m r_O^2}$$

$= \frac{1}{3}g_m r_O^2$

c) If V_o is shorted to ground,

$$R_{in3} = \frac{1}{g_m} + \frac{0}{g_m r_o} = \frac{1}{g_m}$$

Using current division,



$$i_o = g_{m1}V_i\frac{\frac{1}{2}r_o}{\frac{1}{2}r_o + \frac{1}{g_m}} = \frac{g_{m1}V_i}{1 + \frac{2}{g_m r_o}}$$

$$G_m = \frac{i_o}{V_i} = \frac{g_{m1}}{1 + \frac{2}{g_m r_o}} = g_{m1}$$

d) If $R_L = R_{o4}$,

$$R_{in3} = \frac{1}{g_m} + r_o$$

$$i_o = \frac{V_i g_m\left(\frac{r_o}{2}\right)}{r_o/2 + \frac{1}{g_m} + r_o} = \frac{V_i g_m r_o}{3r_o + \frac{2}{g_m}}$$

$$\Rightarrow \frac{V_o}{V_i} = \frac{g_m r_o(g_m r_o^2)}{3r_o + \frac{2}{g_m}}$$

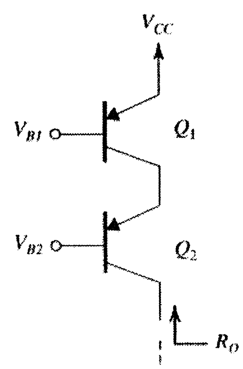
Calculating: $r_o = \frac{A_o}{g_m} = \frac{20}{2\text{ mA/V}} = 10\text{ k}\Omega$

$$\frac{V_o}{V_i} = \frac{-(g_m r_o)^2 r_o}{3r_o + 2/g_m} = -\frac{(20)^2 10\text{ k}\Omega}{3(10\text{ k}\Omega) + \frac{2}{2\text{ mA/V}}}$$

$= -129\text{ V/V}$

6.27

$\beta = 50,\ V_A = 5\text{ V},\ I = 0.5\text{ mA}$
If the base currents are ignored, we can use the same r_o and g_m for each transistor.



$$g_m = \frac{I_c}{V_t} = \frac{0.5\text{ mA}}{25\text{ mV}} = 20\text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{50}{20\text{ mA/V}} = 2.5\text{ k}\Omega$$

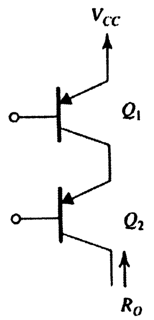
$$r_o = \frac{|V_A|}{I_c} = \frac{5\text{ V}}{0.5\text{ mA}} = 10\text{ k}\Omega$$

$R_o = (g_{m2}r_{o2})(r_{o4} \| r_{\pi 3})$

$R_o = \left(\frac{20\text{ mA}}{\text{V}}\right)(10\text{ k}\Omega)(10\text{ k}\Omega \| 2.5\text{ k}\Omega)$

$R_o = 400\text{ k}\Omega$

6.28



If the transistors are identical,

$$r_{O1} = r_{O2} = r_O = \frac{|V_A|}{I_C}$$

$$g_{m1} = g_{m2} = g_m = \frac{|I_C|}{V_T}$$

$$r_{\pi 1} = r_{\pi 2} = r_\pi = \frac{\beta}{g_m} = \frac{\beta V_T}{|I_C|}$$

$$R_o \simeq (g_{m2} r_{O2})(r_{O1} \parallel r_{\pi 2})$$

$$R_o = \left(\frac{|I_C|}{V_T} \cdot \frac{|V_A|}{|I_C|}\right)\left(\frac{|V_A|}{|I_C|} \parallel \frac{\beta V_T}{|I_C|}\right)$$

$$R_o = \frac{|V_A|}{V_T} \cdot \left[\frac{\frac{|V_A|}{|I_C|} \cdot \frac{\beta V_T}{|I_C|}}{\frac{|V_A|}{|I_C|} + \frac{\beta V_T}{|I_C|}}\right] \text{ with } I_C = I,$$

$$IR_o = \frac{|V_A|}{V_T}\left[\frac{|V_A| \cdot \beta V_T}{|V_A| + \beta V_T}\right]$$

$$IR_o = \frac{|V_A|}{V_T} \cdot \frac{\beta V_T}{1 + \frac{\beta V_T}{|V_A|}} = \frac{|V_A|}{V_T} \cdot \frac{1}{\frac{1}{\beta V_T} + \frac{1}{|V_A|}}$$

$$IR_o = \frac{|V_A|}{(V_T / |V_A|) + (1/\beta)}$$

For $|V_A| = 5\text{ V}, \beta = 50$

If $I = 0.1$ mA,

$$R_o = \frac{5\text{ V}}{\frac{0.025\text{ V}}{5\text{ V}} + \frac{1}{50}} \cdot \frac{1}{0.1\text{ mA}} = 2\text{ M}\Omega$$

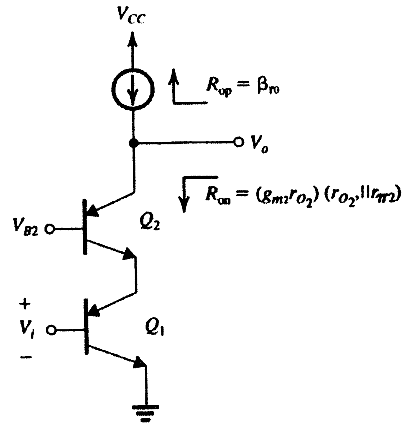
If $I = 0.5$ mA,

$$R_o = 2\text{ M}\Omega\left(\frac{.1\text{ mA}}{.5\text{ mA}}\right) = 400\text{ k}\Omega$$

If $I = 1.0$ mA,

$$R_o = 2\text{ M}\Omega\left(\frac{.1\text{ mA}}{1\text{ mA}}\right) = 200\text{ k}\Omega$$

6.29



$$r_{o1} = r_{o2} = r_o = \frac{|V_A|}{I} = \frac{100\text{ V}}{0.1\text{ mA}} = 1\text{ M}\Omega$$

$$g_{m1} = g_{m2} = g_m = \frac{|I_C|}{V_T} = \frac{0.1\text{ mA}}{25\text{ mV}}$$

$$= 4\text{ mA/V}$$

$$r_{\pi 1} = r_{\pi 2} = r_\pi = \frac{\beta}{g_m} = \frac{100}{4\text{ mA/V}} = 25\text{ k}\Omega$$

$$A_V = -g_{m1}(R_{on} \parallel R_{op})$$

$$R_{op} = \beta r_o = 100(1\text{ M}\Omega) = 100\text{ M}\Omega$$

$$R_{on} = (g_{m2} r_{o2})(r_{o1} \parallel r_{\pi 2})$$

$$= (4\text{ mA/V} \cdot 1\text{ M}\Omega)(1\text{ M}\Omega \parallel 25\text{ k}\Omega)$$

$$R_{on} = 100\text{ M}\Omega$$

so, $A_V = -4\text{ mA/V}\,(100\text{ M} \parallel 100\text{M})$

$= 200{,}000$ V/V

6.30

$$R_o \approx r_o[1 + g_m(R_e \parallel r_\pi)]$$

$$g_m = \frac{|I_C|}{V_T} = \frac{0.5\text{ mA}}{25\text{ mV}} = 0.02\text{ A/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{0.02\text{ A/V}} = 5\text{ k}\Omega$$

when $R_e = 0$, $R_o = r_o$

a) For $R_o = 5 \cdot r_o$,

$$5 = [1 + g_m(R_e \parallel r_\pi)]$$

$$5 = 1 + 0.02\text{ A/V}\,(R_e \parallel 5\text{ k}\Omega)$$

$R_e \| 5\ k\Omega = \dfrac{4}{0.02\ A/V} = 0.2\ k\Omega$

Solving, $\dfrac{R_e \cdot 5\ k\Omega}{R_e + 5\ k\Omega} = 0.2\ k\Omega$

$R_e = \dfrac{5\ k\Omega(0.2\ k\Omega)}{4.8\ k\Omega} = 208\ \Omega$

b) For $R_o = 10 \cdot r_o$,

$10 = 1 + (0.02\ A/V) \cdot (R_e \| 5k)$

So that $\dfrac{R_e \cdot 5\ k\Omega}{R_e + 5\ k\Omega} = \dfrac{9}{0.02\ A/V} = 450\ \Omega$

Solving, $R_e = 495\ \Omega$

c) For $R_o = 50 \cdot r_o$,

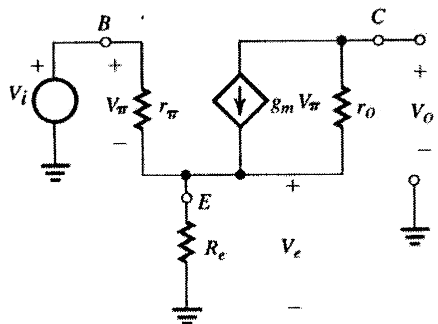
$50 = 1 + 0.02\ A/V(R_e \| 5\ k\Omega)$

$\dfrac{R_e \cdot 5\ k\Omega}{R_e + 5\ k\Omega} = \dfrac{49}{0.02\ A/V} = 2.45\ k\Omega$

$R_e = 4.8\ k\Omega$

6.31

With the output unloaded, the small-signal model can be drawn as follows:



Since no current flows out the collector,

$V_O = -g_m V_\pi r_O + V_e$ By voltage division,

$V_e = \dfrac{V_i R_e}{r_\pi + R_e}$ and $V_\pi = \dfrac{V_i r_\pi}{r_\pi + R_e}$

substituting, we get

$$A_{VO} = \frac{V_O}{V_i} = \frac{-g_m r_O r_\pi + R_e}{r_\pi + R_e}$$

$$A_{VO} = -g_m r_O = \frac{r_\pi - \dfrac{R_e}{g_m r_O}}{r_\pi + R_e}$$

dividing by r_π,

$$A_{VO} = -g_m r_O \cdot \frac{1 - \dfrac{R_e}{g_m r_\pi r_O}}{1 + \dfrac{R_e}{r_\pi}}$$

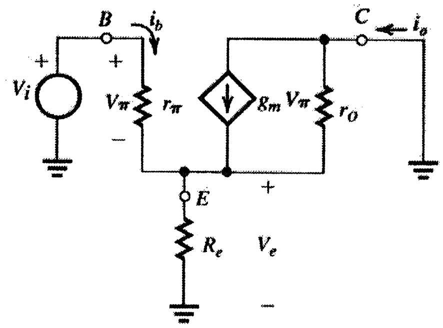
since $g_m r_\pi = \beta$,

$$A_{VO} = -g_m r_O \cdot \frac{1 - \dfrac{R_e}{\beta r_O}}{1 + \dfrac{R_e}{r_\pi}}$$

There are several ways to derive the equation for G_m.

Method 1:

Take the basic small-signal model:



Note that $V_\pi = V_i - V_e$

$$i_O = \frac{0 - V_e}{r_O} + g_m V_\pi$$

$$i_O = -\frac{V_e}{r_O} + g_m(V_i - V_e)$$

Assuming that $i_O >> i_b$

$V_e \simeq i_O R_e$ Then,

$$i_O = \frac{-i_O R_e}{r_O} + g_m V_i + i_O R_e g_m$$

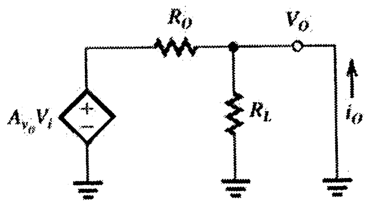
$i_O\left(1 + \dfrac{R_e}{r_O} + R_e g_m\right) = g_m V_i$ so that,

$$G_m = \frac{i_O}{V_i} \simeq \frac{g_m}{1 + \dfrac{R_e}{r_O} + g_m R_e}$$

since $\dfrac{R_e}{r_O} << 1$ usually,

$$G_m = \frac{i_O}{V_i} \simeq \frac{g_m}{1 + g_m R_e}$$

Method 2:
Consider the model



$$R_O = r_O + (Re \,\|\, r_\pi) + (g_m r_O)(R_e \,\|\, r_\pi)$$

or

$$R_O \approx r_O[1 + g_m(Re \,\|\, r_\pi)]$$

Shorting the output removes R_L from the CKT.

$$-A_{VO} = \frac{g_m r_O r_\pi - R_e}{r_\pi + R_e} \text{ (from part 1 above)}$$

$$G_m = \frac{i_O}{V_i} = \frac{-A_{VO}}{R_O} = \frac{\dfrac{g_m r_O r_\pi - R_e}{r_\pi + R_e}}{r_O + g_m r_O \dfrac{R_e r_\pi}{r_\pi + R_e}}$$

$$G_m = \frac{g_m r_O r_\pi - R_e}{r_O(r_\pi + R_e) + g_m r_O R_e r_\pi}$$

Dividing by $r_O r_\pi$, we get

$$G_m = \frac{g_m - \dfrac{R_e}{r_O r_\pi}}{\dfrac{r_O(r_\pi + R_e)}{r_O r_\pi} + g_m R_e}$$

since $\dfrac{r_\pi + R_e}{r_\pi} \simeq 1$ and $\dfrac{R_e}{r_O r_\pi} << g_m$,

$$G_m \simeq \frac{g_m}{1 + g_m R_e}$$

with $\beta = 100$, $r_O = 100\ \text{k}\Omega$,

$I_C = 0.2\ \text{mA}$, and $R_e = 250\ \Omega$,

$$g_m = \frac{|I_C|}{V_T} = \frac{0.2\ \text{mA}}{25\ \text{mV}} = 8\ \text{mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{8\ \dfrac{\text{mA}}{\text{V}}} = 12.5\ \text{k}\Omega$$

$$R_O \simeq r_O + (Re \,\|\, r_\pi)(1 + g_m)$$

$$\simeq r_O + r_O g_m(Re \,\|\, r_\pi)$$

$$R_O \simeq 100\ \text{k}\Omega + (0.25\ \text{k}\Omega \,\|\, 12.5\ \text{k}\Omega)$$

$$(100\ \text{k}\Omega)\left(8\ \frac{\text{mA}}{\text{V}}\right)$$

$$= 296\ \text{k}\Omega$$

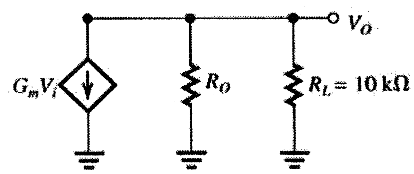
$$A_{VO} = -g_m r_O \frac{1 - \dfrac{R_e}{\beta r_O}}{1 + \dfrac{R_e}{r_\pi}}$$

$$= -(8\ \text{mA/V})(100\ \text{k}\Omega) \cdot \frac{1 - \dfrac{0.25\ \text{k}\Omega}{100(100\ \text{k}\Omega)}}{1 + \dfrac{0.25\ \text{k}\Omega}{12.5\ \text{k}\Omega}}$$

$$A_{VO} = -784\ \text{V/V}$$

$$G_m \simeq \frac{g_m}{1 + g_m R_e} = \frac{8\ \text{mA/V}}{1 + 8\ \text{mA/V}(12.5\ \text{k}\Omega)}$$

$$= 2.67\ \text{mA/V}$$



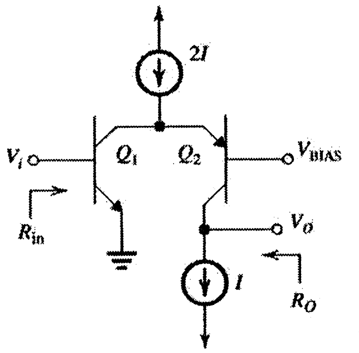
$$A_V = \frac{V_O}{V_i} = -G_m(R_O \,\|\, R_L)$$

$$= -2.67\ \text{mA/V}\ (296\ \text{k}\Omega \,\|\, 10\ \text{k}\Omega)$$

$$A_V = -25.9\ \text{V/V}$$

Note: Depending upon the approximations taken, the values of A_v may very slightly.

6.32



$$g_{m1} = g_{m2} = \frac{|I_C|}{V_T} = \frac{0.1\text{ mA}}{25\text{ mV}} = 4\text{ mA/V}$$

$$r_{\pi 1} = r_{\pi 1} = \frac{\beta}{g_m} = \frac{100}{4\text{ mA/V}} = 25\text{ k}\Omega$$

$$r_{O1} = r_{O2} = \frac{|V_A|}{I} = \frac{5\text{ V}}{0.1\text{ mA}} = 50\text{ k}\Omega$$

$R_{in} = r_{\pi 1} = 25\text{ k}\Omega$

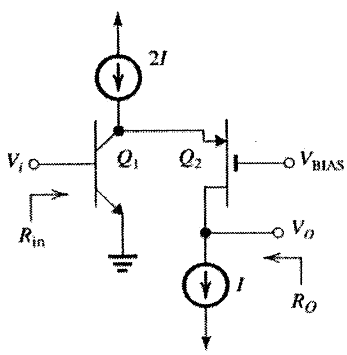
$R_O \simeq (g_{m2}r_{O2})(r_{O1} \| r_{\pi 2})$

$R_O \simeq (4\text{ mA/V})(50\text{ k}\Omega)(50\text{ k}\Omega \| 25\text{ k}\Omega)$

$= 3.33\text{ M}\Omega$

$A_{VO} = -G_m R_O \approx -g_{m1}R_O$

$= -(4\text{ mA/V})(3.33\text{ M}\Omega) = -13.3 \times 10^3\text{ V/V}$



$g_{m1} = 4\text{ mA/V}$

$$g_{m2} = \frac{|I_D|}{\frac{|V_{OV}|}{2}} = \frac{0.1\text{ mA}}{0.2\text{ V/2}} = 1\text{ mA/V}$$

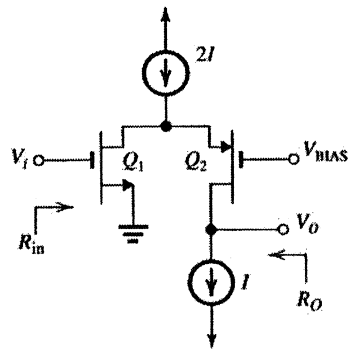
Again, $R_{in} = r_{\pi 1} = 25\text{ k}\Omega$

$R_O = (g_{m2}r_{O2})r_{O1} = (1\text{ mA/V})(50\text{ K})^2$

$= 2.5\text{ M}\Omega$

$A_{VO} = -G_m R_O \approx -g_{m1}R_O$

$= -(4\text{ mA/V})(2.5\text{ M}\Omega) = -10 \times 10^3\text{ V/V}$



From part (b),

$g_{m1} = g_{m2} = 1\text{ mA/V}$

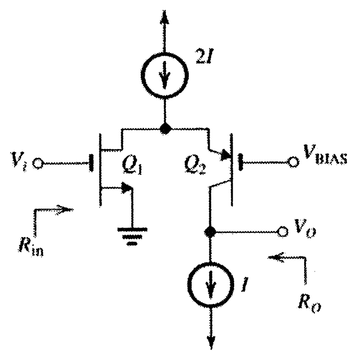
$R_{in} = \infty$

$R_O = (g_{m2}r_{O2})r_{O1}$

$R_O = (1\text{ mA/V})(50\text{ k}\Omega)^2 = 2.5\text{ M}\Omega$

$A_{VO} = -G_m R_O = -g_{m1}R_O$

$A_{VO} = -(1\text{ mA/V})(2.5\text{ M}\Omega) = -2,500\text{ V/V}$



From above,

$g_{m1} = 1\text{ mA/V}$

$g_{m2} = 4\text{ mA/V}, \; r_{\pi 2} = 25\text{ k}\Omega$

$R_{in} = \infty$

$R_O = (g_{m2}r_{O2})(r_{O1} \| r_{\pi 2})$

$R_O = (4\text{ mA/V})(50\text{ k}\Omega)(50\text{ k}\Omega \| 25\text{ k}\Omega)$

$= 3.33\text{ M}\Omega$

$A_{VO} = -G_m R_O$

$A_{VO} \approx -g_{m1}R_O = -1\text{ mA/V}(3.33\text{ M}\Omega)$

$-3.33 \times 10^3\text{ V/V}$

Comments:
(1) A MOSFET for Q_1 makes $R_{in} \to \infty$.
(2) The output resistance when Q_2 is a BJT is limited by $r_{\pi 2}$. In cases (a) and (d), R_o was higher due to the value of r_O and g_{m2}.
(3) In these four cases, A_{VO} was highest with two BJTs A_{vo} was lowest with two MOSFETs. These results could be changed with different biasing.

6.33

$I_o = I_{REF} = 50\mu A$, $L = 0.5\mu m$, $W = 5\mu m$, $V_t = 0.5^V$

$I_o = I_D = \frac{1}{2} k_n' \frac{W}{L} V_{ov}^2$ $\quad k_n' = 250\mu A/V^2$

$50 = \frac{1}{2} \times 250 \times \frac{5}{0.5} (V_{GS} - 0.5)^2 \Rightarrow V_{GS} = 0.7^V$, 0.3^V

$V_{GS} = 0.3V < V_t$ is not acceptable, therefore $\underline{V_{GS} = 0.7^V}$

$I_D = I_{RE} = \frac{V_{DD} - V_{GS}}{R} \Rightarrow \frac{1.8 - 0.7}{R} = 0.050 \Rightarrow \underline{R = 22k\Omega}$

Q_1 and Q_2 have the same V_{GS}. The lowest value of V_o or V_{DS2} is when $V_{DS} = V_{GS} - V_t = 0.7 - 0.5 = 0.2^V$

hence $V_{omin} = 0.2^V$

$r_o = \frac{V_A}{I_D} = \frac{V_A' L}{I_D} = \frac{20 \times 0.5}{0.05} = 200k\Omega$

$\Delta I_o \cong \frac{\Delta V_o}{r_o} = \frac{1}{200^k} = 5\mu A \Rightarrow \underline{\Delta I_o = 5\mu A}$

6.34

$\mu_n C_{ox} = 250\mu A/V^2$, $V_A' = 20V/\mu m$, $V_t = 0.6V$

$\frac{\Delta I_o}{I_o} = 5\% \Rightarrow \Delta I_o = 5\mu A$ for $\Delta V_o = 1.8 - 0.25 = 1.55^V$

$r_o = \frac{\Delta V_o}{\Delta I_o} = \frac{1.55}{5^\mu} = 310k\Omega$

$r_o = \frac{V_A' L}{I_O} \Rightarrow L = I_D \times \frac{r_o}{V_A'} = 0.1 \times \frac{310}{20} = \underline{1.55\mu m}$

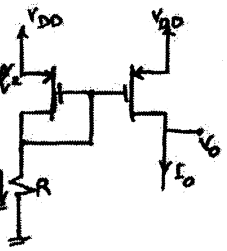
$V_{omin} = V_{GS} - V_t = 0.25 \Rightarrow V_{GS} = 0.25 + 0.6 = 0.85^V$

$R = \frac{V_{DD} - V_{GS}}{I_D} = \frac{1.8 - 0.85}{0.1} = \underline{9.5k\Omega}$

$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_t)^2 \Rightarrow W = \frac{2 L I_D}{\mu_n C_{ox} (V_{GS} - V_t)^2}$

$\Rightarrow W = \frac{2 \times 1.55 \times 100}{250 (0.85 - 0.6)^2} = \underline{19.84\mu m}$

6.35



$V_{DD} = 1.8V$, $|V_t| = 0.6V$, $\mu_p C_{ox} = 100\mu A/V^2$

$I_{REF} = 80\mu A$ $\quad V_{omax} = 1.6^V$

$V_{DS} \leq V_{GS} - V_t$

$V_{omax} = V_{DSmax} = V_{GS} - V_t \Rightarrow$

$1.6 - 1.8 = V_{GS} + 0.6 \Rightarrow V_{GS} = -0.8^V$

$\Rightarrow V_G = 1.8 - 0.8 = 1^V$

$R = \frac{V_G}{I_D} = \frac{1}{0.080} = \underline{12.5k\Omega}$

$I_D = \frac{1}{2} \mu_p C_{ox} (V_{GS} - V_t)^2 \frac{W}{L} \Rightarrow W = \frac{2 L I_D}{\mu_p C_{ox} (V_{GS} - V_t)^2}$

$\frac{W}{L} = \frac{2 \times 80}{100(-0.8 + 0.6)^2} = \underline{40}$

6.36

$W_2 = 4W_1$ $\quad L_1 = L_2$ $\quad V_{ov} = 0.3^V$, $I_{REF} = 20\mu A$

$I_o = I_{REF} \frac{(W/L)_2}{(W/L)_1} = 20 \times 4 = \underline{80\mu A}$

$V_{omin} = V_{ov} = \underline{0.3V}$

$V_t = 0.5V$. According to Eq. 6.11 $I = \frac{(W/L)_2}{(W/L)_1} I_{REF} \left(1 + \frac{V_o - V_{GS}}{V_{A2}}\right)$

$V_{ov} = V_{GS} - V_t \Rightarrow V_{GS} = 0.3 + 0.5 = 0.8V$

$1 + \frac{V_o - V_{GS}}{25} = 1 \Rightarrow \underline{V_o = 0.8V}$

Or we could simply say $V_{DS1} = V_{DS2} = V_o$ and

Since $V_{DS1} = V_{GS1} = 0.8^V \Rightarrow \underline{V_o = 0.8^V}$

$r_{o2} = \frac{V_A}{I_{D2}} = \frac{25}{0.08} = 312.5k\Omega$

$r_{o2} = \frac{\Delta V_o}{\Delta I_o} = \frac{1}{\Delta I_o} \Rightarrow \Delta I_o = \frac{1}{312.5^k} = \underline{3.2\mu A}$

6.37

$V_{GS1} = V_{GS2}$ so that $\frac{I_{D2}}{I_{D1}} = \frac{(W/L)_2}{(W/L)_1}$ and

$$I_{D2} = I_{REF}\frac{(W/L)_2}{(W/L)_1}$$

$$I_{D2} = I_{D3}$$

$V_{GS3} = V_{GS4}$ so that $\frac{(W/L)_3}{(W/L)_4} = \frac{I_{D3}}{I_{D4}} = \frac{I_{D2}}{I_{D4}}$

$$I_O = I_{D4} = I_{REF}\frac{(W/L)_2}{(W/L)_1}\cdot\frac{(W/L)_4}{(W/L)_3}$$

6.38

If the transistor with $W = 10$ is diode-connected, then:
$I_2 = 100 \times \frac{20}{10} = 200\,\mu A$
$I_3 = 100 \times \frac{40}{10} = 400\,\mu A$

If the transistor with $W = 20$ is diode-connected then:
$I_2 = 100 \times \frac{10}{20} = 50\,\mu A$
$I_3 = 100 \times \frac{40}{20} = 200\,\mu A$

If the transistor with $W = 40$ is diode-connected, then:
$I_2 = 100 \times \frac{10}{40} = 25\,\mu A$
$I_3 = 100 \times \frac{20}{40} = 50\,\mu A$

So for cases that only one transistor is diode connected, 4 different output currents are possible. (depending on the configuration we choose).

If 2 transistors are diode-connected, then they act as an equivalent transistor whose width is the sum of the widths of each transistor:

If $W_{eff} = 10 + 20$ then $I_o = 100 \times \frac{40}{30} = 133\,\mu A$
If $W_{eff} = 20 + 40$ then $I_o = 100 \times \frac{10}{60} = 16.7\,\mu A$
If $W_{eff} = 40 + 10$ then $I_o = 100 \times \frac{20}{50} = 40\,\mu A$

So 3 different output currents are possible depending on which two transistors are diode-connected. Now we calculate V_{SG}:

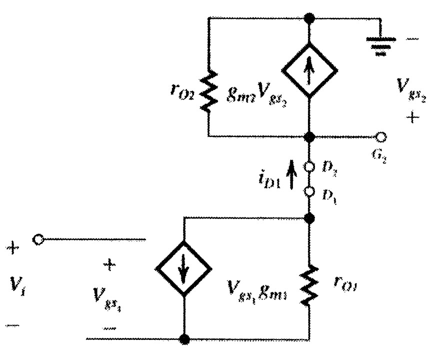
$100 = \frac{1}{2} \times 80 \times \frac{30}{1}(V_{SG} - 0.7)^2 \Rightarrow V_{SG} = 1\,V$ for $W_{eff} = 30\,\mu m$

all have the same V_{SG} for any given configuration.

For $W_{eff} = 60 \Rightarrow 100 = \frac{1}{2} \times 80 \times \frac{60}{1}(V_{SG} - 0.7)^2$
$\Rightarrow V_{SG} = 0.9\,V$

For $W_{eff} = 50 \Rightarrow 100 = \frac{1}{2} \times 80 \times \frac{50}{1}(V_{SG} - 0.7)^2$
$\Rightarrow V_{SG} = 0.93\,V$

6.39 , the small-signal model can be drawn as follows:



(1) $V_O = -g_{m3}(r_{O3} \| R_L)V_{gs2}$

$V_{gs2} = (i_{D1} - g_{m2}V_{gs2})\, r_{O2}$

(2) $V_{gs2} = i_{D1}\, r_{O2} - g_{m3}\, V_{gs2}\, r_{O2}$

(3) $i_{D1} = -V_{gs1}\, g_{m1} - \frac{V_{gs2}}{r_{O1}}$

substituting (3) into (2), we get

$$V_{gs2} = -V_{gs1}\, g_{m1}\, r_{O2} - \frac{V_{gs2}\, r_{O2}}{r_{O1}} - g_{m2}\, V_{gs2}\, r_{O2}$$

(4) $V_{gs2} = \dfrac{-V_{gs1}\, g_{m1}\, r_{O2}}{\left(1 + \frac{r_{O2}}{r_{O1}} + g_{m2}\, r_{O2}\right)}$

substituting (4) into (1), we get

$$V_O = -g_{m3}(r_{O3} \| R_L)\left[\frac{-V_{gs1}\, g_{m1}\, r_{O2}}{\left(1 + \frac{r_{O2}}{r_{O1}} + g_{m2}\, r_{O2}\right)}\right]$$

since $V_{gs1} = V_i$,

$$\frac{V_O}{V_i} = g_{m3}(r_{O3} \| R_L)\left[\frac{g_{m1}\, r_{O2}}{\left(1 + \frac{r_{O2}}{r_{O1}} + g_{m2}\, r_{O2}\right)}\right]$$

divide out r_O:

$$\frac{V_O}{V_i} = \frac{g_{m1}\, g_{m3}(r_{O3} \| R_L)}{\left(\frac{1}{r_{O2}} + \frac{1}{r_{O1}} + g_{m2}\right)}$$

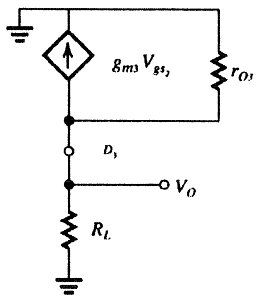
Assuming all r_O values arc >> 1,

$$\frac{V_O}{V_i} = \frac{g_{m1}\, g_{m3}\, R_L}{g_{m2}}$$

Since $I_D = \frac{1}{2}k_p'\left(\frac{W}{L}\right)V_{OV}^2$ and $V_{GS2} = V_{GS3}$,

$V_{OV2} = V_{OV3}$. Also, $g_m = \dfrac{I_D}{V_{OV}/2}$

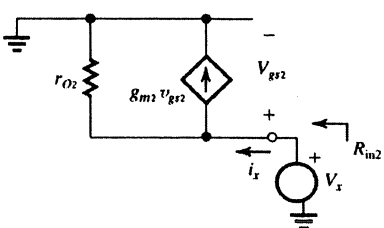
making $g_m \propto I_D$ which is $\propto \dfrac{W}{L}$



Here, $L_2 = L_3$, so we could also express the gain as

$$\frac{V_O}{V_i} = g_{m1} R_L\left(\frac{W_3}{W_2}\right)$$

Now, to find the resistance looking into the diode-connected drain of Q_2, we apply a test voltage V_X:



$$i_X = \frac{V_X}{V_{O2}} + g_{m2}\, V_{gs2}$$

since $V_{gs2} = V_X$, $i_X = \dfrac{V_X}{r_{O2}} + g_{m2}\, V_X$

$$\frac{i_X}{V_X} = \frac{1}{r_{O2}} + g_{m2}$$

$$R_{in2} = \frac{V_X}{i_X} = r_{O2} \| \frac{1}{g_{m2}}$$

The CS gain is

$$\frac{V_{d1}}{V_i} = -g_{m1}\left(\frac{1}{g_{m2}} \| r_{O2} \| r_{O1}\right)$$

6.40

$I_S = 10^{-15}$ A

a) $I_{REF} = I_S\, e^{V_{BE}/V_T} \Rightarrow V_{BE} = V_T \ln \dfrac{I_{REF}}{I_S}$

$I_{REF} = 10\mu A \Rightarrow V_{BE} = 0.025 \ln \dfrac{10\times10^{-6}}{10^{-15}} = 0.576^V$

$I_{REF} = 10mA \Rightarrow V_{BE} = 0.025 \ln \dfrac{10\times10^{-3}}{10^{-15}} = 0.748^V$

Therefore:

$10\mu A \leq I_{REF} \leq 10mA \Rightarrow 0.576^V \leq V_{BE} \leq 0.748^V$

Since β is very high, I_B is negligible and hence

$I_o \simeq I_{REF}$: $10\mu A \leq I_o \leq 10$ mA

b) $I_o = I_{REF} \dfrac{1}{1 + 2/\beta}$

For $0.1^{mA} \leq I_C \leq 5^{mA}$, β remains constant at 100.

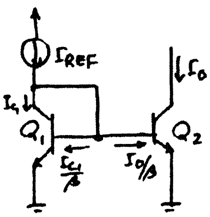
$I_{REF} = 10mA \Rightarrow I_o = \dfrac{10}{1 + \frac{2}{70}} = 9.72mA$

$I_{REF} = 0.1mA \Rightarrow I_o = \dfrac{0.1}{1 + 2/100} = 0.098mA$

$I_{REF} = 1mA \Rightarrow I_o = \dfrac{1}{1 + 2/100} = 0.98mA$

$I_{REF} = 10\mu A \Rightarrow$

6.41



$I_{S2} = I_{S1} \times m$, $I_{c_1} = I_c$

$I_{REF} = I_c + \frac{I_c}{\beta} + \frac{I_o}{\beta}$ ①

$V_{BE1} = V_{BE2} \Rightarrow$

$V_T \ln \frac{I_c}{I_{S1}} = V_T \ln \frac{I_o}{I_{S2}}$

$\Rightarrow \frac{I_o}{I_c} = \frac{I_{S2}}{I_{S1}} = m \Rightarrow I_c = I_o/m$

by substituting for I_c in ①:

$I_{REF} = \frac{I_o}{m} + \frac{I_o}{m\beta} + \frac{I_o}{\beta} \Rightarrow \frac{I_o}{I_{REF}} = \frac{m}{1+\frac{1}{\beta}+\frac{m}{\beta}}$

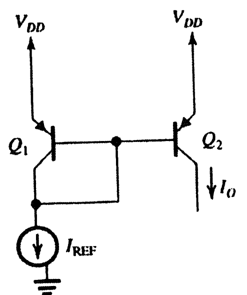
$\frac{I_o}{I_{REF}} = \frac{m}{1+\frac{1+m}{\beta}}$

This result is the same as Eq. 6.22.

For large β, $I_o/I_{REF} = m$, with finite β this ratio drops to $I_o/I_{REF} = \frac{m}{1+\frac{1+m}{\beta}}$. To keep the introduced error within 5%: $0.95m = \frac{m}{1+\frac{1+m}{\beta}}$

$\beta_{min} = 80 \Rightarrow 0.95 = \frac{1}{1+\frac{1+m}{80}} \Rightarrow \underline{m = 3.21}$

6.42



For identical transistors, the transfer ratio is the same as eq. (7.69):

$$\frac{I_O}{I_{REF}} = \frac{1}{1+2/\beta} = \frac{1}{1+\frac{2}{20}} = 0.91$$

6.43

$I_{c_1} = I_{c_2} = I_{R_1}$

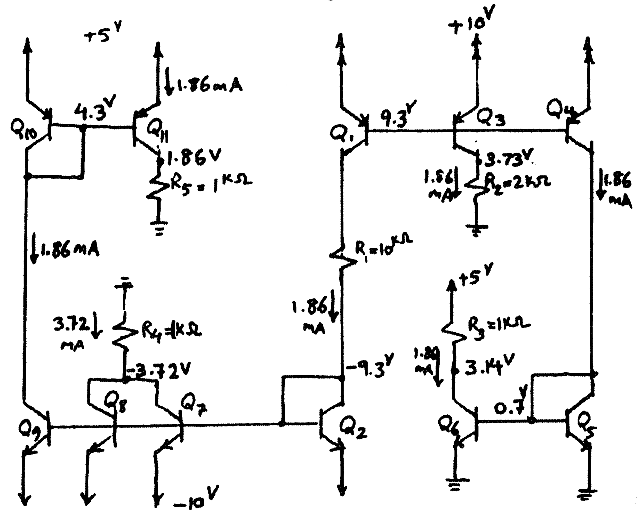
$V_{B1} = 10-0.7 = 9.3V$, $V_{B2} = -10+0.7 = -9.3^V$, $I_{R_1} = \frac{9.3+9.3}{10}$

$\Rightarrow I_{R_1} = 1.86\,mA = I_{c_1} = I_{c_2} = I_{c_3} = I_{c_4} = I_{c_5} = I_{c_6}$

$V_{c_3} = 1.86^m \times 2^K = 3.72V$, $V_{c5} = 0.7^V$

$V_{c6} = 5 - 1.86 \times 1 = 3.14V$, $I_{c9} = I_{c8} = I_{c7} = I_{c2} = 1.86^{mA}$

$I_{R4} = 2 \times 1.86 = 3.72mA \Rightarrow V_{c7} = -3.72 \times 1 = -3.72V$



$I_{c10} = I_{c9} = 1.86mA$

$V_{c9} = V_{c10} = V_{B10} = 5 - 0.7 = 4.3^V$

$I_{c11} = I_{c10} = 1.86mA$

$V_{c11} = 1.86 \times 1 = 1.86V$

6.44

a)

$R = 10k\Omega$

$V_1 = -0.7^V \Rightarrow I_{C_1} = \frac{-0.7-(-10.7)}{10^K} = 1mA$

$I_{C_1} = 1mA$

$V_2 = 5.7 - 0.7 = 5V$

$I = I_{C3} + I_{C4}$, $I_{C3} = I_{C4} = I_{C1} \Rightarrow I = 2\times 1 = 2mA$

$V_3 = 0 + 0.7 = 0.7^V$

$V_4 = -10.7 + 1\times 10^K = -0.7V$

$V_5 = -10.7 + 1\times \frac{10^K}{2} = -5.7^V$

b) $R = 100k\Omega$

$V_1 = -0.7V \Rightarrow I_{C_1} = \frac{-0.7+10.7}{100^K} = 0.1mA$

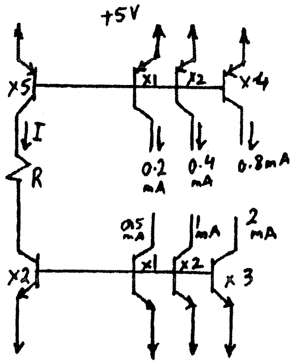
$I = 2 I_{C_1} = 0.2mA$

$V_3 = 0.7V$, $V_2 = 5.7 - 0.7 = 5^V$

$V_4 = -10.7 + \frac{1}{10}\times 100 = -0.7^V$

$V_5 = -10.7 + 0.1\times\frac{100}{2} = -5.7^V$

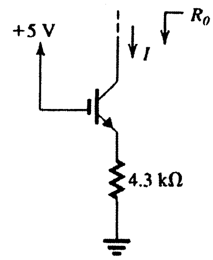
6.45



$I = \frac{10-1.4}{R} = 1mA$

$\Rightarrow R = 8.6K\Omega$

6.46



$$I_E = \frac{5 - V_{BE}}{4.3\ k\Omega} = \frac{5 - 0.7}{4.3\ k\Omega} = 1\ mA$$

since $\beta >> 1$, $I = I_C \approx I_E \approx 1$ mA

To find the output resistance, we can use eq. (7.50) or since $g_m r_O >> 1$,

$R_O \approx r_O + g_m r_O (R_e \| r_\pi)$

In this case,

$$g_m = \frac{I_C}{V_T} = \frac{1\ mA}{25\ mV} = 0.04\ A/V$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{.04} = 2.5\ k\Omega$$

$$r_O = \frac{V_A}{I_C} = \frac{100\ V}{1\ mA} = 100\ k\Omega$$

$R_O = 100\ K + (.04\ A/V)(100\ k\Omega)(4.3\ k\Omega \| 2.5\ k\Omega)$

$R_O = 6.42\ M\Omega$

If the collector voltage changes by 10 V,

$$\Delta I = \frac{\Delta V}{R_O} = \frac{10\ V}{6.42\ M\Omega} = 1.56\ \mu A$$

6.47

All the transistors in this problem are operating at a bias current of 0.5mA and thus have:

$r_e = 50\Omega$, $g_m = 20\,mA/V$, $r_\pi = 5K\Omega$

$C_\pi + C_\mu = \frac{20^m}{2\pi \times 400^M} = 8\,pF$

Since $C_\mu = 2pF \Rightarrow C_\pi = 6pF$, $r_o = \infty$, $r_x = 0$

a) Common-Emitter amplifier:

$R_{sig} = 10K\Omega$, $R_C = 10K\Omega$

$A_M = -\frac{r_\pi}{R_{sig}+r_\pi} g_m R_C = -\frac{5}{10+5}\, 20 \times 10 = \underline{-66.7\,V/V}$

$f_H = \frac{1}{2\pi (R_{sig}//r_\pi)[C_\pi + (1+g_m R_C)C_\mu]} \Rightarrow$

$f_H = \frac{1}{2\pi (10^K//5^K)[6^P + (1+20\times 10)2]} = \underline{117\,KHZ}$

b) Cascode:

$A_M = -\frac{\beta_1 \alpha_2 R_C}{R_{sig}+r_\pi} = -\frac{100 \times 0.99 \times 10}{10+5} = -66\,V/V$

Input pole: $f_{P1} = \frac{1}{2\pi (R_{sig}//r_{\pi 1})(C_{\pi 1} + 2C_{\mu 1})}$

$f_{P1} = \frac{1}{2\pi (10^K//5^K)(6+4)^P} = 4.77\,MHZ$

output pole: $f_{P3} = \frac{1}{2\pi C_{\mu 2} R_C} = \frac{1}{2\pi \times 2^P \times 10^K} = 7.96\,MHZ$

pole at midband node:

$f_{P2} = \frac{1}{2\pi C_{\pi 2} r_{e2}} = \frac{1}{2\pi \times 6^P \times 50} = 530.5\,MHZ$ Very high

$f_H = \sqrt{\frac{1}{(\frac{1}{f_{P1}})^2 + (\frac{1}{f_{P2}})^2}} = 4.1\,MHz$

c) CC-CB Cascade (Modified diff. amplifier)

$A_M = \frac{\beta R_C}{R_{sig}+2r_\pi} = \frac{100 \times 10}{10+10} = 50\,V/V$

Input pole: $f_{P1} = \frac{1}{2\pi (R_{sig} // 2r_\pi)(C_\pi/2 + C_\mu)}$

$f_{P1} = \frac{1}{2\pi (10^K//10^K)(3+2)^P} = 6.4\,MHz$

Output pole: $f_{P2} = \frac{1}{2\pi C_{\mu 2} R_C} = \frac{1}{2\pi \times 2^P \times 10^K} = 7.96\,MHz$

Thus: $f_H = \frac{1}{\sqrt{(\frac{1}{f_{P1}})^2 + (\frac{1}{f_{P2}})^2}} = 5\,MHz$

d) CC-CE Cascade:

$A_M = -\frac{(\beta_1+1)\beta_2 R_C}{R_{sig} + r_{\pi 1} + (\beta_1+1) r_{\pi 2}} = -\frac{101 \times 100 \times 10}{10+5+101\times 5} = -194\,V/V$

Refer to Example 6.13 in:

$R_{\mu 1} = (R_{sig} // R_{in}) = 10^K // (\beta+1)[r_{e1} + r_{\pi 2}]$

$R_{\mu 1} = 10^K // 101 \times [0.05 + 5] = 9.81K\Omega$

$R_{\pi 1} = r_{\pi 1} // \frac{R_s + r_{\pi 2}}{1 + g_{m1} r_{\pi 2}} = 5 // \frac{10+5}{1+20\times 5} = 144\Omega$

$R_T = r_{\pi 2} // \frac{r_{\pi 1} + R_{sig}}{\beta+1} = 5^K // \frac{5+10}{101} = 144\Omega$

where $C_T = C_{\pi 2} + C_{\mu 2}(1 + g_{m2} R_C) = 6 + 2(1+200)$

$C_T = 408pF$

$R_{\mu 2} = R_C = 10K\Omega$

$\tau_H = C_{\mu 1} R_{\mu 1} + C_{\pi 1} R_{\pi 1} + C_T R_T + C_{\mu 2} R_{\mu 2}$

$\tau_H = 2 \times 9.81 + 6 \times 0.144 + 408 \times 0.144 + 2 \times 10$

$\tau_H = 19.62 + 0.86 + 58.75 + 20 = 99.2\,ns$

$f_H = \frac{1}{2\pi \tau_H} = \frac{1}{2\pi \times 99.2^n} = \underline{1.6\,MHZ}$

e) Folded Cascode:

$A_M = -\frac{\beta_1 \alpha_2 R_C}{R_{sig} + r_{\pi 1}} = -\frac{100 \times 0.99 \times 10}{10+5} = -66\,V/V$

Input pole:

$f_{P1} = \frac{1}{2\pi (R_{sig}//r_{\pi 1})(C_{\pi 1} + 2C_{\mu 1})} = \frac{1}{2\pi (10//5)(6+4)}$

$f_{P1} = 4.77\,MHZ$

At middle: $f_{P2} = \frac{1}{2\pi C_{\pi 2} r_{e2}} = \frac{1}{2\pi \times 6^P \times 0.05} = 530\,MHZ$ very high!

At output: $f_{P3} = \frac{1}{2\pi C_{\mu 2} R_C} = \frac{1}{2\pi \times 2 \times 10} \Rightarrow$

$f_{P3} = 7.96\,MHZ$

Thus: $f_H \simeq \frac{1}{\sqrt{\frac{1}{4.77^2} + \frac{1}{7.96^2}}} = \underline{4.1\,MHz}$

f) CC_CB Cascade:

$$A_M = \frac{(\beta_1+1)\alpha_2 R_C}{R_{sig}+(\beta_1+1)2r_e} = \frac{101\times0.99\times10}{10+101\times0.1} \simeq 50\,V/V$$

Input pole: $f_{P1} = \frac{1}{2\pi(R_{sig}\|2r_\pi)(C_\pi/2+C_\mu)}$

$$f_{P1} = \frac{1}{2\pi(10^k\|10^k)(3^p+2^p)} = 6.4\,MHz$$

Output pole: $f_{P2} = \frac{1}{2\pi R_C C_\mu} = \frac{1}{2\pi\times10^k\times2^p} = 7.96\,MHz$

$$f_H \simeq \frac{1}{\sqrt{\frac{1}{6.4^2}+\frac{1}{7.96^2}}} = 5\,MHz$$

Summary of results:

Configuration	A_M (V/V)	f_H (MHz)	G.B. (MHz)
a) CE	−66.7	0.117	7.8
b) Cascode	−66	4.1	271
c) CC_CB cascade	+50	5.0	250
d) CC_CE cascade	−194	1.6	310
e) Folded cascode	−66	4.1	271
f) CC_CB cascade	+50	5.0	250

6.48

$I_{REF} = 80\mu A = I_4 = I_1 = I_2 = I_3$

All transistors have the same g_m, r_o, V_{OV} values.

$I = \frac{1}{2}k_n'\frac{W}{L}V_{OV}^2 \Rightarrow 0.08 = \frac{1}{2}\times4\times V_{OV}^2 \Rightarrow V_{OV}=0.2V$

$V_{GS} = V_{OV}+V_t = 0.2+0.5 = 0.7V$

$V_{G1} = V_{GS} = 0.7V = V_{S4} \Rightarrow V_{G4} = 0.7+V_{GS4} = 1.4^V$

$\Rightarrow V_{G3} = 1.4^V \Rightarrow V_{S3} = 1.4^V - V_{GS} = 0.7^V$

$\Rightarrow V_{O\,min} = V_{D3} = V_{S3}+V_{OV} = 0.9V$

As explained, the voltage at the gate of Q_3 is $2V_{GS}$ which implies voltage of $V_{GS} = V_{OV}+V_t$ at the source of Q_3. For minimum allowable voltage at the output: $V_{DS} = V_{OV}$ or equivalently $V_{O\,min} = V_{OV}+V_{GS}$

$V_{O\,min} = V_{OV}+V_{OV}+V_t = 2V_{OV}+V_t$.

$g_m = \frac{2I_D}{V_{OV}} = \frac{2\times0.08}{0.2} = 0.8\,mA/V$ $\quad r_o = \frac{V_A}{I_D} = \frac{8}{0.08} = 100k\Omega$

Using Eq. 6.189: $R_o = r_{o3}+[1+(g_{m3}+g_{mb3})r_{o3}]r_{o2}$

$R_o = 100K+[1+0.8\times100]\times100 = 8.2M\Omega$

6.49

$I_{REF} = 25\ \mu A$,

$I_4 = 25\ \mu A = I_1$ $W_1 = W_4 = 2\ \mu m$

$W_2 = W_3 = 40\ \mu m$

$$I_1 = \frac{1}{2}k_n'\frac{W_1}{L_1}V_{OV1}^2 \Rightarrow 25$$

$$= \frac{1}{2}\times200\times\frac{2}{1}V_{OV_1}^2 \Rightarrow V_{OV_1} = 0.354\ V$$

$$V_{OV1} = V_{OV2} \Rightarrow \frac{I_2}{I_1} = \frac{(W/L)_2}{\left(\frac{W}{L}\right)_1}$$

$$\Rightarrow I_2 = 25\times\frac{40}{2} = 500\ \mu A$$

$I_2 = 0.5\ mA = I_3$

$I_O = 0.5\ mA$

$V_{GS1} = V_{OV1}+V_t = 0.354+0.6 = 0.954\ V$

$V_{G1} = 0.954\ V$

$V_{G4} = V_{GS1}+V_{GS4}$,

Since $I_1 = I_4$ and $W_1 = W_4$ then

$V_{GS1} = V_{GS4} \Rightarrow V_{G4} = 2V_{GS1}$

$= 1.191\ V = V_{G3}$

The lowest possible voltage for the output is when Q_3 has $V_{DS3} = V_{OV3}$ or

$V_{O\,min} = V_{G3} - V_{GS3} + V_{OV3}$

since $V_{GS1} = V_{GS2}$ and $I_2 = I_3$ then

$V_{GS3} = V_{GS1}$

$\Rightarrow V_{O\,min} = 1.91 - 0.954 + 0.354 = 1.31\ V$

$$g_{m2} = g_{m3} = \frac{2I_D}{V_{OV}} = \frac{2\times0.5}{0.354}$$

$= 2.82\ mA/V$

6.50

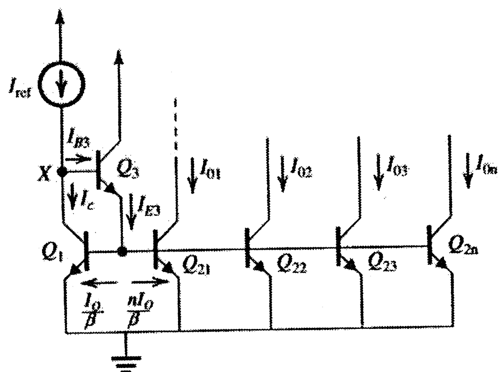
$$r_{o2} = r_{o3} = \frac{V_A}{I_D} = \frac{20}{0.5} = 40\ \text{k}\Omega$$

Eq. 6.189:

$$R_O = r_{o3} + [1 + (g_{m3} + g_{mb3})r_{o3}]r_{o2}$$

$$R_O = 40\ \text{k}\Omega + [1 + 2.82\ \text{k}\Omega \times 40\ \text{k}\Omega \times 40\ \text{k}\Omega] \times 40\ \text{k}\Omega = 4.6\ \text{M}\Omega$$

6.51



$I_{01} = I_{02} = I_{03} ... = I_{0n} = I_0$

The emitter of Q_3 supplies the base currents for all transistor so

$$I_{E3} = \frac{(n+1)I_0}{\beta}$$

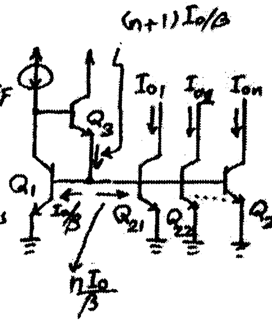
$$I_{REF} = I_{B3}\ rI_0 = \frac{(n+1)I_0}{\beta(\beta+1)} + I_0$$

$$\frac{I_0}{I_{REF}} = \frac{1}{1 + \frac{(n+1)}{\beta(\beta+1)}}$$

for deviation of .1% from unity:

$$\frac{99.9}{100} = \frac{1}{1 + \frac{(n+1)}{100(101)}} \Rightarrow n \cong 9$$

6.52



Since $Q_{21}, Q_{22}, \ldots Q_{2n}$ are all matched to Q_1:

$I_{01} = I_{02} = \cdots = I_{0n} = I_0$

The emitter of Q_3 supplies the basecurrent for all trasistors, so $I_{c3} = \frac{(n+1)I_0}{\beta}$

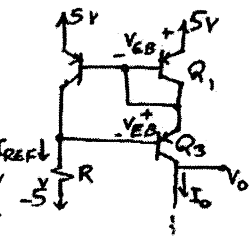
A node equation at the base of Q_3 yields:

$I_{REF} = I_0 + \frac{(n+1)I_0}{\beta(\beta+1)}$, Thus: $\frac{I_0}{I_{REF}} = \frac{1}{1 + \frac{n+1}{\beta^2}}$

For a deviation from unity of less than 0.1%: $\frac{99.9}{100} = \frac{1}{1+\frac{n+1}{\beta^2}} \Rightarrow \frac{n+1}{\beta^2} = \frac{1}{999}$

$\Rightarrow n = \frac{\beta^2}{999} - 1 \Rightarrow n \simeq 9$

6.53

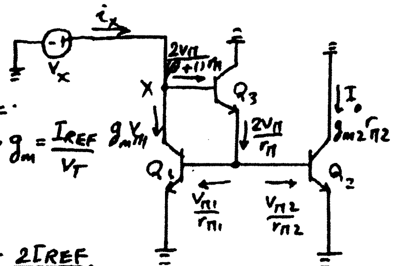


$I_{REF} = 0.1\text{mA} = \frac{5 - 0.7 - 0.7 - (-5)}{R}$

$\Rightarrow R = 86\text{K}\Omega$

V_{omax} is obtained when Q_3 is saturated: $V_{omax} = 5 - 0.7 - 0.2 = 4.1^V$

6.54



$v_{\pi 1} = v_{\pi 2}$

small-signal model

Q_1 and Q_2 are biased at I_{REF}.

$r_{e1} = r_{e2} = \frac{V_T}{I_{REF}} \rightarrow g_m = \frac{I_{REF}}{V_T}$

$r_{\pi 1} = \frac{\beta V_T}{I_{REF}}$

Q_3 is biased at $\frac{2I_{REF}}{\beta}$,

Thus $r_{e3} = \frac{\beta V_T}{2I_{REF}}$

Refer to the small-circuit analysis performed directly on the circuit. Since the current in the emitter of Q_3 is $\frac{2v_{\pi 1}}{r_{\pi 1}}$, the voltage $v_{\pi 3}$ will be: $v_{\pi 3} = \frac{2v_{\pi 1}}{r_{\pi 1}} \times r_{e3}$.

$V_x = v_{\pi 2} + v_{\pi 1} = \frac{2v_{\pi 1} \cdot r_{e3}}{r_{\pi 1}} + v_{\pi 1} = v_{\pi 1}\left(1 + 2\frac{r_{e3}}{r_{\pi 1}}\right)$

$V_x = v_{\pi 1}\left(1 + 2\frac{\beta V_T}{2I_{REF}} \times \frac{I_{REF}}{\beta V_T}\right) = 2v_{\pi 1}$

and $i_x \simeq g_{m1} v_{\pi 1}$. Thus: $R_{in} = \frac{V_x}{I_x} = \frac{2}{g_{m1}} = \frac{2V_T}{I_{REF}}$

For $I_{REF} = 100\,\mu A \Rightarrow R_{in} = \frac{2 \times 0.025}{0.1} = 0.5\,k\Omega$

6.55

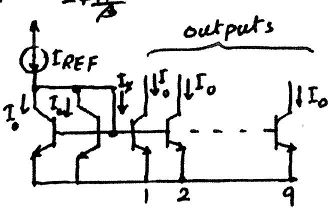
All the output currents are equal to I_o, then we have: $I_{REF} = 2I_o + 11\frac{I_o}{\beta} \Rightarrow \frac{I_o}{I_{REF}} = \frac{1}{2 + 11/\beta}$

I_o is idealy $I_{REF}/2$, For 5% lower I_o:

$0.95 \times \frac{I_{REF}/2}{I_{REF}} = \frac{1}{2 + \frac{11}{\beta}} \Rightarrow \beta = 104.5 \simeq 105$

$\beta = 105$

$I_x = 11 \times \frac{I_o}{\beta}$



6.56

a) See the analysis on the circuit.

$I_{REF} = I + \frac{\beta + 2}{\beta(\beta + 1)}I = I\frac{\beta^2 + 2\beta + 2}{\beta(\beta + 1)}$

$I_{01} = I_{02} = \frac{1}{2}\frac{\beta + 2}{\beta + 1}I$

$\frac{I_{01}}{I_{REF}} = \frac{I_{02}}{I_{REF}} = \frac{1}{2}\frac{\beta(\beta + 2)}{\beta^2 + 2\beta + 2}$

$= \frac{1}{2} \times \frac{1}{1 + 2/(\beta^2/2\beta)}$

$\frac{I_{01}}{I_{REF}} = \frac{1}{2}\frac{1}{1 + 2/\beta^2}$

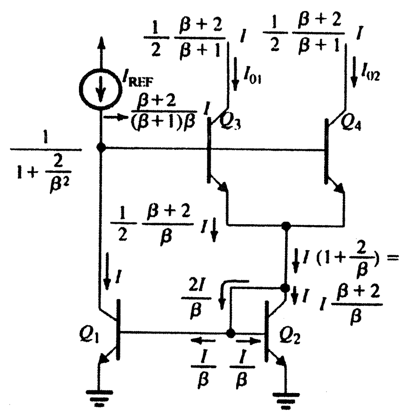
Observe that the deviation factor $\frac{1}{1 + 2/\beta^2}$ is independent of the number of outputs or the value of each output, i.e.:

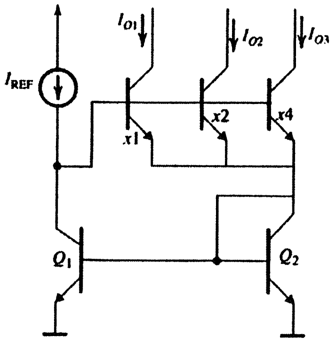
The current I_{REF} can be split into any number of outputs through an appropriate combinations of parallel-connected transistors. (Q_3 and Q_4 in this case) The reason the error factor remains unchanged at $\frac{1}{1 + 2/\beta^2}$ is that the base current that need to be supplied by I_{REF} (subtract from I_{REF}) remains unchanged.

b) The I mA reference current can be used to generate three output currents of 1, 2, 4 mA by using 3 transistors in parallel having relative area ratios of 1, 2, 4 as shown:

$\frac{I_{01}}{I_{REF}} = \frac{1}{7}\frac{1}{1 + 2/\beta^2} \Rightarrow I_{01}$

$= 0.998$ mA (1 mA ideally)





$$\frac{I_{O2}}{I_{REF}} = \frac{2}{7}\frac{1}{1+2/\beta^2} \Rightarrow I_{O2}$$
= (1.996) mA (2 mA ideally)

$$\frac{I_{O3}}{I_{REF}} = \frac{4}{7}\frac{1}{1+2/\beta^2} \Rightarrow I_{O3}$$
= 3.992 mA (4 mA ideally)

6.57

(a) First, we need an estimate for V_{ov} and V_{GS}. Since the currents are all approximately the same, and $I_D = \frac{1}{2}(\mu_n C_{ox})(W/L)V_{OV}^2$,

6.57

$$V_{OV} = \sqrt{\frac{2\,I_D}{\mu_n C_{ox}(W/L)}}$$

$$V_{OV} = \sqrt{\frac{2(100\ \mu A)}{(400\ \mu A/V)(12.5)}} = 0.2\ V$$

since no value is given for V_{tn}, we have to estimate this with $\mu_n C_{ox} = 400\ \mu A/V^2$, this fabrication process is similar to the 0.18 μm technology. We will therefore approximate V_{tn} as approximately 0.5 V.

$V_{GS} = V_{tn} + V_{ov} = 0.5\ V + 0.2\ V = 0.7\ V$

(b) $V_{DS2} = V_{GS1} + V_{GS3} = 1.4$ V, which is $\simeq (2 \cdot V_{gs1})$

$$r_o = \frac{V_A}{I} = \frac{20\ V}{0.1\ mA} = 200\ k\Omega$$

$$\Delta I = \frac{\Delta V_{ds}}{r_o} = \frac{1.4 - 0.7}{200\ k\Omega} = 0.35\ \mu A$$

$I_o \approx I_{REF} - \Delta I$

so that,

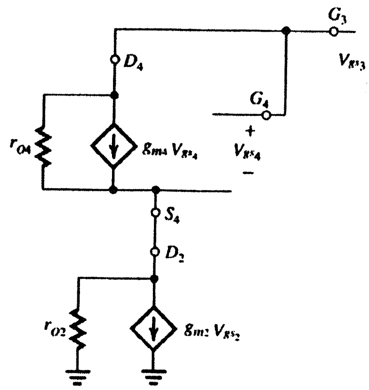
$I_o \approx 100 - 3.5 = 96.5\ \mu A$

(c) $I_o \approx I_{REF}$

(d) $V_{o_{min}} = V_{tn} + 2\,V_{ov}$

$= 0.5 + 2(0.2\ V) = 0.9\ V$

(e) If a small-signal model is added to account for Q_4, the circuit is changed to



Since $V_{DS4} = V_{GS4} = -g_{m4}V_{gs4}r_{o4}$ (no current into gate 3)

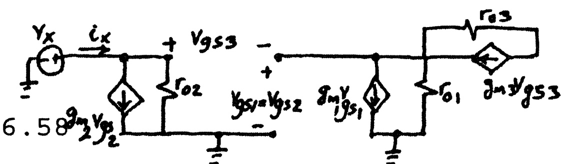
$V_{GS4} = V_{DS4} = 0$ so that $V_{D2} = V_{G3}$ and there is no effect.

$R_o \approx (g_{m3}r_{o3})r_{o2}$

$$g_m = \frac{I_D}{V_{ov}/2} = \frac{0.1\ mA}{0.2\ v/2} = 1\ mA/V$$

$R_o \approx (1\ mA/V)(200\ k\Omega)^2 = 40\ M\Omega$

6.58



$i_x = g_{m2}V_{gs2}$ ①

$V_{gs2} + V_{gs3} = V_x$.

Since Q_2 and Q_3 have the same parameters and same current, therefore $V_{gs2} = V_{gs3}$

$V_x = 2V_{gs2} \Rightarrow V_{gs2} = \frac{V_x}{2}$

Substitute for V_{gs2} in ①:

$i_x = g_{m2} \times \frac{V_x}{2}$

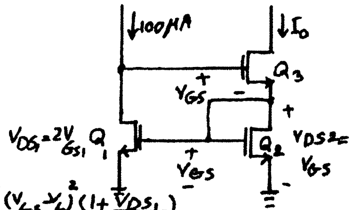
$R_{in} = \frac{V_x}{i_x} = \frac{2}{g_{m2}}$

6.59

$I_{REF} = 100\mu A$

$V_{DS1} = 2V_{GS}$

$V_{DS2} = V_{GS}$



$I_{D1} = I_{REF} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_t)^2(1+\frac{V_{DS1}}{V_A})$

$100 = \frac{1}{2}\times 2000\,(V_{GS}-0.6)^2(1+\frac{2V_{GS}}{20})$

$1 = (V_{GS}-0.6)^2(10+V_{GS})$

$V_{GS} \simeq 0.91V$ (by iteration)

$I_o = I_{D2} = \frac{1}{2}\times 2000\,(V_{GS}-0.6)^2(1+\frac{V_{GS}}{20})$

$I_o = 100.47\mu A$

Thus there is $\frac{0.47}{100}$ or 0.5% error. Modifying the circuit as Fig. 6.61C ensures that Q_1 and Q_2 have the same V_{DS} and thus eliminate the above error.

6.60

$I_{REF} = 100\mu A$ $I_o = 10\mu A$

a) $V_{BE1} = 0.7 + V_t \ln\frac{100}{1000} = 0.642V$

$V_{BE2} = 0.7 + V_t \ln\frac{10}{1000} = 0.585V$

$I_o = \frac{V_{BE1}-V_{BE2}}{R_E} = 10\mu A \Rightarrow R_E = 5.7\ k\Omega$

b) $r_{\pi 2} = (\beta+1)\frac{V_T}{10\mu A} = 503k\Omega >> R_E$

$r_{o2} = \frac{V_A}{I_o} = 10M\Omega \Rightarrow R_o = (1+g_{m2}R_E)r_{o2} = 33M\Omega$

$R_o = 33M\Omega$

$\Delta I_o = \frac{\Delta V_o}{R_o} = \frac{5}{R_o} = 0.15\mu A$

6.61

a) $\frac{I_o}{I_{REF}} = 0.9 \Rightarrow I_o = 90\mu A$

$V_{RE} = V_T \ln\frac{1}{0.9} = 2.63mV$

$R_E = \frac{2.63mV}{90\mu A} = 29.3\Omega$

$r_o = \frac{V_A}{I_o} = 1.11M\Omega$

$g_m = 3.6\ mA/V$

$R_o = (1+g_m R_E)r_o = 1.23M\Omega$ Compare to $r_o = 1.11\ M\Omega$

b) $\frac{I_o}{I_{REF}} = 0.1 \Rightarrow I_o = 10\mu A$

$V_{R_E} = V_T \ln 10 = 57.56\ mV$

$R_E = \frac{57.56m}{10\mu} = 5.76k\Omega$

$r_o = \frac{100}{10\mu} = 10M\Omega$

$g_m = 0.4m\ A/V$

$R_o = (1+g_m R_E)r_o = 33M\Omega$ Compare to $r_o = 10M\Omega$

c) $\frac{I_o}{I_{REF}} = 0.01 \Rightarrow I_o = 1\mu A$

$V_{RE} = V_T \ln 100 = 115mV$

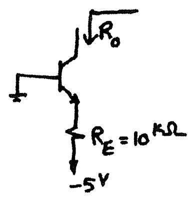
$R_E = \frac{115}{1} = 115k\Omega$

$r_o = \frac{100}{1} = 100M\Omega$

$g_m = 0.04\ mA/V$

$R_o = (1+g_m R_E)r_o = 560M\Omega$ Compare to $r_o = 100\ M\Omega$

6.62



$R_o = [1+g_m(R_E \| r_\pi)]\,r_o$

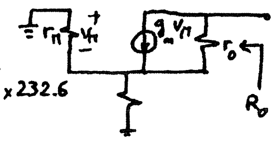
$I_E = \frac{-0.7-(-5)}{R_E} = 0.43mA$

$g_m = \frac{I_C}{V_T} = \frac{0.43}{0.025} = 17.2mA/V$

$r_o = \frac{V_A}{I_C} = \frac{100}{0.43} = 232.6k\Omega$

$r_\pi = \frac{\beta}{g_m} = \frac{100}{17.2} = 5.8k\Omega$

$R_E = 10k\Omega$



$R_o = [1+(10^k \| 5.8^k)\times 17.2]\times 232.6$

$R_o = 14.92M\Omega$

6.63

$I = 2mA \Rightarrow g_m = \frac{2}{0.025} = 80 mA/V \quad , \quad r_\pi = \frac{\beta}{g_m} = 1.25^{K\Omega}$

$r_e = \frac{r_\pi}{\beta+1} = 12.4\Omega$

$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)} \Rightarrow C_\pi + C_\mu = \frac{80^m}{2\pi \times 400 \times 10^6} = 31.85^{PF}$

$\Rightarrow C_\pi = 31.85 - 2 = 29.85 pF$

$A_M = \frac{R_L}{\frac{R_{sig} + r_\pi}{\beta+1} + R_L} = \frac{1}{\frac{R_{sig}}{101} + 0.0124 + 1} = \frac{1}{1.0124 + \frac{R_{sig}}{101}}$

$R_L' = R_L = 1k\Omega \quad , \quad R_{sig}' = R_{sig} + r_x = R_{sig}$

$R_\mu = R_{sig}' \,||\, [r_\pi + (\beta+1)R_L']$ (Eq. 6.179)

$R_\mu = R_{sig} \,||\, (1.25 + 101 \times 1) = R_{sig} \,||\, 102.25^K$

$R_\pi = \frac{R_{sig}' + R_L'}{1 + \frac{R_{sig}'}{r_\pi} + \frac{R_L'}{r_e}}$ (Eq. 6.180)

$R_\pi = \frac{R_{sig} + 1K}{1 + 0.8R_{sig} + 80} = \frac{R_{sig} + 1}{0.8R_{sig} + 81}$

$f_H = \frac{1}{2\pi(R_\pi C_\pi + R_\mu C_\mu)} = \frac{1}{2\pi(29.85R_\pi + 2R_\mu)}$

a) $R_{sig} = 1^{K\Omega}$: $\quad A_M = 0.978 V/V$

$R_\mu = 0.99^{K\Omega}, R_\pi = 24.4\Omega \Rightarrow f_H = 58.8 MHZ$

b) $R_{sig} = 10K\Omega$: $\quad A_M = 0.9 V/V$

$R_\mu = 9.11K\Omega, R_\pi = 124\Omega \Rightarrow f_H = 7.27 MHZ$

C) $R_{sig} = 100K\Omega$: $\quad A_M = 0.499 V/V$

$R_\mu = 50.6K\Omega, R_\pi = 627\Omega \Rightarrow f_H = 1.34 MHZ$

6.64

Each of the transistors is operating at a bias current of approximately 100μA. Thus:

$g_m = \frac{0.1}{0.025} = 4mA/V$, $r_\pi = \frac{100}{4} = 25K\Omega$

$r_e \cong 250\Omega$, $r_o = \frac{100}{0.1} = 1M\Omega$

$C_\pi + C_\mu = \frac{g_m}{2\pi f_T} = \frac{4^m}{2\pi \times 400^M} = 1.59pF \Rightarrow C_\pi = 1.39pF$

a) $R_{in} = (\beta+1)\left[r_{e1} + (r_{\pi2}||r_{o1})\right]$

$R_{in} = 101\left[250\times10^{-3} + 25^K||1M\Omega\right] \cong 2.5M\Omega$

$A_M = -\frac{R_{in}}{R_{in}+R_{sig}} \times \frac{r_{\pi2}||r_{o1}}{r_{e1}+(r_{\pi2}||r_{o1})} \times g_{m2} r_{o2}$

$A_M = -\frac{2.5^M}{2.5+0.01} \times \frac{25^K||1^M}{0.25+(25^K||1M)} \times 4\times1^M$

$A_M = -3943.6\ V/V$

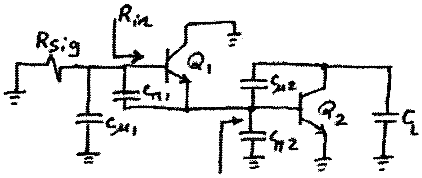
b) To calculate f_H, refer to

$R_{\mu1} = R_{sig}||R_{in} = 10^K||2.5^M \cong 10K\Omega$

$R_{in2} = r_{\pi2}||r_{o1}$

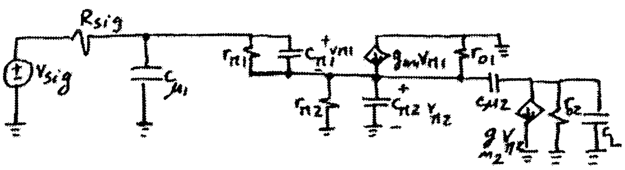
$R_{in2} = 25^K||1^M$

$R_{in2} = 24.4K\Omega$

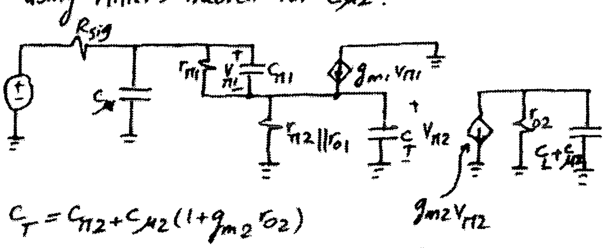


$R_{\pi1} = \frac{R_{sig}+R_{in2}}{1+\frac{R_{sig}}{r_{\pi1}}+\frac{R_{in2}}{r_{e1}}}$

$R_{\pi1} = \frac{10+24.4}{1+\frac{10}{25}+\frac{24.4}{0.25}} = 0.35K\Omega$



Using Miller's Theorem for $C_{\mu2}$:



$C_T = C_{\pi2} + C_{\mu2}(1+g_{m2}r_{o2})$

$C_T = 1.39 + 0.2(1+4\times1000) = 801.6pF$

$R_T = r_{\pi2}||r_{o1}||\frac{r_{\pi1}+R_{sig}}{\beta+1} = 25^K||1000^K||\frac{25+10}{101}$

$R_T = 342\Omega$

$R_{\mu2} = r_{o2} = 1000K$

$\tau_H = C_{\mu1}R_{\mu1} + C_{\pi1}R_{\pi1} + C_T R_T + (C_{\mu2}+C_L)R_{\mu2}$

$\tau_H = 0.2\times10 + 1.39\times0.35 + 801.6\times0.342 + (0.2+1)\times1000$

$\tau_H = 2 + 0.49 + 274.15 + 1200\ ns$

Thus $(C_L + C_{\mu2})R_{\mu2}$ is the dominating term. The second most significant term is $C_T R_T$. So $(C_L + C_{\mu2})$ dominates and then C_T or equivalently $C_{\mu2}$.

$f_H = \frac{1}{2\pi\tau_H} = \frac{1}{2\pi\times1476.6^n} = 107.8MHz$

c) Increasing the bias currents by a factor of 10:

$g_m = 40mA/V$, $r_\pi = 2.5K\Omega$

$r_e \cong 25\Omega$, $r_o = 100K\Omega$

$C_\pi = C_{je} + C_{de}\times10 = 0.8 + 0.59\times10 = 6.7pF$

$C_\mu = 0.2pF$

$R_{in} = 101\left[0.025 + (2.5^K||100^K)\right] = 249K\Omega$

R_{in} is almost decreased by a factor of 10.

$A_M = -\frac{249}{249+10} \times \frac{2.5^K||100^K}{0.025+(2.5^K||100^K)} \times 4000$

$A_M = -3807\ V/V$

A_M remains almost constant.

$C_T = 6.7 + 0.2(1+40\times100) = 806.9$ (almost constant)

$R_{\mu1} = R_{sig}||R_{in} = 10^K||249^K = 9.61K\Omega$

$R_{\mu1}$ stays almost the same.

$R_T = 2.5^K||10^K||\frac{2.5+10}{101} = 117.8\Omega$

R_T is almost reduced by a factor of 3.

$R_{in2} = r_{\pi2}||r_{o1} = 2.44K\Omega$

$R_{\pi1} = \frac{10^K+2.44}{1+\frac{10}{2.5}+\frac{2.44}{0.025}} = 120\Omega$

$R_{\pi1}$ is almost decreased by a factor of 3.

$R_{\mu2} = r_{o2} = 100K\Omega$ (decreased by a factor of 10)

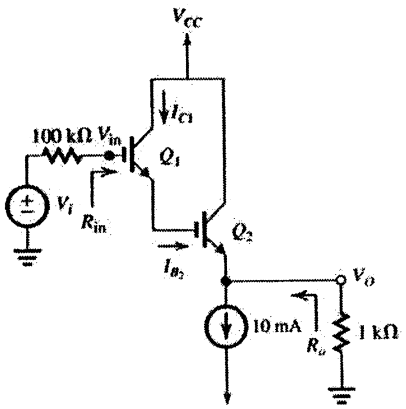
$\tau_H = 0.2\times9.61 + 6.7\times0.120 + 806.9\times0.118 + 1.2\times100$

$\tau_H = 1.92 + 0.8 + 95.2 + 120 = 217.92ns$

Thus the dominant effect, that of the output pole, is reduced by a factor of 10. This occurs because $(C_L + C_{\mu2})$ remains constant while r_{o2} decreases by a factor of 10. The second most significant factor (that due to C_T or $C_{\mu2}$ with Miller Effect) also decreases, but only by a factor of 3. The overall result is an increase in f_H.

Cont.

6.65



$$I_{B2} = \frac{I_{E2}}{\beta + 1} = \frac{10\text{ m}}{101} = 99\ \mu\text{A}$$

$$I_{E1} = I_{B2} = 99\ \mu\text{A}$$

$$I_{C1} = \alpha I_{E1} = \frac{100}{101}(99\ \mu) = 98\ \mu\text{A}$$

$$I_{C2} = \alpha I_{E2} = \frac{100}{101}(10\text{ m}) = 9.9\text{ mA}$$

Neglecting Early effect
using resistance reflection rule:

$$R_{in} = r_{\pi 1} + r_{\pi 2}(\beta_1 + 1) + 1\text{ K}(\beta_2 + 1)(\beta + 1)$$

$$R_{in} = \frac{\beta V_T}{I_{C1}} + \frac{\beta V_T}{I_{C2}}(101) + 1\text{k}(101)^2$$

$$= \frac{100(25\text{ m})}{98\ \mu} + \frac{100(25\text{ m})}{9.9\text{ m}}(101) + 10.2\text{ M}$$

$$R_{in} = 25.5\text{ K} + 25.5\text{ K} + 10.2\text{ M} = 10.252\text{ M}$$

$$R_O = \frac{r_{\pi 2}}{\beta_2 + 1} + \left[\frac{r_{\pi 1}}{\beta_1 + 1} + \frac{100\text{ K}}{\beta_1 + 1}\right]\left(\frac{1}{\beta_2 + 1}\right)$$

$$R_O = \frac{100(25\text{ m})}{(101)(9.9\text{ m})} + \left[\frac{25.5\text{ K}}{101} + \frac{100\text{ K}}{101}\right]\left(\frac{1}{101}\right)$$

$$R_O = 2.5 + (253 + 990)\left(\frac{1}{101}\right) = 14.8\ \Omega$$

$$A_{VO} = 1{,}000\text{ V/V}$$

$$A_V = \frac{1 \times 1{,}000}{14.8 + 1{,}000} = 0.985\text{ V/V}$$

6.66

$I_{E2} = 10\text{mA} \Rightarrow r_{e2} = 2.5\Omega$, $r_{\pi 2} = 253\Omega$

$I_{E1} = \frac{10}{101} \cong 0.1\text{mA} \Rightarrow r_{e1} = 250\Omega$, $r_{\pi 1} = 25.3\text{K}\Omega$

$R_{in} = 101 \times [0.25 + 101(0.0025 + 1)] = 10.3\text{M}\Omega$

$\underline{R_{in} = 10.3\text{M}\Omega}$

$R_{out} = r_{e2} + \frac{1}{\beta_2 + 1}\left[r_{e1} + \frac{R_{sig}}{\beta_1 + 1}\right]$

$R_{out} = 2.5 + \frac{1}{101}\left[250 + \frac{100000}{101}\right] = \underline{14.8\Omega}$

Neglecting r_o:

$A_{v_o} = 1000\text{ V/V}$, $A_v = \frac{1 \times 1000}{14.8 + 1000} = 0.985\text{ V/V}$

6.67

$I_1 = I_2 = I = 1\text{mA} \Rightarrow g_m = 40\text{ mA/V}$, $r_\pi = \frac{120}{40} = 3\text{K}\Omega$

$r_e = \frac{3}{121} \cong 25\Omega$, $C_\pi + C_\mu = \frac{g_m}{2\pi f_T} = \frac{40^m}{2\pi \times 700^M} = 9.1\text{pF}$ ⇓ $\underline{C_\pi = 8.6\text{pF}}$

Using Eq. 6.185:

$A_M = \frac{v_o}{v_{sig}} = \frac{1}{2}\left(\frac{R_{in}}{R_{in} + R_{sig}}\right) g_m R_L$

$R_{in} = 2r_\pi = 2 \times 3\text{K}\Omega = 6\text{K}\Omega$

$A_M = \frac{1}{2} \times \frac{6}{6 + 20} \times 40 \times 10^K = 46.15\text{ V/V}$

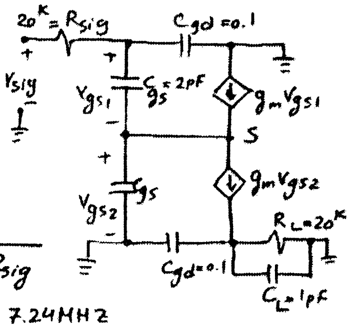
$f_{p1} = \frac{1}{2\pi\left(\frac{C_\pi}{2} + C_\mu\right)(R_{sig} \| 2r_\pi)} = \frac{1}{2\pi\left(\frac{8.6}{2} + 0.5\right)(20^K \| 6^K)}$

$f_{p1} = 7.19\text{MHz}$

$f_{p2} = \frac{1}{2\pi C_\mu R_L} = \frac{1}{2\pi \times 0.5^p \times 10^K} = 31.8\text{ MHz}$

$f_H = \frac{1}{\sqrt{\left(\frac{1}{f_{p1}}\right)^2 + \left(\frac{1}{f_{p2}}\right)^2}} = 7.01\text{MHz}$

6.68



$v_i = 2v_{gs}$

$v_o = R_L \times g_m v_{gs}$

$\frac{v_o}{v_i} = \frac{v_o}{v_{sig}} = g_m R_L/2$

$A_M = \frac{5 \times 20}{2} = \underline{50\text{ V/V}}$

$f_{p1} = \frac{1}{2\pi\left(\frac{C_{gs}}{2} + C_{gd}\right) R_{sig}}$

$f_{p1} = \frac{1}{2\pi(1^p + 0.1^p)20^k} = 7.24\text{MHz}$

$f_{p2} = \frac{1}{2\pi(C_{gd} + C_L)R_L} = \frac{1}{2\pi(0.1 + 1) \times 20^k} = 7.24\text{MHz}$

$f_H = \frac{1}{\sqrt{\left(\frac{1}{f_{p1}}\right)^2 + \left(\frac{1}{f_{p2}}\right)^2}} = \underline{5.12\text{MHz}}$

	0.5 μm		0.25 μm		0.18 μm		0.13 μm	
	NMOS	PMOS	NMOS	PMOS	NMOS	PMOS	NMOS	PMOS
V_{ov}(V)	0.32	−0.54	0.27	−0.46	0.23	−0.48	0.2	−0.42
V_{GS}(V)	1.02	−1.34	0.7	−1.08	0.71	−0.93	0.6	−0.82

	0.5 μm		0.25 μm		0.18 μm		0.13 μm	
	NMOS	PMOS	NMOS	PMOS	NMOS	PMOS	NMOS	PMOS
g_m (mA / V)	0.62	0.37	0.73	0.43	0.88	0.41	1.02	0.48

6.69 If the area of the emitter-base junction is changed by a factor of 10, then I_S is changed by the save factor. If V_{BE} is kept constant, then I_C is also changed by the same factor:

$$I_C = I_S e^{V_{BE}/V_T}$$

$$I_S \propto A,\ I_C \propto I_S \Rightarrow I_C \propto A$$

$$A_2 = 10\,A_1 \Rightarrow I_{C2} = 10\,I_{C1}$$

If I_C is kept constant, then V_{BE} changes:

$$I_{S2} = 10\,I_{S1} \Rightarrow I_S e^{V_{BE}/V_T}$$

$$e^{\frac{V_{BE1}-V_{BE2}}{V_T}} = 10 \Rightarrow V_{BE1} - V_{BE2}$$

$$= V_T \ln 10 = 0.058 \text{ V or } 58 \text{ mV}$$

6.70 $\frac{W}{L} = 10,\ I_D = 100\ \mu\text{A},$

$$I_D = \frac{1}{2} k' \frac{W}{L} V_{OV}^2$$

$$V_{OV} = \sqrt{\frac{2I_D}{k'_n \frac{W}{L}}} = \sqrt{\frac{2 \times 100}{k'_n \times 10}} = \sqrt{\frac{20}{k'_n}}$$

$$V_{GS} = V_t + V_{OV}$$

6.71

$$|V_{OV}| = 0.25 \text{ V},\ I_D = 100\ \mu\text{A}$$

$$I_D = \frac{1}{2} k'_n \frac{W}{L} V_{OV}^2 \Rightarrow \frac{W}{L} = \frac{2I_D}{k'_n V_{OV}^2}$$

For NMOS:

$$k'_n = 267 \frac{\mu\text{A}}{\text{V}^2} \Rightarrow \left(\frac{W}{L}\right)_n$$

$$= \frac{2 \times 100}{267 \times 0.25^2} = 11.98 \simeq 12$$

For PMOS:

$$k'_n = 93 \frac{\mu\text{A}}{\text{V}^2} \Rightarrow \left(\frac{W}{L}\right)_p$$

$$= \frac{2 \times 100}{93 \times 0.25^2} = 34.4 \simeq 34$$

6.72

$$i_{Dn} = i_{Dp} \Rightarrow \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_n V_{OVn}^2$$

$$= \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_p V_{OVp}^2 \quad (1)$$

we also have $g_{mn} = g_{mp}$

$$g_m = \frac{2I_D}{V_{OV}} \Rightarrow V_{OVn} = V_{OVp} \quad (2)$$

$$①, ② \Rightarrow \frac{\left(\frac{W}{L}\right)_p}{\left(\frac{W}{L}\right)_n} = \frac{\mu_n}{\mu_p} = \frac{460}{160} = 2.88$$

6.73

$V_{OV} = 0.25$ V

for an npn transistor:

$$g_m = \frac{I_C}{V_T} = \frac{0.1}{0.025} = 4 \text{ mA/V}$$

For an NMOS with the same g_m, i.e.

$g_m = 4$ mA/V

we will have :

$$g_m = \frac{2I_D}{V_{OV}} \Rightarrow I_D = g_m \times \frac{V_{OV}}{2} = 0.5 \text{ mA}$$

$I_D = 0.5$ mA

6.74

Assuming large r_o. For both transistors, for case (a) we have $r = \frac{1}{g_m} = \frac{1}{\sqrt{2\mu_n C_{ox} \frac{W}{L} I_D}}$

$$r = \frac{10^3}{\sqrt{2 \times 200 \times 10 \times 0.1 \times 10^3}} = 1.58 \text{ k}\Omega$$

For case (b) we have

$$r = r_\pi \| \frac{1}{g_m} = \frac{\beta}{(\beta+1)g_m}$$

$$r = \frac{\beta V_T}{(\beta+1)I_C} \approx \frac{V_T}{I_c} = \frac{0.025}{0.1} = 0.25 \text{ k}\Omega$$

$r = 250\ \Omega$

6.75

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 100 \times 10^{-3}}{0.5} = 0.4 \text{ mA/V}$$

$$r_o = \frac{V_A}{I_D} = \frac{V_A' L}{I_D} = \frac{25 \times 1}{0.1} = 250 \text{ k}\Omega$$

$$A_\phi = g_m r_o = 0.4 \times 250 = 100 \text{ V/V}$$

$$g_m = \mu_n C_{ox} \frac{W}{L} V_{OV} \Rightarrow$$

$$W = \frac{g_m \times L}{\mu_n C_{ox} V_{OV}} = \frac{0.4 \times 1}{127 \times 10^{-3} \times 0.5}$$

$W = 6.3\ \mu$m

6.76

$L = 0.3\ \mu$m, $I_D = 100\ \mu$A,

$V_{OV} = 0.2$ V

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 100 \times 10^{-3}}{0.2} = 1 \text{ mA/V}$$

$$r_o = \frac{V_A}{I_D} = \frac{V_A \times L}{I_D} = \frac{5 \times 0.3}{0.1} = 15 \text{ k}\Omega$$

$$A_v = g_m r_o = 1 \times 15 = 15 \text{V/V}$$

$$g_m = \mu_n C_{ox} \frac{W}{L} V_{OV} \Rightarrow$$

$$W = \frac{g_m \times L}{\mu_n C_{ox} V_{OV}} = \frac{1 \times 0.3}{387 \times 10^{-3} \times 0.2}$$

$W = 3.88\ \mu$m

6.77

$L = 0.3\ \mu$m, $W = 6\ \mu$m

$V_{OV} = 0.2$ V

$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} V_{OV}^2$$

$$= \frac{1}{2} \times 387 \times \frac{6}{0.3} \times 0.2^2 = 155\ \mu\text{A}$$

$I_D = 0.155$ mA

$$g_m = \frac{2I_D}{V_{OV}} = 1.55 \text{ mA/V}$$

$$C_{gs} = \frac{2}{3} \frac{W}{L} C_{ox} + C_{OV} = \frac{2}{3} WLC_{ox}$$

$$+ WL_{OV} C_{ox}$$

$$C_{gs} = \frac{2}{3} \times 6 \times 0.3 \times 8.6 + 6 \times 0.37$$

$= 12.54$ fF

$$C_{gd} = C_{OV} W = 0.37 \times 6 = 2.22 \text{ fF}$$

$$f_T = \frac{g_m}{2\pi(C_{gs} + C_{gd})}$$

$$= \frac{1.55 \times 10^{-3}}{2\pi(12.54 + 2.22) \times 10^{-15}} = 16.7 \text{ GHz}$$

If we use the approximation formula:

$f_T \approx \frac{1.5\ \mu_n V_{OV}}{2\pi L^2}$ when

$$C_{gs} >> C_{gd},\ C_{gs} \simeq \frac{2}{3} \frac{W}{L} C_{ox}$$

$$f_T \simeq \frac{1.5 \times 450 \times 10^{-4} \times 0.2}{2 \times \pi \times 0.3^2 \times 10^{-12}} = 23.9 \text{ GHz}$$

The approximation formula over estimates f_T because it ignores $WL_{OV} C_{ox}$ or C_{OV} in C_{gs} and C_{gd} calculation.

6.78

$I_C = 10\ \mu A$, High-voltage process:

$g_m = \frac{I_C}{V_T} = \frac{10 \times 10^{-3}}{0.025} = 0.4\ mA/V$

$C_{de} = \tau_F g_m = 0.35 \times 10^{-9} \times 0.4 \times 10^{-3}$

$= 140 \times 10^{-15}\ F = 140\ fF$

$C_{je} = 2C_{jeo} = 2 \times 1 = 2 pF = 2000\ fF$

$C_\pi = C_{je} + C_{je} = 2140\ fF$

$C\mu = C\mu_O = 0.3\ pF = 300\ fF$

$f_T = \frac{g_m}{2\pi(C_\pi + C\mu)}$

$= \frac{0.4 \times 10^{-3}}{2\pi(2140 + 300 \times 10^{-15})} = 26.1\ MHz$

$I_C = 100\ \mu A$, High-voltage process:

$g_m = 10 \times 0.4 = 4\ mA/V$,

$C_{de} = 10 \times 140 = 1400\ fF$

$C_\pi = 3400\ fF \Rightarrow$

$f_T = \frac{4 \times 10^{-3}}{2\pi(3400 + 300) \times 10^{-15}} = 172.1\ MHz$

$I_C = 10\ \mu A$, Low-voltage process

$g_m = \frac{10 \times 10^{-3}}{0.025} = 0.4\ mA/V$

$C_{de} = 10 \times 10^{-12} \times 0.4 \times 10^{-3} = 4\ fF$

$C_{je} = 2 \times 5\ fF = 10\ fF$

$C_\pi = C_{de} + C_{je} = 14\ fF$

$C\mu \simeq C\mu_O = 5\ fF$

$f_T = \frac{g_m}{2\pi(C_\mu + C_\pi)} = \frac{0.4 \times 10^{-3}}{2\pi(5 + 14) \times 10^{-15}}$

$= 3.35\ GHz$

$I_C = 100\ \mu A$, Low-voltage process

$g_m = \frac{100 \times 10^{-3}}{0.025} = 4\ mA/V$

$C_{de} = 10 \times 4 = 40\ fF$

$C_\pi = 40 + 10 = 50\ fF$, $C_\mu = 5\ fF$

$f_T = \frac{4 \times 10^{-3}}{2\pi(50 + 5) \times 10^{-15}} = 11.6\ GHz$

In Summary:

	Standard High-Voltage npn		Standard low-Voltage npn	
	$I_C =$ 10 μA	$I_C =$ 100 μA	$I_C =$ 10 μA	$I_C =$ 100 μA
f_T	26.1 MHz	172.1 MHz	3.35 GHz	11.6 MHz

6.79

$I_C = 1\ mA \Rightarrow g_m = \frac{I_C}{V_T} = 40\ mA/V$

<u>For pnp</u>:

$C_{de} = \tau_F g_m = 30 \times 10^{-9} \times 40\ mA/V = 1200\ pF$

$C_{je} = 2C_{jeo} = 2 \times 0.3 = 0.6\ pF$

$C_\pi = 1200.6\ pF$

$C_\mu \simeq 1\ pF$

$\Rightarrow f_T = \frac{g_m}{2\pi(c_\pi + c_\mu)} = \frac{40\ mA/V}{2\pi(1200.6 + 1)\ pF}$

$f_T = 5.3\ MHz$

<u>For npn</u>:

$C_{de} = \tau_F g_m = 0.35\ ns \times 40\ mA/V = 14\ pF$

$C_{je} = 2 \times 1 = 2\ pF$

$C_\mu = \simeq 0.3\ pF$

$C_\pi = 14 + 2 = 16\ pF$

$\Rightarrow f_T = \frac{40\ mA/V}{2\pi(16 + 0.3)\ pF} = 391\ MHz$

6.80

$A_O = g_m r_O = \frac{2I_D}{V_{OV}} \times \frac{V_A}{I_D} = \frac{2V_A}{V_{OV}} = \frac{2V'_A L}{V_{OV}}$

Therefore A_O is only determined by setting values for L and V_{OV}.

$f_T = \frac{g_m}{2\pi(C_{gs} + C_{gd})}$

$= \frac{2I_D/V_{OV}}{2\pi\left(\frac{2}{3}WLC_{ox} + C_{OV} + C_{OV}\right)}$

If we assume that C_{OV} is very small or equivalently $C_{gs} >> C_{gs}$ and $C_{gs} = \frac{2}{3}WLC_{ox}$:

(replace I_D with $\frac{1}{2}k'_n\frac{W}{L}V_{OV}^2$)

$f_T \simeq \frac{k'_n W/L\ V_{OV}}{2\pi \times \frac{2}{3}WLC_{ox}} = \frac{3\mu_n}{4\pi}V_{OV}/L^2$

$= \frac{3}{4\pi}\mu_n\frac{V_{OV}}{L^2}$

As we can see f_T can be determined after knowing V_{OV} and L, it is not dependent on either I_D or W.

6.81

$V_{OV} = 0.2\text{V}, L = 0.2\ \mu\text{m}, 0.3\ \mu\text{m}, 0.4\ \mu\text{m}$

$$A_O = g_m r_O = \frac{2V_A}{V_{OV}} = \frac{2V_A' L}{V_{OV}} = \frac{2 \times 5 \times L}{0.2}$$

$$= 50\ \text{LV/V}$$

$$f_T \simeq \frac{1.5\mu_n V_{OV}}{2\pi L^2} = \frac{1.5 \times 450 \times 10^{-4} \times 0.2}{2 \times 3.14 \times L^2 \times 10^{-2}}$$

$$= \frac{2.15}{L^2}\ \text{GHz}$$

$L(\mu\text{m})$	0.2	0.3	0.4
A_O (V / V)	10	15	20
f_t (GHz)	53.75	23.9	13.4

6.82

$L = 0.5\ \mu\text{m}, V_{OV} = 0.3\ \text{V}, C_L = 1\ \text{pF},$
$f_T = 100\ \text{MHz}$

$$f_T = \frac{g_m}{2\pi C_L} \Rightarrow g_m = 2\pi C_L f_T$$

$$= 2\pi \times 1\ \text{pF} \times 100\ \text{MHz} = 628\ \mu\text{A/V}$$

$$g_m = \frac{2I_D}{V_{OV}} \Rightarrow I_D = g_m \times V_{OV}/2 = 6.28 \times \frac{0.3}{2}$$

$$I_D = 94.2\ \mu\text{A}$$

$$I_D = \frac{1}{2} k_n' \frac{W}{L} V_{OV}^2 \Rightarrow W = \frac{2LI_D}{k_n' V_{OV}^2}$$

$$= \frac{2 \times 0.5 \times 94.2}{190 \times 0.3^2} = 5.51\ \mu\text{m}$$

$$W = 5.51\ \mu\text{m}$$

$$r_O = \frac{V_A}{I_D} = \frac{V_A' L}{I_D} = \frac{20 \times 0.5}{94.2 \times 10^{-3}} = 106.2\ \text{k}\Omega$$

$$A_O = g_m r_O = \frac{628}{1000} \times 106.2 = 66.7\ \text{V/V}$$

$$f_{3db} = \frac{1}{2\pi C_L r_O} = \frac{1}{2\pi \times 1\text{pF} \times 106.2\ \text{k}\Omega} = 1.5\ \text{MHz}$$

7.1

$V_{DD} = V_{SS} = 2.5V$

$K_n' \frac{W}{L} = 3 \frac{mA}{V^2}$; $V_{tn} = 0.7V$

$I = 0.2mA$; $R_D = 5K\Omega$

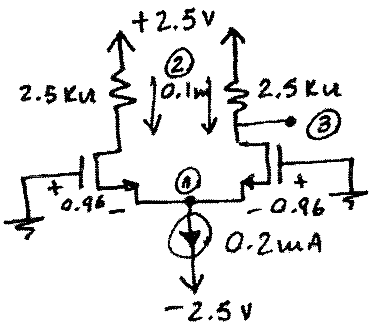
(a) $V_{OV} = \sqrt{I / K_n' W/L}$

$= \sqrt{0.2/3} = \underline{\underline{0.26V}}$

$V_{GS} = V_{OV} + V_t = 0.26 + 0.7$

$= \underline{\underline{0.96V}}$

(b)



① $V_{S1} = V_{S2} = V_{CM} - V_{GS}$

$= 0 - 0.96 = \underline{\underline{-0.96V}}$

② $I_{D1} = I_{D2} = \frac{I}{2} = 0.1mA$

③ $V_{D1} = V_{D2} = V_{DD} - \frac{I}{2} \times R_D$

$= +2.5 - 0.1 \times 2.5 = \underline{\underline{2.25V}}$

(c) If $V_{CM} = +1V$

$V_{S1} = V_{S2} = +1 - 0.96 = \underline{\underline{0.04V}}$

$I_{D1} = I_{D2} = \underline{\underline{0.1mA}}$

$V_{D1} = V_{D2} = \underline{\underline{2.25V}}$

(d) If $V_{CM} = -1V$

$V_{S1} = V_{S2} = -1 - 0.96 = \underline{\underline{-1.96V}}$

$I_{D1} = I_{D2} = \underline{\underline{0.1mA}}$

$V_{D1} = V_{D2} = \underline{\underline{2.25V}}$

(e) $V_{CMAX} = V_t + V_{DD} - \frac{I}{2} R_D$

$= 0.7 + 2.5 - 0.1 \times 2.5 = \underline{\underline{+2.95V}}$

(f) $V_{CMIN} = -V_{SS} + V_{CS} + V_t + V_{OV}$

$= -2.5 + 0.3 + 0.7 + 0.26$

$= \underline{\underline{-1.24V}}$

$V_{S\,min} = V_{CMmin} - V_{GS}$

$= -1.24 - 0.96 = \underline{\underline{-2.2V}}$

7.2

(a) $V_{OV} = -\sqrt{I / K_p' (W/L)}$

$= -\sqrt{0.7/3.5} = \underline{\underline{-0.45V}}$

$V_{GS} = V_{OV} + V_t = -0.45 - 0.8$

$= \underline{\underline{-1.25V}}$

$V_{S1} = V_{S2} = V_G - V_{GS}$

$= 0 + 1.25 = \underline{\underline{+1.25V}}$

$V_{D1} = V_{D2} = \frac{I}{2} \times R_D - V_{DD}$

$= \frac{0.7}{2} \times 2 - 2.5 = \underline{\underline{-1.8V}}$

(b) For Q_1 and Q_2 to remain in saturation:

$V_{DS} \leq V_{GS} - V_t$

$\rightarrow V_{CM} \geq \left(\frac{I}{2} R_D - V_{DD}\right) + V_t$

$V_{CM\,min} = \frac{0.7}{2} \times 2 - 2.5 - 0.8$

$= \underline{\underline{-2.6V}}$

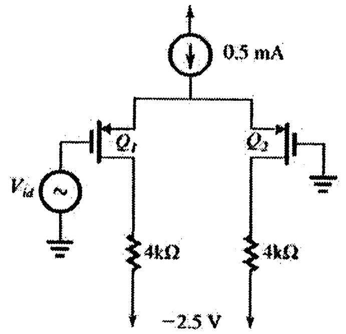
To allow sufficient voltage for the current source to operate properly:

$V_{CM} \leq V_{SS} - V_{CS} + (V_t + V_{OV})$

$\rightarrow V_{CM\,max} = 2.5 - 0.5 - 1.25$

$= 0.75V$ //

7.3



$V_{G2} = 0$

$V_{G1} = v_{id}$

When all the current is on Q_1:

$I = \frac{1}{2}\left(k_p'\frac{w}{L}\right)(V_{GS1} - V_t)^2$

$\Rightarrow V_{GS1} = V_t + \sqrt{\frac{2I}{k_p' w/L}}$

$= V_t + \sqrt{2}V_{OV}$

and V_{GS2} is reduced to V_t thus $V_S = -V_t$.

Then $v_{id} = v_{GS1} + v_S$

$= V_t + \sqrt{2}\, V_{OV} - V_t = \sqrt{2}\, V_{ov}$

In a similar manner as for the NMOS Differential Amplifier, as v_i reaches $-\sqrt{2}\, V_{OV}$ Q_1 turns off and Q_2 on. Thus the steerine range is

$\sqrt{2}\, V_{OV} \le V_i \le -\sqrt{2}\, V_{OV}$

For this particular case

$V_{ov} = \sqrt{\frac{0.25 \text{ mA}}{4 \text{ mA}/\text{V}^2}} = 0.25 \text{ V}$

$\sqrt{2} \times -0.25 \le v_{id} \le \sqrt{2} \times 0.25$

$-0.35 \le v_{id} \le 0.35$

when $V_{id} = -0.35$ V,

$i_{D1} = 0.5 \text{ mA}, i_{D2} = 0$

$V_S = -V_{t2} = +0.8 \text{ V}$

$V_{D1} = 4 \text{ k}\Omega \times 0.5 \text{ mA} - 2.5 = -0.5 \text{ V}$

$V_{D2} = 0 - 2.5 \text{ V} = -2.5 \text{ V}$

when $v_{id} = +0.35$ V,

$i_{D1} = 0$; $i_{D2} = 0.5 \text{ mA}$

$V_S = v_{id} - v_{GS1} = v_{id} - V_{t1}$

$= 0.35 \text{ V} + 0.8 \text{ V} = 1.15 \text{ V}$

$V_{D1} = -2.5 \text{ V}$

$V_{D2} = -0.5 \text{ V}$

7.4

$V_{G1} = v_{id}\ i_{D1} = 0.11 \text{ mA}$

$V_{G2} = 0 \quad i_{D2} = 0.09 \text{ mA}$

$I_D = \frac{1}{2}k_n'\frac{W}{L}(V_{GS} - V_t)^2$

For Q_1:

$0.11 \text{ m} = \frac{1}{2} 5 \text{ m}(V_{GS1} - 0.5)^2$

$\rightarrow V_{GS1} = 0.71 \text{ V}$

For Q_2:

$0.09 \text{ m} = \frac{1}{2} 5 \text{ m}(V_{GS2} - 0.5)^2$

$\rightarrow V_{GS2} = 0.69 \text{ V}$

$V_S = -V_{GS2} = -0.69 \text{ V}$

$v_{id} = V_S + V_{GS1} = -0.69 + 0.71$

$= 0.02 \text{ V}$

$V_{D2} - V_{D1} = 10 \text{ k}\Omega\ (i_{D1} - i_{D2})$

$= 10 \text{ kV } (0.11\ 2\ 0.09) \text{ m}$

$= 0.2 \text{ V}$

thus

$\frac{V_{D2} - V_{D1}}{v_{id}} = \frac{0.2}{0.02} = 10$

when $i_{D1} = 0.09$ mA and

$i_{D2} = 0.11$ mA

is the reverse condition from the case we just studied, thus $v_{id} = -0.02$ V

7.5

$V_{GS} = V_{tn} + V_{OV} = 0.5 \text{ V} + 0.2 \text{ V} = 0.7 \text{ V}$

$V_{D4} = V_{G4} = -V_{SS} + V_{GS} = -1.2 \text{ V} + 0.7 \text{ V}$

$= -0.5 \text{ V}$

$R = \frac{V_{DD} - V_{D4}}{0.1 \text{ mA}} = \frac{1.2\text{V} - (-0.5 \text{ V})}{0.1 \text{ mA}} = 17 \text{ k}\Omega$

$R_D = \frac{V_{DD} - V_{D1}}{0.4 \text{ mA}/2} = \frac{1.2\text{V} - 0.2 \text{ V}}{0.2 \text{ mA}} = 5 \text{ k}\Omega$

$\left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = \frac{0.4 \text{ mA}}{2}\left[k_n' V_{ov}^2\right]$

$= 0.2 \text{ mA}[(0.25 \text{ mA}/\text{V}^2)(0.2\text{V})^2]^{-1} = 20$

$\left(\frac{W}{L}\right)_3 = 0.4 \text{ mA}[0.01 \text{ mA}]^{-1} = 40$

$\left(\frac{W}{L}\right)_4 = 0.1 \text{ mA}[0.01 \text{ mA}]^{-1} = 10$

$V_{Cm(max)} = V_{tn} + V_{DD} - (I/2)R_D$

$= 0.5 \text{ V} + 1.2 \text{ V} - (0.2 \text{ mA})(5 \text{ k}\Omega) = 0.7 \text{ V}$

$V_{Cm(min)} = -V_{SS} + V_{OV3} + V_{tn} + V_{OV1}$

$= -1.2 \text{ V} + 0.2 \text{ V} + 0.5 \text{ V} + 0.2 \text{ V} = -0.3 \text{ V}$

7.6

We know that there is a linear relationship between Vov & Vid since:

$$V_{ov} = \frac{V_{id}/2}{\sqrt{0.1}}$$

Then from the data in table 7.3 we can tell that for $V_{imax} = 150\,mV$

$$V_{ov} = 0.2 \times \frac{150}{126} = \underline{\underline{0.238\,V}}$$

For W/L: $\frac{W}{L} = \frac{1}{(V_{ov})^2} \cdot \frac{I}{K}$

where I and K are constant thus, for W/L:

$$\left(\frac{W}{L}\right)_2 = \frac{50}{\left(\frac{150}{126}\right)^2} = 35.3$$

For gm: $g_m = \frac{I}{V_{ov}}$ where I is constant

$$\rightarrow g_{m2} = \frac{g_{m1}}{\left(\frac{150}{126}\right)} = \frac{2}{\frac{150}{126}} = \underline{\underline{1.68\,\frac{mA}{V}}}$$

7.7

$$\left(\frac{v_{idmax}/2}{V_{OV}}\right)^2 = K$$

$$\Rightarrow 2V_{OV}\sqrt{K} = v_{idmax}$$

Q.E.D.

$$i_{D1} = \frac{I}{2} + \left(\frac{I}{V_{OV}}\right)\frac{v_{id}}{2}\sqrt{1-K}$$

$$i_{D1} = \frac{I}{2} \pm \frac{I}{V_{OV}} \cdot \frac{2}{2}V_{OV}\sqrt{K} \cdot \sqrt{1-K}$$

$$\rightarrow i_{D1} = \frac{I}{2} \pm I\sqrt{K(1-K)}$$

thus $\Delta I = 2I\sqrt{K(1-K)}$

Q.E.D.

For K = 0.01

$$\Delta I = 2I\sqrt{0.01(1-0.01)}$$
$$= 0.198 \times I$$

$$V_{idmax} = 2V_{OV}\sqrt{0.01} = 0.2V_{OV}$$

For K = 0.1

$$\Delta I = 2I\sqrt{0.1(1-0.1)} = 0.8I$$

$$V_{idmax} = 2V_{OV}\sqrt{0.2}$$
$$= 0.894.\ V_{OV}$$

7.8

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_t)^2$$

$$\frac{200}{2} = \frac{1}{2} \times 90 \times \frac{100}{1.6}(V_{GS}-0.8)^2$$

$$\Longrightarrow V_{GS} = \underline{\underline{1.19\,V}}$$

$$g_m = \frac{2I_D}{V_{GS}-V_t} = \frac{2 \times 100}{(1.19-1)} = \underline{\underline{1.06\,\frac{mA}{V}}}$$

$$V_{id}\Big|_{\text{full current switching}} = \sqrt{2}\,(V_{GS}-V_t)$$
$$= \underline{\underline{0.27\,V}}$$

To double this value, $V_{GS}-V_t$ must be doubled which means that I_D should be quadrupled. i.e. I changed to: $\underline{\underline{800\,\mu A}}$

7.9

$$g_m = \frac{2I_0}{V_{OV}} \rightarrow 1m = \frac{I}{0.2}$$

$$\rightarrow I = 0.2mA$$

$$I_0 = \frac{1}{2}\mu_n C_{ox} \frac{W}{L} V_{OV}^2$$

$$100 = \frac{1}{2} \times 90 \times \frac{W}{L} \times (0.2)^2$$

$$\Rightarrow \frac{W}{L} = 55.6$$

7.10

$$i_D = \frac{1}{2} k_n \frac{W}{L}(V_{GS} - V_t)^2$$

$$50 = \frac{1}{2} \times 400(V_{GS} - 1)^2$$

$$\Rightarrow V_{GS} = 1.5 \text{ V}$$

For $v_{G1} = v_{G2} = 0$, $v_S = -1.5$ V

For $v_{G1} = v_{G2} = 2$ V, $v_S = +0.5$ V

The drain currents are equal in both cases.

For $V_{G2} = 0$:

To reduce i_{D2} by 10%,

$i_{D2} = 0.9 \times 50 = 45$ μA

$i_{D1} = 55$ μA

$$v_{GS2} = \sqrt{\frac{2i_{D2}}{400}} + 1 = 1.47 \text{ V}$$

$$v_{GS1} = \sqrt{\frac{2 \times 55}{400}} + 1 = 1.52 \text{ V}$$

Thus, $V_{G1} = v_{GS1} - v_{GS2} = 0.05$ V

To increase i_{D2} by 10%

$i_{D2} = 55$ μA

$i_{D1} = 45$ μA

$v_{GS2} = 1.52$ V

$v_{GS1} = 1.47$ V

$\Rightarrow V_{G1} = -0.05$ V

i_{D2}/i_{D1}	i_{D2} (μA)	i_{D1} (μA)	V_{GS2} (V)	V_{GS1} (V)	$V_G - V_{G1}$ (V)
1	50	50	1.5	1.5	0
0.5	33.3	66.7	1.408	1.577	−0.17
0.8	47.4	52.6	1.487	1.513	−0.026
0.99	47.75	50.25	1.4886	1.5012	−0.013

For $i_{D1}/i_{D2} = 20 \Rightarrow i_{D2} = 4.76$ μA

$i_{D1} = 95.24$ μA

$V_{GS2} = 1.154$ V, $V_{GS1} = 1.690$

Thus $V_{G1} - V_{G2} = 0.536$ V

7.11

(a) $V_{od} = V_{D2} - V_{D1} =$

$(V_{DD} - i_{D2}R_D) - (V_{DD} - i_{D1}R_D) = (i_{D1} - i_{D2})R_D$

$$V_{od} = \left[\left(\frac{I}{V_{OV}}\right)\left(\frac{V_{id}}{2}\right)\sqrt{1 - \left(\frac{V_{id}/2}{V_{OV}}\right)^2} + \left(\frac{I}{V_{OV}}\right)\left(\frac{V_{id}}{2}\right)\sqrt{1 - \left(\frac{V_{id}/2}{V_{OV}}\right)^2}\right]R_D$$

$$= IR_D \frac{V_{id}}{V_{OV}}\sqrt{1 - \left(\frac{V_{id}/2}{V_{OV}}\right)^2}$$

(b) see plot

slope of linear portion

$$= \frac{d}{dV_{id}}\left(\frac{IR_D}{V_{OV}} \cdot V_{id}\right) = IR_D/V_{OV}$$

(c) see plot

when the bias current is doubled, V_{OV} so

$$V_{od}/V_{id} = \frac{2IR_D}{\sqrt{2}\,V_{OV}}\sqrt{1 - \left(\frac{V_{id}/2}{\sqrt{2}\,V_{OV}}\right)^2}$$

increases by a factor of $\sqrt{2}$ the slope of the linear part has increased by a factor of $\sqrt{2}$

(d) see plot

If W/L is doubled, V_{OV} reduces by a factor at $\sqrt{2}$

$$\text{so } V_{od}/V_{id} = \frac{2IR_D}{\sqrt{2}\,V_{OV}}\sqrt{1 - \left(\frac{V_{id}/\sqrt{2}}{V_{OV}}\right)^2}$$

The slope of the linear part has increased by factor of $\sqrt{2}$ compared to (b)

7.12

$V_{OV} = \sqrt{I / K_n' \frac{W}{L}} = \sqrt{\frac{0.5}{0.25 \times 50}} = \underline{0.2V}$

$g_m = \frac{I}{V_{OV}} = \frac{0.5mA}{0.2V} = \underline{2.5 \frac{mA}{V}}$

$r_o = \frac{V_A}{I_D} = \frac{10}{(0.5m/2)} = \underline{40K\Omega}$

$A_d = g_m \times (R_D \| r_o)$

$= 2.5 \frac{mA}{V} (4K\Omega \| 40K\Omega)$

$= \underline{9.09} \; V/V$

7.13

$\left(\frac{V_{id}/2}{V_{OV}}\right)^2 = 0.1 \rightarrow \left(\frac{0.2/2}{V_{OV}}\right)^2 = 0.1$

$\rightarrow V_{OV} = \sqrt{0.1} = \underline{0.316 \; V}$

$g_m = \frac{I}{V_{OV}} \rightarrow 3 \frac{mA}{V} = \frac{I}{0.316}$

$\rightarrow I = \underline{0.95mA}$

also: $V_{OV} = \sqrt{\frac{I}{K_n' W/L}}$

$\Rightarrow (0.316)^2 = \frac{0.95 mA}{0.1 \frac{mA}{V^2} \times \left(\frac{W}{L}\right)}$

$\rightarrow \frac{W}{L} = \underline{95}$

If $R_D = 5K\Omega \Rightarrow$

$A_d = g_m R_D = 3 \frac{mA}{V} \times 5K\Omega = \underline{15 \; V/V}$

if $V_{id} = 0.2 \Rightarrow V_{od} = V_{id} \times A_d$

$= 0.2 \times 15 = \underline{3V}$

7.14

(a) $g_m = \frac{A_d}{R_D} = \frac{20}{47 \text{ k}\Omega} = 0.426 \text{ mA / V}$

(b) $I = g_m V_{OV} = (0.426 \text{ mA / V})(0.2\text{V})$

$= 85 \; \mu\text{A}$

(c) $V_{RD} = \frac{I}{2} R_D = (85 \; \mu\text{A}/2)(47 \text{ k}\Omega)$

$= 2 \text{ V}$

(d) $V_{id(MAX)} = V_{CM} + 10 \text{ mV} = 0.51 \text{ V}$

$V_{DD} \geq V_{id(MAX)} - V_t + I_D R_D$

$= 0.51 \text{ V} - V_t + (85 \; \mu\text{A}/2)(47 \text{ k}\Omega)$

$= 2.51 \text{ V} - V_t$

7.15

For a CS amplifier $A_v = -g_m R_D$

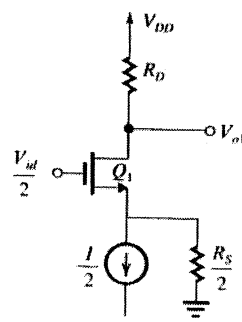
For a differential amplifier $A_d = g_m R_D$ with $I = 2I_D$

So the differential pair requires twice the bias current as the CS amplifier.

The power dissipation at the diff amp is also twice as high.

7.16

HALF-CIRCUIT



small-signal analysis

$V_{gs} = \frac{V_{id}}{2} - g_m V_{gs} \frac{R_s}{2}$

$V_{gs} = \frac{V_{id}/2}{1 + g_m \frac{R_s}{2}}$

$V_{O1} = -g_m V_{gs} R_D = -g_m \left[\frac{V_{id}/2}{1 + g_m R_s/2}\right] R_D$

$$A_d = \frac{V_{od}}{V_{id}} = \frac{g_m R_D}{1 + g_m R_s/2}$$

when $R_S = 0$ $A_d = g_m R_D$ (agrees with Eqtn. 8.35)

when $R_S = \frac{2}{g_m}$ the differential gain is reduced by half

7.17

(a) $V_{G1} = V_{G2} = 0V$

$V_{S1} = V_{S2}$ assuming matching components

$$V_{S1} = V_{G1} - V_{GS1} = 0V - (V_t + V_{OV})$$
$$= -(V_t + V_{OV})$$

(b) zero current flows through Q_3

$$V_{OV3} = V_C - V_{S1} - V_t = V_C - (-(V_t + V_{OV})) - V_t$$
$$= V_C + V_t$$
$$= V_C + V_{OV}$$

(c) $V_{G1} = -V_{G2} = V_{id}/2$

V_{S1} is now more negative than in (a) and V_{S2} is now less negative than in (a) so there is a voltage across Q_3. If this voltage is small and if V_C is such that $V_{GS3} > V_t$ then Q_3 will operate in triode.

$$r_{Ds3} = \left[k_n' \frac{W}{L} V_{ov3}\right]^{-1}$$

$$g_{m1} = g_{m2} = \frac{1/2\, k_n' \frac{W}{L} V_{ov}^2}{V_{os}} = 1/2\, k_n' \frac{W}{L} V_{ov}$$

$$\text{so } r_{Ds3} = \left[g_{m1} \frac{V_{ov3}}{V_{ov}}\right]^{-1} = \frac{V_{ov}}{V_{ov3}\, g_{m1}}$$

(d) $r_{DS3} = \frac{V_{OV}}{V_{OV3}} \cdot \frac{1}{g_{m1}}$

(*i*) $R_S = \frac{1}{g_{m1}}$ $\therefore V_{OV3} = V_{OV}$

From (b) $V_{OV3} = V_C + V_{OV}$ so $V_C = 0V$

(*ii*) $R_s = \frac{1}{2 g_{m1}}$ $\therefore V_{Os3} = 2 V_{OV}$

so $V_C = V_{OV}$

7.18

(a) $V_{G1} = V_{G2} = 0V$

$V_{S1} = V_{S2} = -(V_t + V_{OV})$

Zero current flows through Q_3 and Q_4

Q_3 and Q_4 have the same overdrive voltage as Q_1 and Q_2

$$r_{DS3} = r_{DS4} = \left[k_n'\left(\frac{W}{L}\right)_{3,4} V_{OV3,4}\right]^{-1}$$

$$= \left[k_n'\left(\frac{W}{L}\right)_{3,4} V_{OV1,2}\right]^{-1}$$

$$g_{m1,2} = \frac{1}{2}k_n'\left(\frac{W}{L}\right)_{1,2} V_{OV1,2}$$

$$V_{OV1,2} = g_{m1,2}\left[\frac{1}{2}k_n'\left(\frac{W}{L}\right)_{1,2}\right]^{-1}$$

$$r_{DS3} = r_{DS4} = r_{DS3,4}$$

$$= \left[k_n'\left(\frac{W}{L}\right)_{3,4} g_{m1,2}\left[\frac{1}{2}k_n'\left(\frac{W}{L}\right)_{1,2}\right]^{-1}\right]^{-1}$$

$$= \left[2g_{m1,2}\left(\frac{W}{L}\right)_{3,4} / \left(\frac{W}{L}\right)_{1,2}\right]^{-1}$$

$$= \frac{\left(\frac{W}{L}\right)_{1,2}}{\left(\frac{W}{L}\right)_{3,4} \cdot 2 \cdot g_{m1,2}}$$

$$R_S = 2r_{DS3,4} = \frac{\left(\frac{W}{L}\right)_{1,2}}{g_{m1,2}\left(\frac{W}{L}\right)_{3,4}}$$

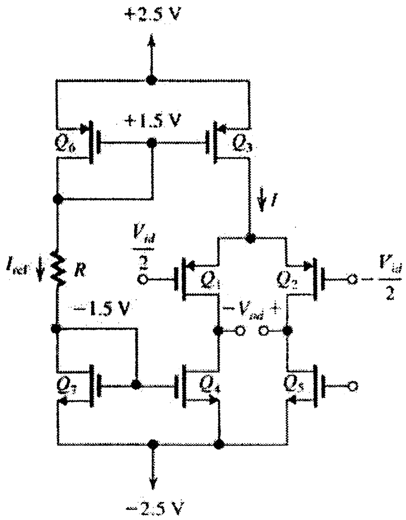
(b) $A_d = V_{od}/V_{id} = \frac{g_m R_D}{1 + g_m R_S/2}$

(See solution to 8.21)

$$= \frac{g_{m1,2} R_D}{1 + g_{m1,2}\left[\frac{\left(\frac{W}{L}\right)_{1,2}}{\left(\frac{W}{L}\right)_{3,4} \cdot 2 \cdot g_{m1,2}}\right]}$$

$$= \frac{g_{m1,2} R_D}{1 + \frac{\left(\frac{W}{L}\right)_{1,2}}{2\left(\frac{W}{L}\right)_{3,4}}}$$

7.19



For $I_{REF} = 100\ \mu A$,

$$R = \frac{V_{D6} - V_{D7}}{I_{REF}} = \frac{1.5-(-1.5)}{0.1\ mA} = 30\ k\Omega$$

$V_{GS7} = V_{GS4} = V_{GS5} = -1.5-(-2.5) = 1\ V$

$V_{OV7} = V_{OV4} = V_{OV5} = V_{GS} - V_{tn} = 1 - 0.7 = 0.3\ V$

$V_{GS6} = V_{GS3} = 1.5 - 2.5 = -1\ V$

$V_{OV6} = V_{OV3} = V_{GS} - V_{tp} = -1-(-0.7) = -0.3\ V$

From section 8.23, we know that

$A_d = g_{m1}(r_{o1} \| r_{o4})$

Since Q_1 and Q_2 circuits are symmetical

With $I = I_{REF} = 100\ \mu A$

$$I_D = \frac{I}{2} = 50\ \mu A$$

$$r_{o1} = r_{o2} = r_{o4} = r_{o5} = \frac{|V_A|}{I_D} = \frac{20\ V}{50\ \mu A} = 400\ k\Omega$$

So,

$80\ V/V = g_{m1}(400\ K \| 400\ K)$

and

$g_{m1} = 400\ \mu A/V$

Since $g_m = \frac{|I_D|}{|V_{OV}|/2}$,

$$|V_{OV1}| = |V_{OV2}| = |V_{OV4}| = |V_{OV5}| = \frac{2I_D}{g_m}$$

$$= \frac{2(50\ \mu A)}{400\ \mu A/V} = 0.25\ V$$

so,

$V_{GS1} = V_{GS2} = V_{OV} + V_{tp} = -0.25 - 0.7$

$= -0.95\ V$

For $\left(\frac{W}{L}\right)$ ratios

$$I_D = \frac{1}{2}\mu C_{ox}\left(\frac{W}{L}\right)(V_{OV})^2$$

So that

$$\frac{W}{L} = \frac{2I_D}{\mu C_{ox} V_{OV}^2}$$

For Q_7,

$$\left(\frac{W}{L}\right)_7 = \frac{2(100\ \mu A)}{90\ \mu A/V^2(0.3\ V)^2} = 24.7$$

For Q_4 and Q_5,

$$\left(\frac{W}{L}\right)_4 = \left(\frac{W}{L}\right)_5 = \frac{2(50\ \mu A)}{90\ \mu A/V^2(0.3)^2} = 12.3$$

For Q_1 and Q_2,

$$\left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = \frac{2(50\ \mu A)}{30\ \mu A/V^2(0.25)^2} = 53.3$$

For Q_6 and Q_3,

$$\left(\frac{W}{L}\right)_6 = \left(\frac{W}{L}\right)_3 = \frac{2(100\ \mu A)}{30\ \mu A/V^2(0.3\ V)^2} = 74.1$$

In summary, the results are as follows:

	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	
μC_{ox}	30	30	30	90	90	30	90	$\mu A/V^2$
I_D	50	50	100	50	50	100	100	μA
V_{OV}	−0.25	−0.25	−0.3	0.3	0.3	−0.3	0.3	V
$\frac{W}{L}$	53.3	53.3	74.1	12.3	12.3	74.1	24.7	
V_{GS}	−0.95	−0.95	−1	1	1	−1	1	V

7.20

(a) $I_{D1} = \frac{1}{2}k_n'\frac{W}{L}(V_{GS1} - V_t)^2$

$I_{D2} = \frac{1}{2}k_n'\left(2 \times \frac{W}{L}\right)(V_{GS2} - V_t)^2$

Since $V_{GS} - V_t$ is equal for both transistors :

$\Rightarrow \frac{I_{D1}}{I_{D2}} = \frac{1}{2} ; I_{D2} = 2I_{D1}$

but $I = I_{D1} + I_{D2} = 3I_{D1}$

$I_{D1} = I/3$

$I_{D2} = 2I/3$

(b) $V_{OV} = V_{GS} - V_t$

$V_{OV1} = V_{OV2} = V_{OV}$

For $Q1: \frac{I}{3} = \frac{1}{2}k_n'\left(\frac{W}{L}\right)V_{OV}^2$

$$\Rightarrow V_{OV} = \sqrt{\frac{2}{3}\frac{I}{k_n' W/L}}$$

(c) $g_m = \frac{2I_D}{V_{OV}} \rightarrow g_{m1} = \frac{2I}{3V_{OV}}$

$g_{m2} = \frac{4}{3}\frac{I}{V_{OV}}$

$v_{O1} = -g_{m1} \times \frac{v_{id}}{2} \cdot R_D$

$= -\frac{2}{3}\frac{I}{V_{OV}} \cdot R_D \cdot v_{id}$

$v_{O2} = +g_{m2} \times \frac{v_{id}}{2} \cdot R_D$

$= \frac{4}{3}\frac{I}{V_{OV}} \cdot R_D \cdot v_{id}$

$\Rightarrow \frac{v_{O2} - v_{O1}}{v_{id}} = \left(\frac{4}{3} + \frac{2}{3}\right)\frac{I}{V_{OV}} \cdot R_D$

$= 2 \times \frac{I}{V_{OV}} \cdot R_D$

7.21

$$A_d = g_{m1}(R_{on} \| R_{op})$$

$= g_{m1}[(g_{m3}r_{O3})r_{O1} \| (g_{m5}r_{O5})r_{O7}]$

If all transistors have the same channel length and the same $|V_{OV}|$ and $|V_A|$ Since $g_m = \frac{2I_D}{V_{OV}}$ and

$r_O = \frac{V_A}{I_D}$ and with g_m and r_o the same for all devices.

$$A_d = \frac{2I_D}{V_{OV}}\left(\left(\frac{2I_D}{V_{OV}} \cdot \frac{V_A}{I_D}\right)\frac{V_A}{I_D}\right) \| \left(\left(\frac{2I_D}{V_{OV}} \cdot \frac{V_A}{I_D}\right)\frac{V_A}{I_D}\right)$$

$= \frac{2I_D}{V_{OV}}\left[\frac{2\,V_A^2}{V_{OV}\,I_D} \| \frac{2\,V_A^2}{V_{OV}\,I_D}\right]$

$= \left(\frac{2I_D}{V_{OV}} \cdot \frac{V_A^2}{V_{OV}\,I_D}\right)$

$= \frac{2\,V_A^2}{V_{OV}^2} = 2\left(\frac{|V_A|}{|V_{OV}|}\right)^2$

For $A_d = 1000$ V/V and $|V_{OV}| = 0.2$ V

$1000 = 2\frac{|V_A|^2}{|V_{OV}|^2}$

$V_A = \sqrt{500} \cdot 0,2\text{ V} = 4.47$ V

If $|V_A'| = 10$ V/µA

$L = \frac{4.47\text{ V}}{10\text{ V}/\mu\text{M}} = 0.447\ \mu\text{m}$

For high g_m the bias current should be high, but with ± 0.9 V Supplies the bias current must not exceed $\frac{1\text{ mW}}{1.8\text{ V}} = 0.556$ mA to keep power dissipation at 1 mW

7.22

$V_{OV} = \sqrt{\frac{I}{K_n'W/L}} = \sqrt{\frac{0.2}{3}} = 0.26V$

$g_m = \frac{I}{V_{OV}} = \frac{0.2mA}{0.26V} = 0.77\frac{mA}{V}$

(a) Single-ended output:

$|A_d| = \frac{1}{2}g_m \times R_D = \frac{0.77 \times 10}{2}$

$= 3.85$ V/V

$|A_{cm}| = \frac{R_D}{2R_{SS}} = \frac{10}{2 \times 100} = 0.05$ V/V

$CMRR = \left|\frac{A_d}{A_{cm}}\right| = \frac{3.85}{0.05} = 77$

i.e 37.7 dB

(b) Differential output, and 1% mismatch in R_D's:

$$|A_d| = g_m R_D = 0.77 \times 10 = 7.7\ V/V$$

$$|A_{cm}| = \frac{R_D}{2R_{SS}} \times \left(\frac{\Delta R_D}{R_D}\right)$$

$$= \frac{10}{2\times 100} \times 0.01 = 0.5\ mV/V$$

$$CMRR = \left|\frac{A_d}{A_{cm}}\right| = \frac{7.7}{0.5\times10^{-3}} = 15,400$$

i.e 83.7 dB

7.23

$$V_{OV} = -\sqrt{\frac{I}{k_p' W/L}} = -\sqrt{\frac{0.7\ mA}{3.5\ mA/V^2}}$$

$$= -0.45\ V$$

$$g_m = \frac{I}{|V_{OV}|} = \frac{0.7\ mA}{0.45\ V} = 1.56\ \frac{mA}{V}$$

$$|A_d| = g_m R_D = 1.56 \times 2 = 3.12\ V/V$$

$$|A_{cm}| = \frac{R_D}{2R_{SS}} \cdot \left(\frac{\Delta R_D}{R_D}\right) = \frac{2}{2\times 30} \times 0.02$$

$$= 6.7\times10^{-4}$$

$$CMRR = \frac{3.12}{6.7\times10^{-4}} = 4680 \rightarrow 73.4\ dB$$

7.24

(a) $R_{D1} = R_D + \frac{\Delta R_D}{2}$ $R_{D2} = R_D - \frac{\Delta R_D}{2}$

$g_{m1} = g_m + \frac{\Delta g_m}{2}$ $g_{m2} = g_m - \frac{\Delta g_m}{2}$

$i_{d1} = \frac{g_{m1}V_{icm}}{g_m R_{SS}}$ $i_{d2} = \frac{g_{m2}V_{icm}}{2g_m R_{SS}}$

$$i_{d1} - i_{d2} = (g_{m1} - g_{m2})\frac{V_{icm}}{2g_m R_{SS}}$$

$$= \Delta g_m \frac{V_{icm}}{2g_m R_{SS}} \quad (1)$$

$$i_{d1} + i_{d2} = (g_{m1} + g_{m2})\frac{V_{icm}}{2g_m R_{SS}}$$

$$= (2g_m)\frac{V_{icm}}{2g_m R_{SS}} = \frac{V_{icm}}{R_{SS}} \quad (2)$$

$$V_{od} = V_{O2} - V_{O1} = -i_{d2}R_{D2} + i_{d1}R_{D1}$$

$$= -i_{d2}\left(R_D - \frac{\Delta R_D}{2}\right) + i_{d1}\left(R_D + \frac{\Delta R_D}{2}\right)$$

$$V_{Od} = R_D(i_{d1} - i_{d2}) + \frac{\Delta R_D}{2}(i_{d2} + i_{d1})$$

Now substitute (1) and (2)

$$V_{Od} = R_D\left(\Delta g_m \frac{V_{icm}}{2g_m R_{SS}}\right) + \frac{\Delta R_D}{2}\left(\frac{V_{icm}}{R_{SS}}\right)$$

$$A_{CM} = \frac{V_{Od}}{V_{icm}} = \frac{R_D}{R_{SS}}\cdot\frac{\Delta g_m}{2g_m} + \frac{\Delta R_D}{2R_{SS}}$$

$$= \frac{R_D}{2R_{SS}}\left[\frac{\Delta g_m}{g_m} + \frac{\Delta R_D}{R_D}\right]$$

(b) $R_D = 5\ K\Omega$ $R_{SS} = 25\ K\Omega$

If $A_{cm} = 0.002$ V/V, use the result of (a)

$$A_{cm} = 0.002 = \frac{R_D}{2R_{SS}}\left[\frac{\Delta g_m}{g_m} + \frac{\Delta R_D}{R_D}\right]$$

So, ΔR_D can compensate for Δg_m

$$0.002 = \frac{5\ k\Omega}{2.25\ k\Omega}\cdot\frac{\Delta R_D}{5\ k\Omega}$$

$$\Delta R_D = 0.002(50\ k\Omega) = 100\ \Omega$$

so a 100 ohm compensation in R_D(a 2% adjustment) is sufficient.

7.25

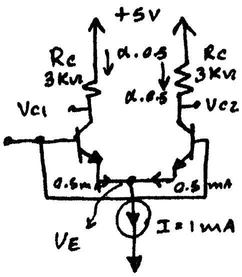
If $A_O = 100$ (40dB), R_{SS} and therefore $CMRR$ will increase by 40 dB.

$$A_O = \frac{V_A}{V_{OV}/2}$$

$$V_A = 100\cdot\frac{V_{OV}}{2} = 100\left(\frac{0.2\ V}{2}\right) = 10\ V$$

for $V_A = \frac{10\ V}{\mu m}$, $L = 1\ \mu m$

7.26



$V_{BE} = 0.7$ @ $i_c = 1\text{mA}$

→ at $i_c = 0.5\text{mA}$

$V_{BE} = 0.7 + 25\text{m} \ln\left(\frac{0.5}{1}\right)$

$= 0.683\text{V}$

Thus,

$V_E = V_{CM} - V_{BE}$

$= -2 - 0.683 = -2.683\text{V}$

$i_{c1} = i_{c2} = \alpha \times 0.5 = \frac{100}{101} \times 0.5$

$= 0.495\text{mA}$

$V_{c1} = V_{c2} = V_{cc} - i_c R_c$

$= 5 - 0.495 \times 3$

$= +3.515\text{V}$

7.27

$I = 0.5\text{ mA}$ So $I_{C1} = I_{C2} = 0.25\text{ mA}$

$V_E = V_B - V_{BE}$ $V_{BE} = 0.7 + 0.025 \cdot \ln\left(\frac{i_E}{I}\right)$

for $i_E = 0.5\text{ mA}$, $V_{BE} = 0.683\text{V}$

if $V_{B1} = 0.5\text{ V}$ and $V_{B2} = 0\text{V}$, $V_{id} = 0.5\text{ V}$

$i_{E1} = \frac{I}{1 + e^{-V_{id}/V_T}} = \frac{0.5\text{ mA}}{1 + e^{-0.5\text{ V}/.025\text{ V}}}$

$= \frac{0.5\text{ mA}}{1 + e^{-20}} = 0.5\text{ mA}$

$i_{E2} = \frac{I}{1 + e^{V_{id}/V_T}} = \frac{0.5\text{ mA}}{1 + e^{0.5/.025}}$

$= \frac{0.5\text{ mA}}{4.85 \times 10^{8}} \approx 1 \times 10^{-12}\text{A}$

$i_{C1} = \frac{100}{101} 0.5\text{ mA} = 0.495\text{ mA}$

$V_{C1} = 2.5\text{ V} - (0.495\text{ mA})(8\text{ k}\Omega)$

$= -1.46\text{ V}$

$i_{C2} \approx 0$ $V_{C2} = 2.5\text{ V}$

$V_E = 0.5\text{ V} - 0.683\text{ V} = -0.183\text{ V}$

if $V_{B1} = -0.5\text{ V}$ and $V_{B2} = 0\text{ V}$

$V_{id} = -0.5\text{ V}$ $i_{E1} \approx 0$ $i_{E2} \approx 0.5\text{ mA}$

(Same equations as above)

$V_{C1} = 2.5\text{ V}$ $V_{C2} = -1.46\text{ V}$

$V_E = 0 - 0.683\text{ V} = -0.683\text{ V}$

7.28

$V_{cm\,max} = V_{CC} - \alpha \frac{I}{2} R_C + 0.4\text{ V}$

$= 2.5\text{ V} - \frac{100}{101}\left(\frac{0.5\text{ mA}}{2}\right) 8\text{ k}\Omega + 0.4\text{V} = 0.92\text{ V}$

$V_{CM\,min} = -V_{EE} + V_{CS} + V_{BE}$

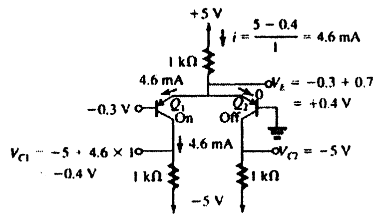
$= -2.5\text{ V} + 0.3\text{ V} + V_{BE}$

$V_{BE} = 0.7\text{ V} + 0.025 \ln\left(\frac{0.25\text{ mA}}{1\text{ mA}}\right) = 0.665\text{ V}$

$V_{CM\,min} = -2.2\text{ V} + 0.665\text{ V} = -1.53\text{ V}$

So $-1.53\text{ V} < V_{CM} < 0.92\text{ V}$

7.29



7.30

$V_{BE} = 690\text{ mV}$ at $i_C = 1\text{ mA}$ $\beta = 50$

$V_{CE}(SAT) = 0.3\text{V}$

$R_C = 82\text{ k}\Omega$ $V_{CC} = -V_{EE} = 1.2\text{ V}$

$I = 20\ \mu\text{A}$

(a)

$V_{BE} = 690\text{ mV} + 25\text{ mV} \ln\left(\frac{10\mu\text{A}}{1000\mu\text{A}}\right) = 575\text{ mV}$

$V_E = V_B - V_{BE} = -575\text{ mV}$

$V_{C1} = V_{C2} = 1.2\text{ V} - (10\ \mu\text{A})(82\text{ k}\Omega) = 0.38\text{ V}$

(b) .

$V_{CM\,MAX} = V_{CC} - \alpha\frac{I}{2}R_C + 0.4\text{ V}$

$= 1.2\text{V} - \frac{50}{51}10\ \mu\text{A} \cdot 82\text{ k}\Omega + 0.4\text{ V}$

$= 0.8\text{V}$

$V_{CM\,MIN} = V_{EE} + V_{CS} + V_{BE}$

$= -1.2\text{ V} + 0.3\text{ V} + 0.575\text{ V} = -0.325\text{ V}$

So $-0.325\text{ V} < V_{CM} < 0.80\text{ V}$

(c)

$i_{E1} = 1.1\left(\frac{I}{2}\right) = \frac{1}{1+e^{-V_{id}/V_T}}\ \frac{1}{0.55} = 1 + e^{-V_{id}/V_T}$

$0.82 = e^{-V_{id}/V_T}$

$-V_T\ln(0.82) = V_{id} = 5\text{ mV}$

if $V_{B2} = 0, V_{B1} = +5\text{ mV}$

(c) $v_{CM\,max} = 3 = 5 - \frac{I}{2}R_C$

$\Rightarrow IR_C = 4V$

(d) $\frac{I/2}{\beta+1} \leq 2\,\mu A$

$\Rightarrow I \leq 4(\beta+1)\,\mu A$

Thus, $I = 4 \times 101\,\mu A = 0.404\,\mu A$

Select $I = 0.4\,mA$

$R_C = \frac{4V}{I} = \frac{4V}{0.4mA} = 10K\Omega$

7.31

With only common-mode at the inputs

$V_{C1} = V_{C2} = V_{CC} - \alpha\frac{I}{2}R_C + V_r$

therefore the ripple voltage directly appears at the single-ended output V_{C1} and V_{C2}

However, because the differential output $V_{od} = V_{C2} - V_{C1}$ does not include the common-mode output, the ripple voltage does not appear on the differential output.

This is an advantage of using the differential output compared to using the single ended output.

7.32

(a) $v_{CM\,max} = v_{C1,2} = V_{CC} - \frac{I}{2}\cdot R_C$

(b) If the current is steered to Q_1, then

$v_{C1} = V_{CC} - IR_C$, a change of: $-\frac{I}{2}R_C$

$v_{C2} = V_{CC}$, a change of $+\frac{I}{2}R_C$

7.33

$i_{E1} = \frac{I}{1+e^{-\frac{v_d}{V_T}}}$, $v_d = v_{B1} - v_{B2}$

$\frac{\Delta i_{E1}}{I} = \frac{i_{E1} - I/2}{I} = \frac{I_{E1}}{I} - 0.5$

Define normalized Gain

$G_n = \frac{\Delta i_{E1}I}{v_d}$

vd (mV)	5	10	20	30	40
G_n	9.97	9.87	9.50	8.95	8.30

Observe that the gain stays relatively constant upto V_d nearly 20 mV. Then it decreases significantly with the increase in signal level. Whenever gain depends on signal level, nonlinear, distortion occurs.

7.34

With:

$V_{B1} - V_{B2} = 10mV$

$i_{E1} = \frac{I}{1+e^{-10/25}} = 0.598 I$

Since $i_{E1} + i_{E2} = I$

$i_{E2} = 0.402 I$

For a collector resistance R_C

$V_o = V_{C1} - V_{C2} = (V_{CC} - i_{C1} R_C) - (V_{CC} - i_{C2} R_C)$

$= -(i_{C2} - i_{C1}) R_C$

$= -\alpha (i_{E2} - i_{E1}) R_C$

$\simeq -0.196 I R_C$

Thus, for $V_o = 1V$; $0.196 I R_C = 1$

$I R_C = 5.102$

Now $I = 2mA$, thus

$R_C = \underline{2.5 K\Omega}$

DC (bias) voltage at each collector

$= V_{CC} - \frac{I}{2} R_C = 10 - 1 \times 2.5$

$= 7.5V$

For a $-1V$ output swing, the minimum voltage at each collector is:

$7.5 - 0.5 = 7.0V$

Thus, $V_{ICM}|_{max} = \underline{7V}$

7.35

$i_{E1} = \frac{I}{1+e^{-V_{id}/V_T}}$ and $i_{E2} = \frac{I}{1+e^{V_{id}/V_T}}$

with $V_{id} = v_{B1} - v_{B2} = 5$ mV, and $\alpha = 1$,

$i_{C1} \simeq i_{E1} = \frac{I}{1+e^{-5mV/25mV}} = 0.55 I$

$i_{C2} \simeq i_{E2} = \frac{I}{1+e^{5mV/25mV}} = 0.45 I$

$V_{C2} - V_{C1} = (V_{CC} - i_{C2} R_C) - (V_{CC} - i_{C1} R_C)$

$= -0.45 IR_C + 0.55 IR_C = 0.1 IR_C$

$A_v = \frac{v_O}{V_{id}} = \frac{(0.1) IR_C}{0.005 \text{ V}} = (20 IR_C)\text{V/V}$

(b) Each collector is biased at $V_{CC} - \frac{I}{2} R_C$

If we want to maintain the same differential input, each collector should be allowed to fall by $\frac{0.1 IR_C}{2}$ below its bias value.

so,

$V_{C(min)} = V_{CC} - 0.5 IR_C - 0.05 IR_C$

$= V_{CC} - 0.55 IR_C$

If this is permitted until $v_{CB} = 0$,

$V_{ICM(max)} = V_{C(min)} = V_{CC} - 0.55 IR_C$

If the gain is $20 IR_C$

$IR_C = \frac{A_v}{20}$ so that

$V_{ICM(max)} = V_{CC} - \frac{0.55 A_v}{20} = V_{CC} - 0.0275 A_v$

so, for a given V_{CC}, A_v reduces the maximum allowed V_{ICM}.

A_v (V/V)	100	200	300	400
$V_{ICM(max)}$ (V)	$V_{CC} -$ 2.75	$V_{CC} -$ 5.5	$V_{CC} -$ 8.25	$V_{CC} -$ 11
IR_C (V)	5	10	15	20
R_C (kΩ)	5	10	15	20

For example, if $V_{CC} = 10$ V, a gain of 200 can be achived by increasing R_C to 10 k Ω, the maximum common-mode input voltage would be $V_{CC} - 5.5 = 4.5$ V. If a gain of 300 is required, if can be achieved by changing RC to 15 kΩ. Howerver this means that $V_{ICM(max)} = V_{CC} - 8.25 = 1.75$ V.

7.36

$I = 6\,mA$

The current will divide between the two transistors in proportion to their emitter areas. Thus with no input,

$I_{E1} = 1.5 I_{E2}$

$I_{E1} + I_{E2} = 2.5\, I_{E2} = 6\,mA$

$I_{E2} = 2.4\,mA$

$I_{E1} = 3.6\,mA$

For $\alpha \simeq 1$

$I_{C1} = \underline{\underline{3.6\,mA}}$

$I_{C2} = \underline{\underline{2.4\,mA}}$

To equalize the collector currents we apply a difference signal $v_d = v_{B2} - v_{B1}$ whose value can be determined as follows:

$i_{E1} = I_{SE1}\, e^{((v_{B1} - v_E)/V_T)}$

$i_{E2} = I_{SE2}\, e^{((v_{B2} - v_E)/V_T)}$

where $I_{SE1}/I_{SE2} = 1.5$

Now, $i_{E1} = i_{E2}$ when

$1 = 1.5\, e^{(v_{B1} - v_{B2})/V_T}$

$v_d = v_{B2} - v_{B1} = V_T \ln 1.5 = \underline{\underline{10.1\,mV}}$

7.37

(a)

$$V_{BE} = 690\text{ mV} + 25\text{ mV}\ln\left(\frac{0.2/2}{1}\right) = 632\text{ mV}$$

$R_e = 0, V_{id} = 0$

(b) Eqn 8.73

$$i_{C1} = \alpha\, i_{E1} \approx i_{E1} = \frac{200\ \mu A}{1 + e^{-20/25}} = 138\ \mu A$$

$$i_{C2} = \alpha\, i_{E2} \approx i_{E2} = \frac{200\ \mu A}{1 + e^{20/25}} = 62\ \mu A$$

$R_e = 0, V_{id} = 20\text{ mV}$

(c)

$$V_{BE1} = 690\text{ mV} + 25\text{ mV}\ln\left(\frac{.138}{1}\right) = 640\text{ mV}$$

$$V_{BE2} = 690\text{ mV} + 25\text{ mV}\ln\left(\frac{.062}{1}\right) = 620\text{ mV}$$

$$V_{BE1} - V_{BE2} = 20\text{ mV}$$

$$200\text{ mV} = V_{id} = V_{B1} - V_{B2}$$

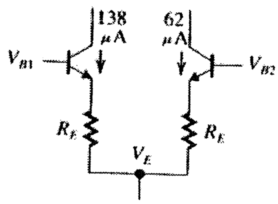
$$= (V_{BE1} + 138\ \mu A\ R_E + V_E)$$

$$-(V_{BE2} + 62\ \mu A\ R_E + V_E)$$

$$200\text{mV} = V_{B1} - V_{B2} + (138\ \mu A - 62\ \mu A)R_E$$

$$180\text{ mA} = 76\ \mu A \cdot R_E$$

$$R_E = 2.37\text{ k}\Omega$$



(d) Without R_E, a V_{id} of 20 mV causes a differential current of 76 μA

$$G_m = \frac{76\ \mu A}{20\text{ mV}} = 3.8\text{ mA/V} = (263\ \Omega)^{-1}$$

with $R_E = 2.37\text{ k}\Omega$, a v_{id} of 200 mV causes a differential current of 76 μA

$$G_m = \frac{76\ \mu A}{200\text{ mV}} = 0.38\text{ mA/V} = (2.63\text{ k}\Omega)^{-1}$$

So G_m has been reduced by a factor of 10. This is the same factor by which V_{id} increased. So we have traded differential gain for a wider usable input range.

7.38

Each device is operating at a current of $150\,\mu A = 0.15\,mA$. Thus,

$$g_m = \frac{0.15\text{ mA}}{25\text{ mV}} = \underline{\underline{6\ \frac{mA}{V}}}$$

$$R_{id} = 2(\beta+1)r_e = 2r_\pi$$

$$= 2 \times \frac{150}{4} = \underline{\underline{75\text{ k}\Omega}}$$

7.39

$R_{id} \geq 10K\Omega$; $A_d = 200\ V/V$;
$\beta \geq 100$; $V_{CC} = 10V$

$R_{id} = 10^4 = 2r_\pi = 2 \times \frac{100}{g_m}$

$\Rightarrow g_m = 20\ mA/V$

Thus each device is operating at 0.5mA and $I = \underline{1mA}$

Voltage gain $= g_m . R_C$

$200 = 20 R_C$

$\Rightarrow R_C = \underline{10K\Omega}$

7.40

$5mV$

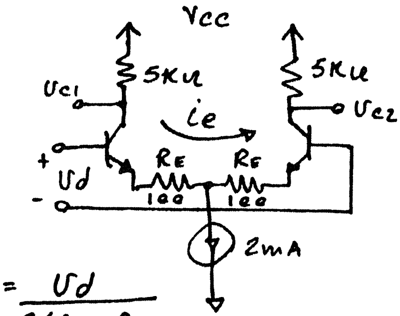
$r_e = \frac{V_T}{I/2} = \frac{25mV}{50\mu A} = \underline{500\Omega}$

Half-circuit gain $= \frac{\alpha R_C}{r_e} \simeq \frac{R_C}{r_e}$

$= \frac{10K}{500} = \underline{20}\ V/V$

At one collector we expect a signal of (+100mV) and at the other a signal of (-100mV)

7.41



$r_e = \frac{V_T}{I_E}$
$= \frac{25m}{1m}$
$= 25\Omega$

(a) $i_e = \frac{v_d}{2(r_e + R_E)}$

$= \frac{0.1\ V}{2(25 + 100)\Omega} = \underline{0.4mA}$

(b) $i_{E1} = 1 + 0.4 = \underline{1.4}\ mA$

$i_{E2} = 1 - 0.4 = \underline{0.6mA}$

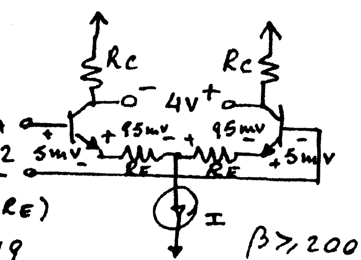
(c) $v_{C1} = -i_e R_C \simeq -0.4 \times 5 = \underline{-2V}$

$v_{C2} = \underline{+2V}$

(d) $v_{od} = 4V$

$A_d = v_{od} / v_{id} = \frac{4}{0.1} = \underline{40}\ V/V$

7.42



$\frac{2R_C}{2(r_e + R_E)} \simeq \frac{4}{0.2}$

$R_C = 20(r_e + R_E)$

$\frac{R_E}{r_e} = \frac{95}{5} = 19$

$\beta \geq 200$

$R_{in} \geq 80K$

$R_{in} = 2(\beta+1)(r_e+R_E)$
$= 2 \times 201 \times 20 r_e = 80K\Omega$
$\Rightarrow r_e \simeq \frac{80000}{8000} = 10\Omega$

Thus each device is operating at a current of $\frac{25mV}{10\Omega} = 2.5mA$

$\Rightarrow I = 5mA$

$R_E = 19 \times 10 = 190\Omega$
$R_C = 20 \times 200 = 4K\Omega$

7.43

(a) $V_{BC} \le 0.4$ V

$V_B - V_C \le 0.4$ V

$(V_{CM} + V_{id}/2) - (V_{CC} - i_{C1}R_C) \le 0.4$ V

So $V_{CM\,max} = V_{CC} + 0.4\text{V} - \frac{\hat{V}_{id}}{2} - i_{C1}R_C$

$= V_{CC} + 0.4\text{ V} - \frac{\hat{V}_{id}}{2} - \left(I_C + g_m\frac{\hat{V}_{id}}{2}\right)R_C$

$A_d = g_m R_C$ and $g_m = \frac{I_C}{V_T}$

$I_C = g_m V_T$

$V_{CM\,max} = V_{CC} + 0.4\text{V} - \frac{\hat{V}_{id}}{2}$

$-\left[(g_m V_T R_C) + \left(g_m \frac{\hat{V}_{id}}{2} R_C\right)\right]$

$= V_{CC} + 0.4\text{V} - \frac{\hat{V}_{id}}{2} - \left[A_d V_T + A_d \frac{\hat{V}_{id}}{2}\right]$

$= V_{CC} + 0.4\text{V} - \frac{\hat{V}_{id}}{2} - A_d\left[V_T + \frac{\hat{V}_{id}}{2}\right]$

(b)

$V_{CM\,max} = V_{CC} + 0.4\text{V} - \frac{\hat{V}_{id}}{2} - A_d\left(V_T + \frac{\hat{V}_{id}}{2}\right)$

$= 5\text{ V} + 0.4\text{ V} - \frac{10\text{ mV}}{2} - 100\left(25\text{ mV} + \frac{10\text{ mV}}{2}\right)$

$= 5\text{ V} + 0.4\text{ V} - 5\text{ mV} - 100\,(30\text{ mV})$

$= 2.395$ V

$\hat{V}_{od} = A_d \cdot \hat{V}_{id} = 100 \cdot 10\text{ mV} = 1\text{ V}$

$IR_C = 2I_C R_C$ Eqn 8.80 $g_m = \frac{I_C}{V_T}$

$I_C = g_m V_T$ $IR_C = 2(g_m V_T) R_C$ Eqn 8.93

$A_d = g_m R_C$

$IR_C = 2V_T A_d = (2)(25\text{ mV})(100) = 5\text{ V}$

$I = \frac{\text{quiescent power}}{V_{CC} - (-V_{EE})} = \frac{5\text{ mW}}{10\text{ V}} = 0.5\text{ mA}$

$R_C = \frac{5\text{ V}}{1} = 10\text{ k}\Omega$

(c) For $V_{CM\,max} = OV$

$= 5\text{ V} + 0.4\text{ V} - \frac{\hat{V}_{id}}{2} - A_d\left(25\text{ mV} + \frac{\hat{V}_{id}}{2}\right)$

$OV = 5.4\text{ V} - 5\text{ mV} - A_d\,(30\text{ mV})$

$A_d = \frac{5.395\text{ V}}{30\text{ mV}} = 180\text{V/V}$

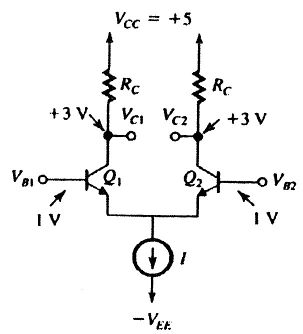
(for $\hat{V}_{id} = 10$ mV)

7.44

with $IR_C = 4$V, and assuming that $\alpha = 1$,

$V_{C1} = V_{C2} = V_{CC} - \frac{I}{2} \cdot R_C$

$= 5 - 2 = 3$ V



(a) $v_{B1} = 1 + 0.005\sin(\omega t)$

$v_{B2} = 1 - 0.005\sin(\omega t)$

we see

that since $\frac{V_{id}}{V_T} = \frac{10.0\text{ mV}}{25\text{ mV}} = 0.4$,

the output will be fairly linear. With the information given,

since $i_C = i_E$

$$i_{C1} \simeq \frac{I}{1 + e^{(-V_{id}/V_T)}} \text{ and } i_{C2} \simeq \frac{I}{1 + e^{(V_{id}/V_T)}}$$

$$v_{od} = v_{C2} - v_{C1}$$

$$= (V_{CC} - i_{C2}R_C) - (V_{CC} - i_{C1}R_C)$$

or

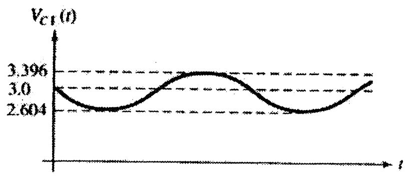
$$V_{od} = \frac{I\,R_C}{1 + e^{-V_{id}/V_T}} - \frac{I\,R_C}{1 + e^{V_{id}/V_T}}$$

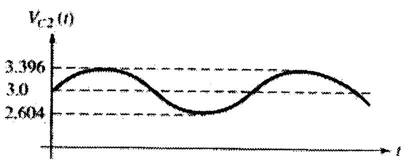
with $I_{RC} = 4$ V and $|V_{id}| = 10$ mV,

$$v_{od\,max} = 5\text{ V}\left(\frac{1}{1 + e^{-10/25}} - \frac{1}{1 + e^{10/25}}\right)$$

$= 989$ mV

so, $A_d = \frac{V_{od\,max}}{V_{id\,max}} = \frac{792\text{ mV}}{10\text{ mV}} = 79.2$





(b) $v_{B1} = 1 + 0.1\sin(\omega t)$

$v_{B2} = 1 - 0.1\sin(\omega t)$

Here, $\frac{V_{id}}{V_T} = \frac{200\text{ mV}}{25\text{ mV}} = 8$

see that this will clearly represent large-signal operation with significant distortion.

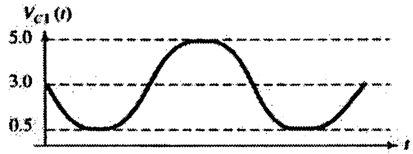
Using the same equation,

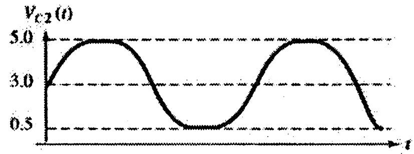
$$v_{od\,max} = 4\text{ V}\left(\frac{1}{1 + e^{-200/25}} - \frac{1}{1 + e^{200/25}}\right)$$

$\simeq 4.0$ V

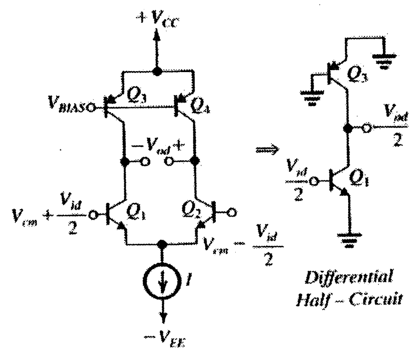
$$A_d = \frac{V_{od}}{V_{id}} = \frac{5\text{ V}}{0.2\text{ V}} = 25$$

waveform is distorted; upper excursions are limited to 5 V.





7.45



$|A_d| = \frac{V_{od}}{V_{id}} = g_{m1}(r_{O1} \parallel r_{O3})$ Assuming that

$I_C = I_E = \frac{I}{2}$,

$r_{O1} = r_{O3} = \frac{|V_A|}{I_C} = \frac{10\text{ V}}{I_C}$ and

$g_m = \frac{|I_C|}{V_T} = \frac{I_C}{25\text{ mV}}$

$$A_d = \frac{I_C}{25\text{ mV}}\left(\frac{1}{2}\right)\left(\frac{10\text{ V}}{I_C}\right) = \frac{5\text{ V}}{25\text{ mV}} = 200$$

7.46

$$-\frac{v_{od}}{2} = i_b \beta R_C$$

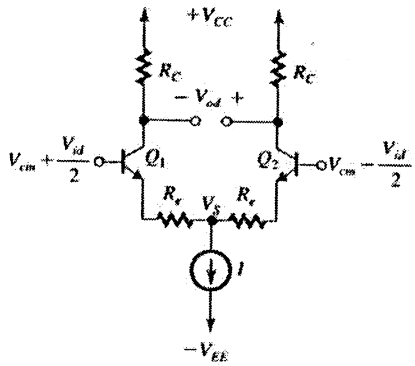
$$i_b = \frac{\frac{v_{id}}{2}}{r\pi + (\beta + 1)R_e}$$

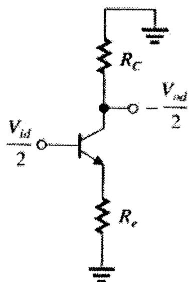
So,

$$-\frac{v_{od}}{2} = \frac{\frac{v_{id}}{2}\beta R_C}{r\pi + (\beta + 1)R_e}$$

$$\frac{V_{od}}{V_{id}} = \frac{-R_C}{\frac{r\pi}{\beta} + \frac{\beta + 1}{\beta}R_e}$$ If $\alpha \simeq 1$ and

(a)





Differential Half – Circuit

noting

$r_e = \frac{V_T}{I_E}$,

$|A_d| = \left|\frac{V_{od}}{V_{id}}\right| \simeq \frac{R_C}{r_e + R_e}$ which is identical to

The half circuit has

$R_i = r\pi + (\beta + 1)R_e$

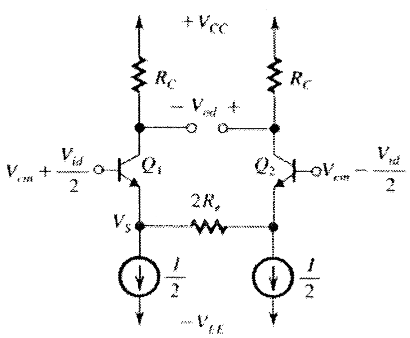
$R_{id} = 2r\pi + (\beta + 1)(2R_e)$

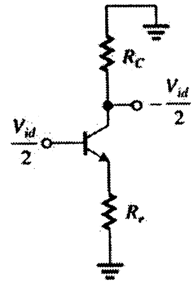
This is equivalent to

$R_{id} = (\beta + 1)(2r_e + 2R_e)$

$V_{Cm} = V_{BE} + \frac{I}{2}R_e + V_S$

(b)





Differential Half – Circuit is the same as the circuit of Part (a)

So, using the same derivation,

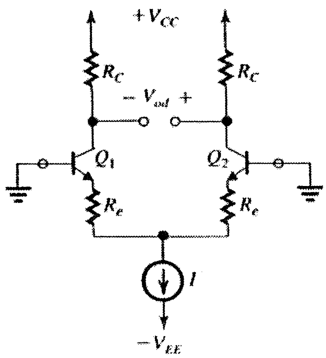
$|A_d| = \left|\frac{V_{od}}{V_{id}}\right| \simeq \frac{R_C}{r_e + R_e}$

$R_{id} = 2r\pi + 2(\beta + 1)R_e = (\beta + 1)(2r_e + 2R_e)$

$V_{Cm} = V_{BE} + V_S$

Since the quiescent emitter currents do not pass through the $2R_e$ resistance, there is no drop so that V_{Cm} can be lower in case (b) than case (a)

7.47



$V_{Re} = 4V_T \qquad V_{RC} = 40V_T$

From Eq. (8.94).

$$A_d \simeq \frac{R_C}{r_e + R_e} = \frac{\frac{40V_T}{I_C}}{\frac{V_T}{I_E} + \frac{4V_T}{I_E}}$$

If $\alpha \simeq 1$, $I_C \simeq I_E$, and

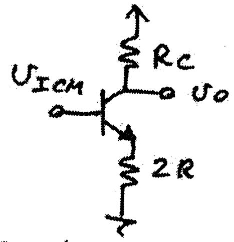
$$A_d = \frac{40V_T}{5V_T} = 8$$

7.48

$2 M\Omega$

$$\frac{v_o}{v_{Icm}} \simeq \frac{R_C}{2R} = \frac{20 K\Omega}{2 M\Omega} = 0.01 \text{ V/V}$$

or -40 dB



7.49

$$\frac{v_o}{v_i} = \frac{\alpha \times 20 K\Omega}{(2r_e + 2 \times 200)\,\Omega}$$

Where $r_e = \frac{V_T}{0.5/2} = \frac{0.05 V}{0.5 mA} = 100 \Omega$

$$\frac{v_o}{v_i} \simeq \frac{20\,000}{600} = 33.3 \text{ V/V}$$

$$R_i = (\beta+1)(2r_e + 2 \times 200) = 101 \times 2 \times 300 \simeq 60 K\Omega$$

7.50

Each transistor is operating at $I_E = 1 mA$, thus

$r_e = 25 \Omega$ and $r_\pi = 101 \times 25 = 2525 \Omega$

$$\frac{v_o}{v_i} = \frac{\alpha \times 7.5 K\Omega}{(2r_e + 200)\,\Omega} \simeq \frac{7500}{250} = 30 \frac{V}{V}$$

$$R_i = (\beta+1)(r_e + 200 + r_e) \simeq 25 K\Omega$$

7.51

with $V_{cm} = 0$, $V_E = -0.7$ V

(a) $I = \frac{V_E - V_{EE}}{R_E} = \frac{-0.7-(-5)}{4.3 K} = 1$ mA

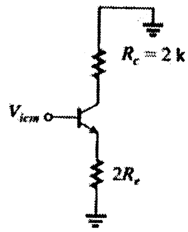
$r_e = \frac{V_T}{I_E} = \frac{25 \text{ mV}}{1 \text{ mA}/2} = 50 \Omega$

Since $R_e >> r_o$,

$\left|\frac{V_o}{V_{id}}\right| \simeq \frac{1}{2} \cdot \frac{\alpha R_C}{r_e}$ If $\alpha \simeq 1$,

$\left|\frac{V_o}{V_{id}}\right| \simeq \frac{R_C}{2r_e} = \frac{2 K}{2(50)} = 20$ V/V

(b) $A_{cm} \simeq \frac{\alpha R_C}{2R_e + r_e}$



common-mode half circuit

If $\alpha = 1$,

$A_{cm} \simeq \frac{2 K}{2(4.3 K) + 50} = 0.23$

(c) CMRR (dB) = 20 $\log_{10}$

$\left|\frac{V_o/V_{id}}{A_{cm}}\right| = 20 \log_{10}\left|\frac{20}{0.23}\right| = 38.8$ dB

(d)

$V_{B1} = 0.1 \sin 2\pi \times 60t + 0.005 \sin 2\pi \times 1000\,t$

$V_{B2} = 0.1 \sin 2\pi \times 60t - 0.005 \sin 2\pi \times 1000t$

$V_{id} = 0.01 \sin 2\pi \times 1000\,t$

$V_{icm} = 0.1 \sin 2\pi \times 60\,t$

so that

$V_o = \left|\frac{V_o}{V_{id}}\right| \cdot V_{id} + A_{cm} \cdot V_{icm}$

$V_o(t) = 20\,[0.01 \sin 2\pi \times 1000\,t] + 0.23$

$[0.1 \sin 2\pi \times 60\,t]$

$V_o(t) = 0.2 \sin 2\pi \times 1000\,t + 0.023 \sin 2\pi \times 60\,t$

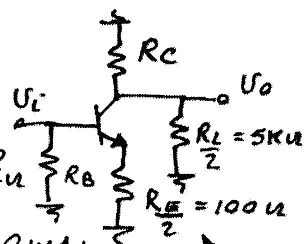
7.52

Each transistor is biased at 1mA. Thus,

$r_e = 25 \Omega$, $g_m = 40$ mA/V

$r_o = 100/1 = 100 K\Omega$

The differential half-circuit is



$$A_d = \frac{v_o}{v_i} = \frac{\alpha\,[R_C \,//\, (R_L/2)]}{r_e + R_E/2}$$

$$\simeq \frac{10//5}{0.025 + 0.100} = \underline{\underline{26.7}}\ \text{V/V}$$

$$R_{id} = 2[R_B // (\beta+1)(r_e + R_E/2)]$$

$$= 2[30 // 101\,(0.025 + 0.100)]$$

$$= \underline{\underline{17.8}}\ \text{k}\Omega$$

The common-mode half circuit

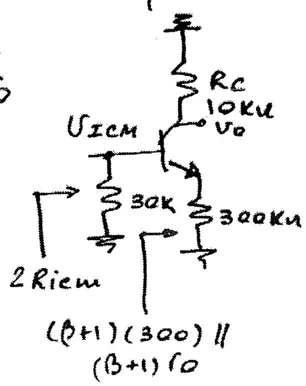
$$A_{cm} = \frac{v_o}{v_{Icm}} \simeq \frac{10}{300}$$

$$= \frac{1}{30} = \underline{\underline{0.033}}\ \text{V/V}$$

$$2R_{icm} = 30\text{K} // 7.5\text{M}\Omega$$

$$= 30\text{k}\Omega$$

$$R_{icm} = \underline{\underline{15\text{k}\Omega}}$$



$\simeq 30 // 10 = 7.5\text{M}\Omega$

Without the R_B resistors $R_{icm} = \underline{\underline{3.75\text{ M}\Omega}}$

7.53

(a) $A_d\big|_{\text{single-ended output}} = \dfrac{\alpha\,(R_C // r_o)}{2r_e}$

where $r_e = \dfrac{0.025\text{V}}{0.25\text{mA}} = 100\Omega$

$r_o = \dfrac{200\text{V}}{0.25\text{mA}} = 800\text{k}\Omega$

$A_d\big|_{\text{single ended}} \simeq \dfrac{20}{2 \times 0.1} = \underline{\underline{100}}\ \text{V/V}$

(b) $A_d\big|_{\text{diff output}} = 2 \times A_d\big|_{\text{single ended}}$

$= \underline{\underline{200}}\ \text{V/V}$

(c) $R_{id} = 2r_\pi = 2 \times 201 \times 100$

$= \underline{\underline{40.2\text{k}\Omega}}$

(d) $A_{cm}\big|_{\text{single-ended output}} = \dfrac{R_C}{2R}$

$= \dfrac{20}{2000} = \underline{\underline{0.1}}\ \text{V/V}$

(e) $A_{cm}\big|_{\text{diff out}} = 0$

7.54

$I = 100\ \mu\text{A}$, $\beta = 50$, $V_A = 20$ V

For Q_3,

$$R_{EE} = r_{o3} = \frac{V_A}{I} = \frac{20\text{ V}}{0.1\text{ mA}} = 200\text{ k}\Omega$$

$$r_o = r_{o1} = r_{o2} = \frac{V_A}{I/2} = \frac{20\text{ V}}{0.05\text{ mA}} = 400\text{ k}\Omega$$

Using Eq. (8.103),

$$R_{icm} \simeq \beta R_{EE}\frac{1 + \dfrac{R_C}{\beta r_o}}{1 + \dfrac{R_C + 2R_{EE}}{r_o}}$$

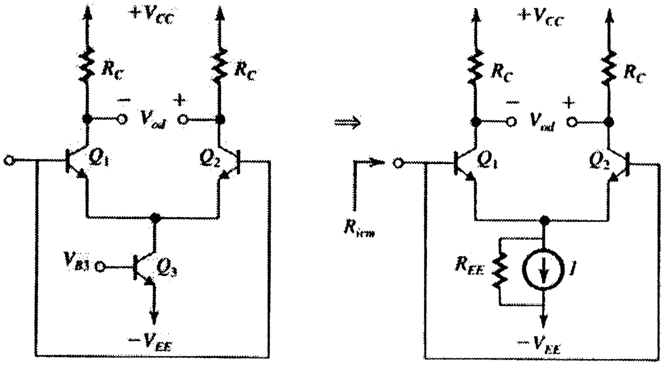
$$R_{icm} \simeq 50(200\text{ k})\cdot\frac{1 + \dfrac{R_C}{(50)(400\text{ k})}}{1 + \dfrac{R_C + 2(200\text{ k})}{400\text{ k}}}$$

If $R_C << R_{EE}$

and $R_C << r_o$,

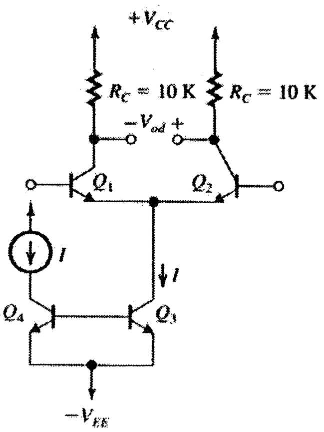
$R_{icm} \simeq 50\ (200\text{ K})\ (.5) = 5\text{ M}\Omega$

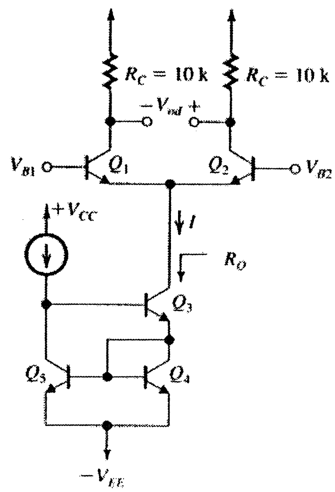
This figure is for 7.54



(c)

7.55





Equivalent

$R_{EE} = r_{o3} = \frac{V_A}{I} = \frac{10\text{ V}}{0.5\text{ mA}} = 20\text{ k}\Omega$

$r_{e2} = r_{e1} = r_e = \frac{V_T}{I/2} = \frac{25\text{ mV}}{0.5\text{ mA}/2} = 100\ \Omega$

Since $\alpha = \frac{\beta}{\beta+1} = \frac{100}{101} = 1$,

$A_d \simeq \frac{R_C}{r_e} = \frac{10\text{ k}}{0.1\text{ k}} = 100\text{ V/V}$

(b) $A_{cm} \equiv \frac{\alpha \Delta R_C}{2R_{EE} + r_e} = \frac{(0.02)(10\text{ k})}{2(20\text{ k}) + 0.1\text{ k}}$

$= 0.00499\text{ V/V}$

$\text{CMRR(dB)} = 20\log_{10}\left|\frac{A_d}{A_{cm}}\right| = 20\log_{10}\left|\frac{100}{0.00499}\right|$

$= 86\text{ dB}$

From Eq. (7.88)

$R_o \simeq \frac{1}{2}\beta_3 r_{o3}$

$R_o \simeq \frac{1}{2}(100)(20\text{ k}) = 1\text{ M}\Omega$

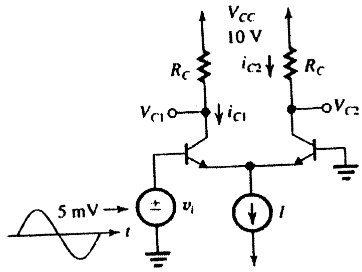
$A_{cm} \simeq \frac{\Delta R_C}{2R_o + r_e} \simeq$

$\frac{(0.02)(10\text{ k})}{2(1\text{ m}) + 0.1\text{ k}} = 0.0001\text{ V/V}$

$\text{CMRR(dB)} = 20\log_{10}\left|\frac{A_d}{A_{cm}}\right| = 20\log_{10}\left|\frac{100}{0.0001}\right|$

$= 120\text{ dB}$

7.56



$$i_{c1} = \frac{I}{2} + \left(\frac{I/2}{V_T}\right)\left(\frac{5}{2}\right)\sin\omega t$$

$$i_{c2} = \frac{I}{2} - \left(\frac{I/2}{V_T}\right)\left(\frac{5}{2}\right)\sin\omega t$$

$$v_{C1} = V_{CC} - \frac{I}{2}R_C - \frac{I/2}{V_T}R_C\frac{5}{2}\sin\omega t$$

$$v_{C2} = V_{CC} - \frac{I}{2}R_C - \frac{I/2}{V_T}R_C\frac{5}{2}\sin\omega t$$

$V_{C1}, V_{C2} \geq 0$

$\Rightarrow 10 - 5I - 0.5I = 0$

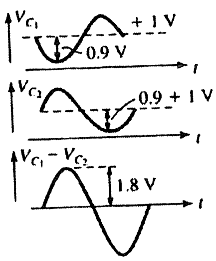
$I = 1.8$ mA

$V_{C1} = V_{C2} = 1$ V

$A_d = \dfrac{20\text{ k}\Omega}{2r_e}$, where $r_e = \dfrac{25}{0.9} = 27.8\ \Omega$

Thus, $A_d = 360$ V/V

$v_{C2} - v_{C1} = 1.8 \sin \omega t$, V



7.57

Taken single-endedly $Acm_s = \dfrac{\alpha R_C}{2R_o}$

Let collector resistors be R_c & $R_c + \Delta R_c$, then

$$Acm = \frac{\alpha}{2R_o}(R_C + \Delta R_C - R_C)$$

$$= \alpha\frac{\Delta R_C}{2R_o}$$

Which can be written as

$$Acm_d = \frac{\alpha R_C}{2R_o}\cdot\frac{\Delta R_C}{R_C} = Acm_s\frac{\Delta R_C}{R_C}$$

$$\text{CMRR} = \frac{A_d}{A_{cmd}} = \frac{2.A_S}{Acm_s\frac{\Delta R_C}{R_C}}$$

$$= \frac{A_s}{Acm_s}\cdot\frac{2}{\frac{\Delta R_C}{R_C}}$$

Thus, $20\log\dfrac{2}{\frac{\Delta R_C}{R_C}} = 40$ dB

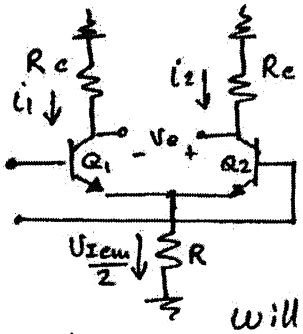
$\rightarrow \Delta R_C/R_C = 2\%$

7.58

The bias current will split between the two transistors according to their area ratio. Thus the large-area device will carry twice the current of the other device. That is, the bias currents will be $2I/3$ and $I/3$.

Now with v_{Icm} applied, the CM signal current will $\hookrightarrow v_{Icm}/R$

split between Q_1 and Q_2 in the same ratio. This is because their r_e values will be related in the same way. Thus, if Q_1 is the large device r_{e1} will be half the value of r_{e2}.



The result will be that

$i_1 = \frac{2}{3}\frac{v_{Icm}}{R}$ and $i_2 = \frac{1}{3}\frac{v_{Icm}}{R}$

Thus the differential output voltage v_o will be

$v_o = 0 - i_2 R_c - (-i_1 R_c) = (i_1 - i_2) R_c$

$= \frac{1}{3}\frac{v_{Icm}}{R} \cdot R_c$

$A_{cm} = \frac{1}{3}\frac{R_c}{R} = \frac{1}{3} \times \frac{12}{1000} = 0.004 \ \frac{V}{V}$

7.59

For $I = 200\,\mu A$:

$g_m = \sqrt{2 k_n' W/L \, I_D} = \sqrt{2 \times 4 \times 0.1}$
$= 0.89$ mA/V

$R_D = 10\,K\Omega$

Thus, $A_d = g_m R_D = 10 \times 0.89 = 8.9 \ V/V$

$V_{os} = \frac{(V_{GS} - V_t)}{2} \cdot \frac{\Delta R_D}{R_D}$

where $\frac{\Delta R_D}{R_D} = 0.02$ (worst case)

and $V_{GS} - V_t = \sqrt{\frac{2 I_D}{k_n' W/L}} = \sqrt{\frac{2 \times 0.1}{4}}$
$= 0.223$ V

Thus, $V_{os} = \frac{1}{2} \times 0.223 \times 0.02$

$= 2.23$ mV

For $I = 400\,\mu A$:

$g_m = \sqrt{2 \times 4 \times 0.2} = 1.265$ mA/V

$A_d = 12.65$ V/V

$V_{ov} = V_{GS} - V_t = 0.316$ V

$V_{os} = \frac{1}{2} \times 0.316 \times 0.02 = 3.16$ mV

Thus both A_d and V_{os} increase by the same ratio since both are proportional to $\sqrt{I}$

7.60

Worst cases: $\Delta V_t = 10$ mV

$\frac{\Delta R_D}{R_D} = 0.04$; $\frac{\Delta (W/L)}{(W/L)} = 0.04$

V_{os1} (due to ΔR_D) $= \frac{V_{ov}}{2}\frac{\Delta R_D}{R_D} = \frac{0.3 \times 0.04}{2}$

$= 6$ mV

V_{OS2} (due to $\Delta W/L$) $= \frac{V_{OV}}{2}\frac{\Delta W/L}{W/L} = \frac{0.3 \times 0.04}{2}$

$= 6mV$

V_{OS3} (due to ΔV_t) $= \Delta V_t = \underline{10mV}$

Since these offsets are not correlated

$$V_{OS} = \sqrt{V_{OS1}^2 + V_{OS2}^2 + V_{OS3}^2}$$

$$V_{OS} = \sqrt{6^2 + 6^2 + 10^2} = \underline{13.11mV}$$

The major contribution is due to the threshold mismatch ΔV_t.

To find the required mismatch ΔR_D that can correct for V_{OS}

$$13.11mV = \frac{V_{OV}}{2} \cdot \frac{\Delta R_D}{R_D}$$

$$\Rightarrow \frac{\Delta R_D}{R_D} = \frac{2 \times 13.11mV}{0.3V}$$

$= 0.087$ or $\underline{8.7\%}$

If ΔV_t is reduced by a factor of 10 to 1mV, V_{OS} reduces to:

$$\sqrt{6^2 + 6^2 + 1^2} = 8.54mV$$

and $\frac{\Delta R_D}{R_D} = \frac{2 \times 8.54mV}{0.3V} = \underline{5.69\%}$

7.61

$$V_{OV} = \sqrt{\frac{I}{k_n' W/L}} = \sqrt{\frac{100}{100 \times 20}} = 0.223V$$

we obtain V_{OS} due to $\Delta R_D/R_D$ as:

$$V_{OS} = \frac{V_{OV}}{2}\frac{\Delta R_D}{R_D} = \frac{0.223 \times 0.05}{2}$$

$= \underline{5.57mV}$

From Eqn. (7.117), V_{OS} due to $\Delta(W/L)/(W/L)$ is:

$$V_{OS} = \left(\frac{V_{OV}}{2}\right)\frac{\Delta W/L}{W/L} = \frac{0.223 \times 0.05}{2}$$

$= \underline{5.57mV}$

The offset arising from ΔV_t is

$$V_{OS} = \Delta V_t = \underline{5mV}$$

Worst case offset is:

$5.57 + 5.57 + 5 = 16.15mV$

Applying the root-sum-of-squares

$$V_{OS} = \sqrt{2(5.57m)^2 + 5m^2} = \underline{9.33mV}$$

7.62

$$\Delta V_C = \Delta R_C \cdot \frac{I}{2}$$

$$A_d = \frac{R_C}{r_e} = \frac{R_C}{V_T / \frac{I}{2}} = \frac{I R_C}{2 V_T}$$

$$\Rightarrow V_{OS} = \frac{\Delta V_C}{A_d} = \frac{\Delta R_C}{R_C} \cdot V_T$$

$= 0.1 \times 25 = \underline{2.5mV}$

7.63 $V_{OS} = V_T \cdot \dfrac{\Delta I_S}{I_S}$

$= 25 \times 0.1 = \underline{2.5\,\text{mV}}$

7.64

$\Delta v_{EE} = \Delta R_C \dfrac{I}{2}$

$$A_d = \frac{R_C}{r_e + R_e} = \frac{R_C}{\dfrac{2V_T}{I} + R_E} = \frac{I R_C}{2V_T + I R_E}$$

$$V_{OS} = \frac{\Delta v_C}{A_d} = \frac{\Delta R_C}{R_C}\left(V_T + \frac{I R_E}{2}\right)$$

7.65

CASE 1: BJT Diff. Amp.

From Eq. (8.121)

$$|V_{OS}| = V_T\left(\frac{\Delta R_C}{R_C}\right) = 25\text{ mV }(0.04) = 1\text{ mV}$$

CASE 2: MOSFET Diff. Amp.

$$V_{OS} = \left(\frac{V_{OV}}{2}\right)\left(\frac{\Delta R_D}{R_D}\right) = \frac{300\text{ mV}}{2}(0.04) = 6\text{ mV}$$

If the MOSFET widths, are increased by a factor of 4, and since I_D must remain constant, we see that since

$$I_D = \frac{1}{2}K_n'\left(\frac{W}{L}\right)V_{OV}^2,$$

The new $V_{OV} = \sqrt{\dfrac{2I_D}{(4)K_n'\left(\dfrac{W}{L}\right)}}$ which is $\sqrt{\dfrac{1}{4}}$ or $\dfrac{1}{2}$

of its orginal value.

So, the new offset voltage is

$$V_{OS} = \left(\frac{150\text{ mV}}{2}\right)(0.04) = 3\text{ mV}$$

7.66

Since the two transistors are matched except for their V_A value, we can express the collector currents when the input terminals are grounded as,

$$I_{C1} = I_C\left(1 + \frac{V_{CE}}{V_{A1}}\right)$$

$$I_{C2} = I_C\left(1 + \frac{V_{CE}}{V_{A2}}\right)$$

Where I_C can be determined from

$I_{C1} + I_{C2} = I$

$$\Rightarrow I_C = \frac{I}{2 + \dfrac{V_{CE}}{V_{A1}} + \dfrac{V_{CE}}{V_{A2}}}$$

Note that for $V_{CE} \ll V_{A1}, V_{A2}, I_C \approx \frac{I}{2}$ Thus, the differential gain A_d can still be written as

$$A_d \approx \frac{R_C}{r_e} = \frac{I R_C}{2V_T}$$

The offset voltage at the output can be found from

$\Delta V_C = v_{C2} - V_{C1} = (I_{C1} - I_{C2})R_e$

$$= I_C R_C\left(\frac{V_{CE}}{V_{A1}} - \frac{V_{CE}}{V_{A2}}\right)$$

$$\approx \frac{I}{2}R_C\left(\frac{V_{CE}}{V_{A1}} - \frac{V_{CE}}{V_{A2}}\right)$$

Thus, $V_{OS} = \dfrac{\Delta V_C}{A_d}$

$$V_{OS} = V_T\left(\frac{V_{CE}}{V_{A1}} - \frac{V_{CE}}{V_{A2}}\right)$$

For

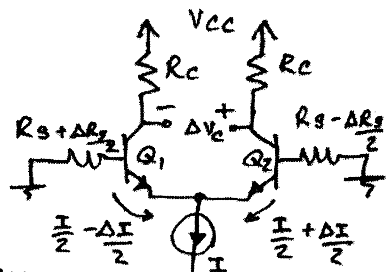
$V_{CE} = 10\text{ V}, V_{A1} = 100\text{ V}$ and

$V_{A2} = 300\text{ V}$

$$V_{OS} = 25\left(\frac{10}{100} - \frac{10}{300}\right)$$

$= 1.7\text{ mV}$

7.67



Equating the incremental changes in voltage from ground to emitter on both sides of the pair (and neglecting second-order terms, i.e. Δ×Δ terms):

$$\frac{I}{2(\beta+1)} \cdot \frac{\Delta R_s}{2} - \frac{\Delta I}{2(\beta+1)} \cdot R_s - \frac{\Delta I}{2} \cdot r_e$$

$$\simeq \frac{-I}{2(\beta+1)} \frac{\Delta R_s}{2} + \frac{\Delta I}{2(\beta+1)} R_s + \frac{\Delta I}{2} r_e$$

$$\Delta I \left[r_e + \frac{R_s}{\beta+1} \right] = \frac{I}{2(\beta+1)} \cdot \Delta R_s$$

$$\Delta I = \frac{I \, \Delta R_s}{2(\beta+1)} \cdot \frac{1}{r_e + \frac{R_s}{\beta+1}}$$

$$\Delta V_c = -\Delta I . R_c$$

$$= -\frac{I R_c \Delta R_s}{2(\beta+1)} \cdot \frac{1}{r_e + R_s/(\beta+1)}$$

$A_d = R_c / r_e$

Thus, $V_{os} = \Delta V_c / A_d$

$$= -\frac{I \Delta R_s}{2(\beta+1)} \cdot \frac{r_e}{r_e + \frac{R_s}{\beta+1}}$$

For $\frac{R_s}{\beta+1} << r_e$ and $\beta >> 1$,

$$|V_{os}| \simeq \frac{I}{2\beta}(\Delta R_s) \quad Q.E.D.$$

7.68

(a) $R_{C1} = 5 \times 1.05 = 5.25 K\Omega$

$R_{C2} = 5 \times 0.95 = 4.75 K\Omega$

Perfect offset nulling will be achieved when x is such that

$R_{C1} + (x \times 1K\Omega) = R_{C2} + (1-x) \times 1K\Omega$

$\Longrightarrow 5.25 + x = 4.75 + 1 - x$

$\Longrightarrow x = \underline{\underline{0.25}}$

(b) $I_{C1} = 1.05 mA$

$I_{C2} = 0.95 mA$

Offset nulling is achieved when x is such that

$1.05\,(x+5) = 0.95((1-x)+5)$

$x = \underline{\underline{0.225}}$

7.69

$$I_{Bmax} = \frac{I/2}{\beta_{min}+1} = \frac{300}{80+1} = \underline{\underline{3.7 \mu A}}$$

$$I_{Bmin} = \frac{I/2}{\beta_{max}+1} = \frac{300}{200+1} = \underline{\underline{1.5 \mu A}}$$

$I_{os} = I_{Bmax} - I_{Bmin} = \underline{\underline{2.2 \mu A}}$

7.70

$I_{E1} = \frac{2}{3} I$ and $I_{E2} = \frac{1}{3} I$

(Q_1 twice the area of Q_2)

$$\Delta V_c = V_{C2} - V_{C1} \simeq \underline{\underline{\frac{1}{3} I R_c}}$$

Nominally,

$$A_d = \frac{R_c}{r_e} = \frac{I R_c}{2V_T}$$

$$V_{os} = \frac{\Delta V_c}{A_d} = \frac{2}{3} V_T = \underline{16.7 mV}$$

Thus, small-signal analysis predicts that a 16.7mV DC voltage applied as $v_{B2} - v_{B1} = 16.7v$ would restore the current balance in the pair and reduce ΔV_C to zero.

Using large-signal analysis:

$$i_{E1} = I_{S1} \cdot e^{\frac{v_{B1} - v_E}{V_T}}$$

$$i_{E2} = I_{S2} \cdot e^{\frac{v_{B2} - v_E}{V_T}}$$

Thus,

$$\frac{i_{E1}}{i_{E2}} = \frac{I_{S1}}{I_{S2}} \cdot e^{\frac{v_{B1} - v_{B2}}{V_T}}$$

To restore balance, $i_{E1} = i_{E2}$, thus

$$1 = 2e^{\frac{V_{B1} - V_{B2}}{V_T}}$$

$$\Rightarrow V_{B1} - V_{B2} = -V_T \ln 2$$

$$V_{B2} - V_{B1} = \underline{17.3 mV}$$

Nominally

$$I_B = \frac{I/2}{\beta + 1} \simeq \frac{100}{2 \times 100} = \underline{0.5 \mu A}$$

But with the imbalance,

$$I_{B1} \simeq \frac{2I/3}{\beta} = \frac{2 \times 100}{300} = 0.67 \mu A$$

$$I_{B2} = \frac{I/3}{\beta} = \frac{100}{300} = 0.33 \mu A$$

$$I_B = \frac{I_{B1} + I_{B2}}{2} = \underline{0.5 \mu A}$$

$$I_{OS} = |I_{B1} - I_{B2}| = \underline{0.34 \mu A}$$

7.71

$R_C = 20K\Omega$; $A_d = 90 V/V$

$V_{OS} = \pm 3mV$

Worst case $|V_{OS}|$ is $3mV$

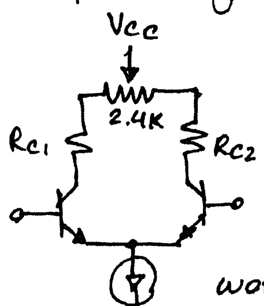
$|V_{OS}| = V_T \left(\frac{\Delta R_C}{R_C} \right)$ $|V_{OS}| = 3mV$

$$\Rightarrow \frac{3m \times 20K}{25m} = 2.4K, \Delta R_C = 2.4K\Omega$$

This is the maximum mismatch that occurs in R_C.

Thus, if the lowest collector resistor is adjusted from $R_{C\,min} + \Delta R$ with ΔR varying between zero and $2.4K\Omega$, then the offset would be eliminated!

This can be achieved with the following circuit:



When R_{C1} & R_{C2} are equal the potentiometer is tuned to the middle point. In the worst case, when either R_C is higher by $2.4K\Omega$, the potentiometer is adjusted to one extreme such as to increase the lowest R_C by $2.4K\Omega$. In all other cases when ΔR_C is distributed between R_{C1} and R_{C2} the potentiometer is adjusted

7.72

For each transistor $I_D = I/2$.

$r_{o2} = r_{o4} = r_o$

$$\Rightarrow A_d = \frac{1}{2} g_m r_o$$

but $g_m = \frac{2I_D}{V_{ov}}$ and $r_o = \frac{V_A}{I_D}$

$$\Rightarrow A_d = \frac{1}{2}\left(\frac{2I_D}{V_{ov}}\right)\frac{V_A}{I_D} = \frac{V_A}{V_{ov}}$$

$$\rightarrow 80\ V/V = 20V / V_{ov}$$

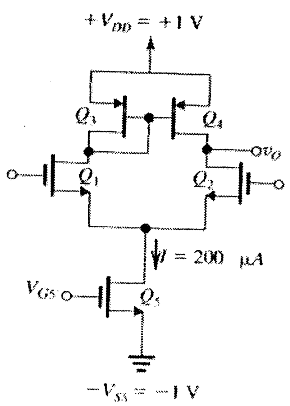
$$\rightarrow V_{ov} = 20/80 = 0.25V$$

Finally,

$$I = 2I_D = K\frac{W}{L} V_{ov}^2$$

$$= 3.2 \frac{mA}{V^2} (0.25V)^2 = \underline{\underline{0.2\,mA}}$$

7.73



(a) $I_{D5} = I = 200\ \mu A$

$$I_{D1} = I_{D2} = I_{D3} = I_{D4} = \frac{I}{2}$$

$$= \frac{200\ \mu A}{2} = 100\ \mu A$$

$$I_{D1} = I_{D2} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)V_{OV}^2$$

so,

$$\left(\frac{W}{L}\right)_{1-2} = \frac{2I_D}{\mu_n C_{ox} V_{OV}^2} = \frac{2(100\ \mu A)}{400\ \mu A / V\ (0.2\ V)^2}$$

$$= 12.5$$

For Q_3 and Q_4,

$$\left(\frac{W}{L}\right)_{3-4} = \frac{2I_D}{\mu_p C_{ox} V_{OV}^2} = \frac{2(100\ \mu A)}{100\ \mu A / V^2\ (0.2\ V)^2}$$

$$= 50$$

(b)

$$A_d = \frac{V_O}{V_{id}} = g_m(r_{o2} \| r_{o4})$$

Since $r_{o2} = r_{o4} = r_o$, $A_d = \frac{1}{2} g_m r_o$

$$g_{m1} = g_{m2} = g_{m3} = g_{m4} = g_m = \frac{I_D}{V_{OV}/2}$$

$$= \frac{100\ \mu A}{0.2\ V/2} = 1\ mA/V$$

The value of r_o needed is

$$r_O = \frac{2A_d}{g_m} = \frac{(2)(50\ V/V)}{1\ mA/V} = 100\ k\Omega$$

Since $r_O = \frac{|V_A'|}{I_D} \cdot L$,

$$L = \frac{r_O I_D}{|V_A'|} = \frac{100\ K(0.1\ mA)}{20\ V/\mu m} = 0.5\ \mu m$$

(c) If $V_{cm} = 0$, the maximum V_o is $V_{DD} - V_{ov} = 1 - 0.2 = +0.8$ V with a single-NMOS current transistor, the lowest V_o should go is $V_{SS} + 2\ V_{ov} = -1 + 2(0.2\ V) = -0.6$ V so, the range of V_o is -0.6 V to $+0.8$ V

(d) Q_5 delivers $I = 200\ \mu A$, and $L = 0.5\ \mu m$, $V_{ov} = 0.2$ V. So,

$$r_{o5} = \frac{|V_A'|}{I} \cdot L = \frac{(20\ V/\mu m)(0.5\ \mu m)}{0.2\ mA} = 50\ k\Omega$$

$$r_{o1} = r_{o4} = r_o = 100\ k\Omega$$

$$A_{cm} = \frac{v_O}{v_{icm}} = -\frac{r_{O4}}{2R_{SS}} \cdot \frac{1}{1 + g_{m3}\, r_{O3}}$$

$$A_{cm} = -\frac{100\ k}{2(50\ k)} \cdot \frac{1}{1 + (1\ mA/V)(100\ k)}$$

$$= -0.01$$

$$CMRR(dB) = 20 \log_{10}\frac{|A_d|}{|A_{cm}|}$$

$$= 20 \log_{10}\left(\frac{50}{0.01}\right) = 74\ dB$$

7.74

CMRR = $(g_m r_o)(g_m R_{ss})$

(a) For a simple current mirror

$R_{SS} = r_{OS} \Rightarrow (\text{for } I_D = I/2)$

CMRR = $(g_m r_o)(g_m r_{os})$

$$= \left(\frac{2I_D}{V_{OV}} \cdot \frac{V_A}{I_D}\right) \cdot \left(\frac{2I_D}{V_{OV}} \cdot \frac{V_A}{2I_D}\right)$$

$$= 2 \cdot \frac{V_A}{V_{OV}} \cdot \frac{V_A}{V_{OV}}$$

$$= 2\left(\frac{V_A}{V_{OV}}\right)^2 \quad \text{Q.E.D.}$$

(b) for the modified Wilson current source of

$RSS \simeq g_{m7} \cdot r_{O7} \cdot r_{O5}$

$\Rightarrow$ CMRR $= (g_m r_O)(g_m \cdot g_{m7} r_{O7} r_{O5})$

For $Q_{5,6,7,8}$:

$$V_{OVS} = \sqrt{\frac{2I}{k' W/L}}$$

while for $Q_{1,2,3,4}$:

$$V_{OV} = \sqrt{\frac{I}{k' W/L}}$$

$\Rightarrow V_{OVS} = \sqrt{2}\, V_{OV}$.

Thus, (for $I = 2I_D$)

$$CMRR = \frac{I}{V_{OV}} \cdot \frac{V_A}{(I/2)} \cdot \frac{I}{V_{OV}} \cdot \frac{2I}{\sqrt{2}V_{OV}} \cdot \frac{V_A}{I} \cdot \frac{V_A}{I}$$

$$= \frac{4}{\sqrt{2}} \frac{VA^3}{V_{OV}^3} = 2 \cdot \sqrt{2} \frac{VA^3}{V_{OV}^3}$$

For $k' W/L = 10 \text{ mA/V}^2$

$I = 1$ mA

$|V_A| = 10$ V

$$V_{OV} = \sqrt{\frac{1 \text{ mA}}{10 \text{ mA/V}^2}} = 0.316 \text{ V}$$

$\Rightarrow$ For the simple current mirror case:

$$\text{CMRR} = 2\left(\frac{10}{0.316}\right)^2 = 2000$$

$\rightarrow$ 66 dB

For the Wilson source:

$$CMRR = 2\sqrt{2}\frac{(10)^3}{(0.316)^3} = 89442$$

$\rightarrow$ 99 dB

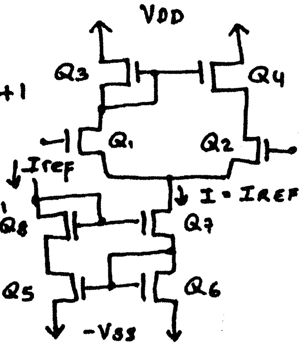
7.75

$$V_{GS_{1-4}} = \sqrt{\frac{50\mu \cdot 2}{800\mu}} + 1$$

$= 1.35$ V

$$V_{GS_{5-8}} = \sqrt{\frac{2 \times 100}{800}} + 1$$

$= 1.5$ V



For $V_{DS} = V_{GS}$

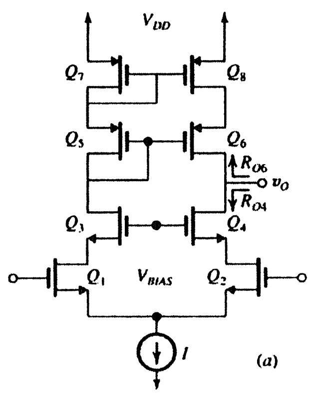
$-V_{SS} + 2V_{GS_{5-8}} + 2V_{GS_{1-4}} = V_{DD}$

Thus,

$V_{DD} + V_{SS} = 2(1.5) + 2(1.35)$

$= \underline{5.7 \text{ V}}$

7.76



(a)

(b) $R_{O4} = (g_{m4} r_{O4}) r_{O2}$

$= g_m r_o^2$

$R_{O6} = (g_{m6} r_{O6}) r_{O8}$

$= g_m r_o^2$

$A_d = g_m (R_{O4} \| R_{O6})$

$= g_m \cdot \frac{1}{2} g m^2 r_o^2$

$g_m = \frac{2I_D}{V_{OV}} \qquad r_O = \frac{V_A}{I_D}$

thus, $g_m r_o = 2V_A / V_{ov}$

$\Rightarrow A_d = 2(V_A / V_{OV})^2$

Q.E.D.

For $V_{ov} = 0.25$ V & $V_A = 20$ V

$A_d = 2\,(20 / 0.25)^2 = 12800$ V/V

7.77

$$i_1 = \frac{V_O}{r_O} = \frac{\frac{1}{2}(g_m r_O)\, v_{id}}{r_O} = \frac{1}{2} g_m v_{id}$$

$$i_2 = g_{m4} v_{sg4} = \frac{g_m v_{id}}{4}$$

$$i_3 = i_1 - i_2 = \frac{g_m v_{id}}{2} - g_m \frac{v_{id}}{4} = \frac{g_m v_{id}}{4}$$

$$i_4 = -g_{m2} v_{gs2} = -g_m \left[-\frac{v_{id}}{2} - \frac{v_{id}}{4} \right]$$

$$= \frac{3}{4} g_m v_{id}$$

$$i_5 = i_4 = \frac{3}{4} g_m v_{id}$$

$$i_6 = i_4 - i_3 = \frac{3}{4} g_m v_{id} - \frac{1}{4} g_m v_{id} = \frac{1}{2} g_m v_{id}$$

However, if we use KVL,

$$i_6 = \frac{v_o - v_s}{r_o} = \frac{\frac{1}{2} g_m r_o v_{id} - \frac{V_{id}}{4}}{r_o}$$

$$= \frac{1}{2} g_m V_{id} - \frac{V_{id}}{4 r_o} \quad \text{inconsistent}$$

$$i_7 = i_5 - i_6 = \frac{3}{4} g_m V_{id} - \frac{g_m V_{id}}{2} = \frac{g_m V_{id}}{4}$$

(which is the same as i_3)

$$i_8 = g_m v_{gs1} = g_m \left(\frac{V_{id}}{2} - \frac{V_{id}}{4} \right) = \frac{1}{4} g_m v_{id}$$

$$i_9 = i_8 = \frac{1}{4} gm\, V_{id}$$

$$i_{10} = i_8 - i_7 = \frac{g_m V_{id}}{4} - \frac{g_m V_{id}}{4} = 0$$

$i_{11} + i_{10} = i_9$ or

$$i_{11} = i_9 - i_{10} = i_9 = \frac{g_m V_{id}}{4}$$

(which is the same as i_7)

$$i_{12} = g_m v_{gs3} = \frac{1}{4} g_m V_{id}$$

$$i_{13} = i_{11} - i_{12} = \frac{1}{4} g_m V_{id} - \frac{1}{4} g_m V_{id} = 0$$

Note, through, that this is inconsistent with KVL.
If $i_{13} = 0$, $V_{D3} = 0$, but $V_{D3} = V_{G3} = -V_{id}/4$.

If $i_{10} = 0$, $V_{D1} = \frac{V_{id}}{4}$, but this conflicts with V_{D3} being $-\frac{V_{id}}{4}$.

It appears that the approximations for V_{gs} and v_s prevent a clean solution. If these were more exact, all current and voltage relutionships should be consistent.

7.78

$$I_{D1} = I_{D2} = I_{D3} = I_{D4} = \frac{I}{2} = \frac{100\ \mu\text{A}}{2} = 50\ \mu\text{A}$$

$$g_{m1} = g_{m2} = \frac{I_D}{V_{ov}/2} = \frac{50\ \mu\text{A}}{0.2\ \text{V}/2} = 0.5\ \text{mA/V}$$

$$G_m = g_{m1} = 0.5\ \text{mA/V}$$

$$r_{o1} = r_{o2} = \frac{V_{An}}{I_D} = \frac{20\ \text{V}}{0.05\ \text{mA}} = 400\ \text{k}\Omega$$

$$r_{o3} = r_{o4} = \frac{|V_{Ap}|}{I_D} = \frac{12\ \text{V}}{0.05\ \text{mA}} = 240\ \text{k}\Omega$$

$$R_o = r_{o2} \| r_{o4} = 400\ \text{k} \| 240\ \text{k} = 150\ \text{k}\Omega$$

$$A_d = G_m R_o = (0.5\ \text{mA/V})(150\ \text{k}) = 75\ \text{V/V}$$

Gain will be reduced by a factor of 2 if
$R_L = R_o = 150\ \text{k}\Omega$

7.79

$R_{id} = (\beta+1)\, 2r_e$; $r_e = \frac{25\text{mV}}{50\mu A} = 500\Omega$

$\rightarrow R_{id} = 101 \times 1000 = \underline{101 K\Omega}$

$R_o = r_{o4} \| r_{o2} = \frac{r_o}{2}$; $r_o = \frac{V_A}{I_c}$

$\rightarrow r_o = \frac{160V}{50\mu A} = 3.2M\Omega$

Thus, $R_o = \underline{1.6M\Omega}$

$G_m = g_{m1} = g_{m2} = \frac{50\mu A}{25mV} = \underline{2mA/V}$

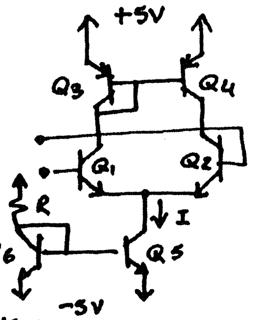
$Ad = G_m R_o = 2 \times 1600 = \underline{\underline{3200\ V/V}}$

With a subsequent stage having a 100 KΩ input resistance,

$Ad = G_m (R_o \| 100K\Omega)$

$= \underline{\underline{188.2\ V/V}}$

7.80



$G_m = \frac{I/2}{V_T} = 5\frac{mA}{V}$

$I = \underline{\underline{250\mu A}}$

$R = \frac{5-(-5)-V_{BE}}{I}$

$= \frac{9.3}{0.25} = \underline{\underline{37.2K\Omega}}$

$R_{id} = (\beta+1)2r_e$ where,

$r_e = \frac{V_T}{I/2} = \frac{25mV}{0.125mA} = 200\Omega$

$\rightarrow R_{id} = 151 \times 2 \times 0.2 = \underline{\underline{60.4K\Omega}}$

$r_o = \frac{V_A}{I_C} = \frac{100}{0.125} = 800K\Omega$

$R_o = \frac{r_o}{2} = \underline{\underline{400K\Omega}}$

$Ad = g_m R_o = 5 \times 400 = \underline{\underline{2000\ V/V}}$

$I_B = \frac{I/2}{\beta+1} = \frac{125}{151} = \underline{\underline{0.83\mu A}}$

$V_{ICM}|_{max} \simeq V_{C1} + 0.4V$

$= 5 - 0.7 + 0.4 = \underline{\underline{4.7V}}$

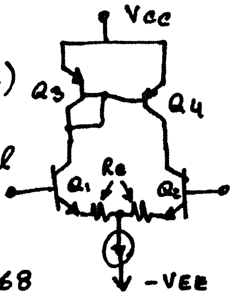
$V_{ICM}|_{min} = V_{B5} - 0.4 + 0.7$

$= -5 - 0.4 + 0.7$

$= -4V$

Thus, the input common-mode range is $-4V$ to $+4.7V$ (where we have assumed that a transistor remains active

7.81



$R_{id} = (\beta+1)2(r_e+R_E)$

Gm is still equal to: $g_m = 5mA/V$

$\Rightarrow I = \underline{\underline{250\mu A}}$

(From Problem 7.68 above)

and $r_e = \frac{V_T}{I/2} = 200\Omega$, $r_o = 800K\Omega$

If $R_{id} = 100K\Omega \Rightarrow$

$100K = 151 \times 2 \times (200 + R_E)$

$\rightarrow R_E = 131\Omega$

To obtain Ad:

$Ad = G_m \cdot R_o$ (Equ. 7.165)

As in the derivation of R_{o2} in Equ. (7.162), R_{o2} can be found using Equ. (6.159), but this time noting that Re at the emitter of Q2 is:

$r_{e1} + 2R_E$

Thus,

$R_{o2} = r_{o2}\left[1 + g_m\left((r_{e1} + 2R_E) \| r_{\pi 2}\right)\right]$

$R_{o2} = 800K\left[1 + 5m\left((200 + 2\times 131) \| 30.2K\right)\right]$ ← $((\beta+1)r_e)$

$R_{o2} = 2620\ K\Omega$

$R_o = R_{o2} \| r_{o4}$

$= (2620 \| 800)K = 613K\Omega$

$\Rightarrow A_d = 5m \times 613K = \underline{3065}\ V/V$

$A_{cm} = \dfrac{-r_{o4}}{\beta_3 R_{EE}} = -\dfrac{(2\times 240K)}{150\times 240K}$

$= -13.3\ mV/V$

and, $CMRR = \left|\dfrac{2400}{-13.3m}\right| = 180{,}451$

i.e. $\underline{105\ dB}$

$\dfrac{v_i}{v_s} = \dfrac{R_{id}}{R_{id} + R_s} = \dfrac{7.5K}{7.5K + 10K} = 0.43\dfrac{V}{V}$

$\Rightarrow$ Overall gain A:

$A = \dfrac{v_i}{v_s}\cdot\dfrac{v_o}{v_i} = 0.43 \times 2400$

$= \underline{1032}\ V/V$

7.82

$G_m = g_m = \dfrac{I/2}{V_T} = \dfrac{0.5m/2}{25m}$

$g_m = \underline{10mA/V}$

$R_o = r_{o2} \| r_{o4} = \dfrac{V_A}{I_{C2}} \| \dfrac{V_A}{I_{C4}} = \dfrac{1}{2}\dfrac{V_A}{I/2}$

$= \dfrac{120}{0.5m} = \underline{240K\Omega}$

$A_d = G_m R_o = 10 \times 240 = \underline{2400\ \dfrac{V}{V}}$

$R_{id} = 2r_\pi \simeq 2\dfrac{V_T}{I/2}\beta = \dfrac{25m \times 150}{0.5m}$

$R_{id} = \underline{7.5K\Omega}$

For a simple current mirror the output resistance (thus R_{EE}) is r_o

$\Rightarrow R_{EE} = \dfrac{V_A}{I} = \dfrac{120}{0.5m} = \underline{240K\Omega}$

7.83

(a) If R_{o1} and R_{o2} can be ignored,

$i_i = G_{mcm} V_{icm}$

$v_o \simeq [A_m i_i - G_{mcm} V_{icm}] R_{om}$

substituting in for i_i,

$v_o = [A_m G_{mcm} V_{icm} - G_{mcm} V_{icm}] R_{om}$

$A_{cm} = \dfrac{v_o}{V_{icm}} = G_{mcm} R_{om}(A_m - 1)$

(b) , $i_i = i_{ro3} + A_m i_i$

$A_m = \dfrac{i_i - i_{ro3}}{i_i} = 1 - \dfrac{i_{ro3}}{i_i} = 1 - \dfrac{V_{sg3}}{r_{o3} i_i}$

$g_m + V_{sg3} = A_m i_i$ so,

$A_m = 1 - \dfrac{A_m i_i}{g_{m4} r_{o3} i_i}$ since $g_{m4} = g_{m3}$,

$A_m = \dfrac{1}{1 + \dfrac{1}{g_{m3} r_{o3}}}$

Continuing, we can substitute this into the equation of part (a):

$A_{cm} = G_{mcm} R_{om}\left(\dfrac{1}{1 + \dfrac{1}{g_{m3} r_{o3}}} - 1\right)$

Since $V_{icm}G_{mcm} = \frac{V_{icm}}{2R_{ss}}$, $G_{mcm} = \frac{1}{2R_{ss}}$

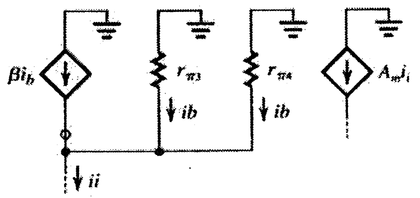
substituting,

$$A_{cm} = \left(\frac{R_{om}}{2R_{ss}}\right)\left[\frac{1-\left(1+\frac{1}{g_{m3}r_{o3}}\right)}{1+\frac{1}{g_{m3}r_{o3}}}\right]$$

since $R_{om} = r_{o4}$,

$$A_{cm} = \frac{-r_{o4}}{2R_{ss}}\left(\frac{1}{g_{m3}r_{o3}+1}\right)$$

(c)



$$i_i = \beta i_b + 2i_b \Rightarrow i_b = \frac{i_i}{\beta+2}$$

$$A_m i_i = i_b\beta$$

$$A_m i_i = \left(\frac{i_i}{\beta+2}\right)\beta$$

$$A_m = \left(\frac{\beta}{\beta+2}\right) = \frac{1}{1+2/\beta}$$

Now, substituting into the resulting equation of part (a),

$$A_{cm} = G_{mcm}R_{om}(A_m - 1)$$

$$A_{cm} = G_{mcm}r_{O4}\left(\frac{\beta}{\beta+2} - 1\right)$$ and since

$$G_{mcm} = \frac{1}{2R_{EE}},$$

$$A_{cm} = \frac{r_{O4}}{2R_{EE}}\cdot\left(\frac{\beta-\beta-2}{\beta+2}\right) = \frac{-r_{O4}}{2R_{EE}}\cdot\left(\frac{2}{\beta+2}\right)$$

since $\beta >> 2$,

$$A_m \simeq \frac{-r_{O4}}{\beta_p R_{EE}}$$

7.84

for a wilson current mirror,

$$\frac{I_O}{I_{REF}} = \frac{1}{1+\frac{2}{\beta(\beta+2)}}$$

As an active load, this means that one collector current will be $\frac{\alpha I}{2}$, while the other is

$$\frac{\alpha I}{2}\left(1+\frac{2}{\beta(\beta+2)}\right)$$

$$|\Delta i| = \frac{\alpha I}{2}\left(1+\frac{2}{\beta(\beta+2)} - 1\right) = \alpha I\left[\frac{1}{\beta(\beta+2)}\right]$$

$$G_m = g_m = \frac{\frac{\alpha I}{2}}{V_T} = \frac{\alpha I}{2V_T}$$

$$|V_{OS}| = \frac{\Delta i}{G_m} = \frac{\frac{\alpha I}{\beta(\beta+2)}}{\frac{\alpha I}{2V_T}} = \frac{2V_T}{\beta(\beta+2)}$$

For $\beta_P = 50$,

$$|V_{OS}| = \frac{2(25\text{ mV})}{50(50+2)} = 19.2\ \mu\text{V}$$

7.85

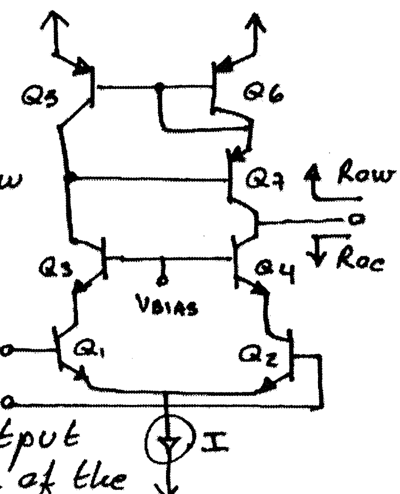
Avo = Gm Ro

Gm = gm.

Ro = Roc // Row

where Roc is the output resistance of the cascode stage &

Row the output resistance of the wilson stage



$R_{oc} = \beta r_o$ & $R_{ow} = \frac{\beta r_o}{2}$

$\Rightarrow R_o = \beta r_o \parallel \frac{\beta r_o}{2} = \frac{\beta r_o \cdot \frac{\beta r_o}{2}}{\beta r_o (1 + \frac{1}{2})}$

$= \frac{\beta r_o}{2 \times \frac{3}{2}} = \frac{\beta r_o}{3}$

$\Rightarrow A_{vo} = G_m R_o = \frac{g_m \beta r_o}{3}$ Q.E.D

For: $I = 0.4mA$, $\beta = 100$, $V_A = 120V$

$A_{vo} = \frac{I/2}{V_T} \cdot \frac{\beta}{3} \cdot \frac{V_A}{I/2} = \frac{\beta}{3} \frac{V_A}{V_T}$

$= \frac{100}{3} \times \frac{120V}{25mV} = 160000$

i.e. 104 dB

7.86

To obtain maximum positive swing V_{bias} must be as low as possible.

To keep the top current sources out of saturation:

$V_{CC} - 0.2 - 0.7 = V_{bias\ max}$

$V_{bias\ max} = 4.1$ V

And: $V_O - V_{bias\ min} = +0.4$ V

Since $V_O \sim 0 \Rightarrow V_{bias\ min} = -0.4$ V

$\Rightarrow$ Range of V_{bias} is:

$(-0.4 << V_{bias} \leq 4.1)$ V

For: $I = 0.4$ mA, $\beta_P = 50$, $\beta_N = 150$ & $V_A = 120$

$G_m = g_{m1} = \frac{0.2\text{ mA}}{25\text{ mV}} = 8\ \frac{\text{mA}}{\text{V}}$

For the folded cascode: $R_{O4} = \beta_4 r_{O4}$

For the Wilson mirror: $R_{O5} = \beta_5 \frac{r_{O5}}{2}$

$\Rightarrow R_O = \left[\beta_4 \cdot r_{O4} \parallel \beta_5 \cdot \frac{r_{O5}}{2}\right]$

$r_{O4} = r_{O5} = 120/0.2\text{ mA} = 600\text{ k}\Omega$

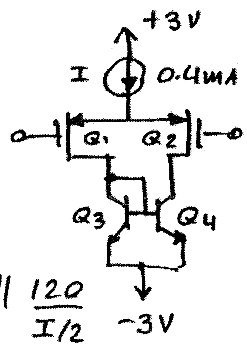
$\rightarrow R_O = \left[50 \times 600\text{ k} \parallel 150 \times \frac{600\text{ k}}{2}\right]$

$= [30\text{ M} \parallel 45\text{ M}]$

$= 18\text{ M}\Omega$

$A_d = G_m R_O = 8\ \frac{\text{mA}}{\text{V}} \times 18\text{ M}\Omega = 144000$

7.87



$K_p' W/L = 6.4 mA/V^2$

$|V_{Ap}| = 10$ V

$V_{A\,NPN} = 120$.

$R_o = r_{o2} \parallel r_{o4} = \frac{V_{Ap}}{I/2} \parallel \frac{120}{I/2}$

$R_o = (10/0.2m) \parallel (120/0.2m) = 46k\Omega$

$G_m = g_{m1} = \sqrt{I \times K_p' W/L}$

$= \sqrt{0.4mA \times 6.4mA/V^2}$

$\Rightarrow G_m = 1.6 \frac{mA}{V}$

$A_d = G_m \times R_o = 1.6 \frac{mA}{V} \times 46k\Omega$

$\longrightarrow A_d = 73.6$ V/V

7.88

$I_{D5} = I_{D8} = I_{D7} = I_{D6} = I = I_{REF} = 225\ \mu A$

$I_{D1} = I_{D2} = I_{D3} = I_{D4} = \frac{I}{2} = \frac{225\ \mu A}{2}$

$= 112.5\ \mu A$

From Eq. (8.180), systemic balance will occur in this circuit when

$$\frac{\left(\frac{W}{L}\right)_6}{\left(\frac{W}{L}\right)_4} = 2\frac{\left(\frac{W}{L}\right)_7}{\left(\frac{W}{L}\right)_5}$$

$$\left(\frac{W}{L}\right)_6 = 2\frac{\left(\frac{W}{L}\right)_7}{\left(\frac{W}{L}\right)_5}\cdot\left(\frac{W}{L}\right)_4 = (2)\frac{\left(\frac{60}{0.5}\right)}{\left(\frac{60}{0.5}\right)}\cdot\left(\frac{10}{0.5}\right)$$

$$= \frac{20}{0.5}$$

so, $W_6 = 20$

To find $|V_{OV}|$, we use $I_D = \frac{1}{2}\mu C_{ox}\left(\frac{W}{L}\right)V_{OV}^2$

$$|V_{OV}|_{1,2} = \sqrt{\frac{2(I_{D1})}{\mu_p C_{Ox}\left(\frac{W}{L}\right)_{1,2}}} = \sqrt{\frac{225\ \mu A}{60\ \mu A\,V^2\left(\frac{30}{0.5}\right)}}$$

$= 0.25$ V

$$|V_{OV}|_{3,4} = \sqrt{\frac{2I_{D3}}{\mu_n C_{Ox}}} = \sqrt{\frac{2(112.5\ \mu A)}{180\ \mu A/V^2\left(\frac{10}{0.5}\right)}}$$

$= 0.25$ V

$$|V_{OV}|_{5,7,8} = \sqrt{\frac{2I}{\mu_p C_{Ox}\left(\frac{W}{L}\right)_{5,7,8}}} = \sqrt{\frac{2(225\ \mu A)}{60\ \mu A/V\left(\frac{60}{0.5}\right)}}$$

$= 0.25$ V

$|V_{GS}| = |V_t| + |V_{OV}|$, so all are

$|V_{GS}| = 0.75 + 0.25 = 1.0$ V

$$g_{m_{1-4}} = \frac{I/2}{1\ V_{OV}1/2} = \frac{225\ \mu A}{0.25\ V} = 0.9\ mA/V$$

$$g_{m_{5-8}} = \frac{I}{1\ V_{OV}1/V} = \frac{2(225\ \mu A)}{0.25\ V} = 1.8\ mA/V$$

$$r_{o_{1-4}} = \frac{|V_A|}{I/2} = \frac{9\ V}{0.225\ mA} = 80\ k\Omega$$

$$r_{o_{5-8}} = \frac{|V_A|}{I/2} = \frac{9\ V}{0.1125\ mA} = 40\ k\Omega$$

$A_1 = -g_{m1}(r_{O2} \| r_{O4})$

$= -(0.9\ mA/V)(80\ k \| 80\ k) = -36$ V/V

$A_2 = -g_{m6}(r_{O6} \| r_{O7})$

$= -(1.8\ mA/V)(40\ k \| 40\ k) = -36$ V/V

$A_O = A_1 \times A_2 = (-36)(-36) = 1296$ V/V

$= 20\log_{10}(1296) = 62.25$ dB

The input common-mode range is determined as follows:

The lower limit is when the input is such that Q_1 and Q_2 leave the saturation region:

$V_{D1} = -V_{SS} + V_{GS3} = -1.5 + 1 = 0.5$ V

with $|V_{DS}| = |V_{OV}|$, this would be when

$V_{S1} = -0.5 + 0.25 = -0.25$ V

$V_{in\,min} = V_{S1} - V_{SG} = -0.25 - 1 = -1.25$ V

The upper limit is when Q_5 leaves saturation:

$V_{DS_{max}} = V_{DD} - |V_{OV}| = 1.5 - 0.25 = 1.25$ V

$V_{in\,max} = V_{S\,max} - V_{SG} = 1.25 - 1.0 = +0.25$ V

so, range is (-1.25 V to +0.25 V)

For the output range, $V_{O\,max}$ is

$V_{O\,max} = V_{DD} - |V_{OV}| = 1.5 - 0.25 = 1.25$ V

$V_{O\,min} = -V_{SS} + |V_{OV}| = -1.5 + 0.25$

$= -1.25$ V

so the output range is (-1.25 V +1.25 V.)

	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_8
$I_D(\mu A)$	112.5	112.5	112.5	112.5	225	225	225	225
$\lvert V_{OV}\rvert(V)$	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
$\lvert V_{GS}\rvert(V)$	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
$g_m\left(\frac{mA}{V}\right)$	0.9	0.9	0.9	0.9	1.8	1.8	1.8	1.8
$r_O(k\Omega)$	80	80	80	80	40	40	40	40

7.89

$I_{D8} = I_{D1-4} = I_{REF} = 200\ \mu A$

$I_{D5} = 2I_{D1} = 400\ \mu A$

No requirements are given for Q_6 and Q_7, so choose

* $I_{D6} = I_{D7} = 2I_{REF} = 400\ \mu A$

	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_8
$\left(\frac{W}{L}\right)$	25	25	100	100	50	200	50	25

$I_D = \frac{1}{2}k'(W/L)V_{OV}^2$ so,

$$\left(\frac{W}{L}\right)_{1,2,8} = \frac{2I_{REF}}{k_n'(V_{OV})^2} = \frac{2(200\ \mu A)}{400\ \mu A/V^2(0.2\ V)^2}$$

$= 25$

$$\left(\frac{W}{L}\right)_{3,4} = \frac{2I_{REF}}{k_p'(V_{OV})^2} = \frac{2(200\ \mu A)}{400\ \mu A/V^2(0.2\ V)^2}$$

$= 100$

$$\left(\frac{W}{L}\right)_{5,7} = \frac{2(2I_{REF})}{k_n'(V_{OV})^2} = \frac{2(400\ \mu A)}{400\ \mu A/V^2(0.2\ V)^2}$$

$= 50$

$$\left(\frac{W}{L}\right)_C = \frac{4I_{REF}}{k_p'(V_{OV})^2} = \frac{4(200\ \mu A)}{100\ \mu A/V^2(0.2\ V)^2}$$

$= 200$

Ideally, $V_O(dc) = 0$

(b) For the common-mode input range:
The lower limit is when Q_5 is leaving saturation.

$V_{D5} = -V_{SS} + |V_{OS}| = -1\ V + 0.2\ V = -0.8V$

$V_{in(min)} = V_{GS1} + V_{D5} = V_{tn} + V_{OV} + V_{D5}$

$= 0.4 + 0.2 - 0.8 = -0.2\ V$

The upper input limit is when Q_1 and $Q2$ leave the saturation region:

$V_{D1} = V_{DD} - V_{SD3} = 1 - (0.4 + 0.2) = 0.4\ V$

$V_{DS1} = |V_{OV}| = 0.2\ V,$ so

$V_{in(max)} = V_{D1} - V_{OV} + V_{GS1}$

$= V_{D1} + V_{tn} = 0.4\ V = 0.8\ V$

so, the range of input voltage is
(-0.2 V to +0.8 V)

(c) The maximum output voltage is

$V_{O_{(max)}} = V_{DD} - |V_{OV}| = 1 - 0.2 = +0.8\ V$

$V_{O_{(min)}} = -V_{SS} + |V_{OV}| = -1 + 0.2$

$= -0.8\ V$

so range is (–0.8 V to +0.8 V)

(d) $r_{O2} = r_{O4} = \frac{|V_A|}{I_{D2}} = \frac{5\ V}{0.2\ mA} = 25\ k\Omega$

$r_{O6} = r_{O7} = \frac{|V_A|}{I_{D6}} = \frac{5\ V}{0.4\ mA} = 12.5\ k\Omega$

$g_{m1} = g_{m2} = \frac{|I_D|}{V_{OV}/2} = \frac{0.2\ mA}{0.2\ V/2} = 2\ mA/V$

$g_{m6} = \frac{|I_{D6}|}{V_{OV}/2} = \frac{0.4\ mA}{0.2/2} = 4\ mA/V$

$A_1 = g_{m1}(r_{O2} \| r_{O4}) = (2\ mA/V)(25\ k \| 25\ k)$

$= 25\ V/V$

$A_2 = -g_{m2}(r_{O6} \| r_{O7}) = -4\ mA/V(12.5\ k \| 12.5\ k)$

$= -25\ V/V$

$A_O = A_1 \cdot A_2 = 25(-25) = -625\ V/V$

7.90

$I = \frac{1}{2}k\ V_{OV}^2$

(a) $V_{OV} = \sqrt{\frac{2I}{k}}$

If K increases by 4 $\rightarrow V_{OV}$ decreases by 1/2

$g_m = 2I/V_{OV} = k \cdot V_{OV}$

$\rightarrow$ if k increases by 4

gm increases by $\times$ 2

(b) $A_1 = gmR_{O1}$

$\rightarrow A_1$ increases $\times$ 2 as does A_O

(c) Offsets due to V_t mismatch are unaffected.

Others reduced $\times \frac{1}{2}$ since A_O increases $\times$ 2

7.91

$I_{D7} = \frac{W_7}{W_8} I_{REF} = \frac{50}{40} \times 90\ \mu A$

$= 112.5\ \mu A$

Output offset current $= I_{D7} - I_{D6}$

$= 112.5 - 90 = 22.5\ \mu A$

$\Rightarrow V_o = 22.5\ \mu\ (r_{o6} \| r_{o7})$

$r_{o7} = \frac{10}{112.5\ \mu} = 88.9\ k\Omega$

$\Rightarrow V_o = 22.5\ \mu\ (111\ k \| 88.9\ k)$

$= 1.11\ V$

$V_{os} = \frac{V_o}{A_v} = \frac{1.11V}{1109} = 1\ mV$

7.92

Offset current $= I_{O2} - I_{O4}$
$= I_{D3} - I_{D4}$

$$I_{D3} = \frac{K}{2}(V_{GS} - V_t)^2$$

$$I_{D4} = \frac{K}{2}(V_{GS} - (V_t + \Delta V_t))^2$$

$$I_O = I_{D3} - I_{D4}$$
$$= \frac{K}{2}\left[(V_{GS} - V_t - V_{GS} + V_t + \Delta V_t) \times (V_{GS} - V_t + V_{GS} - V_t - \Delta V_t)\right]$$
$$= \Delta V_t \cdot \frac{K}{2}(2V_{GS} - 2V_t - \Delta V_t)$$
$$\simeq K(V_{GS} - V_t) \cdot \Delta V_t$$
$$I_O = g_{m3} \Delta V_t$$

Recall $I_O = G_{m1} \cdot V_{OS}$
and $G_{m1} = g_{m1}$

$$\Rightarrow V_{OS} = \frac{g_{m3}}{g_{m1}} \cdot \Delta V_t$$

For $\Delta V_t = 2mV$

$$V_{OS} = \frac{0.3m}{0.3m} \times 2m = \underline{\underline{2mV}}$$

7.93

(a) $\underline{\underline{I_{E1} = I_{E2} = 0.1mA \simeq I_{E3}, I_{E4}}}$

$\underline{\underline{I_{E5} \simeq 1mA}}$ and since the output is held at 0V $\underline{\underline{I_{E6} = 2mA}}$

(b) $r_{e1} = r_{e2} = \frac{25mV}{0.1mA} = 250\Omega$

$$r_{e5} = \frac{25mV}{1mA} = 25\Omega$$

$$r_{e6} = \frac{25mV}{2mA} = 12.5\Omega$$

For the active loaded differential pair; Recall from Eqn. (7.161) $G_{m1} = g_{m1}$

$$\simeq \frac{1}{r_{e1}} = \frac{1}{250} = 4\frac{mA}{V}$$

$R_{O1} = (\beta+1) r_{e5}$ Since all r_o's $= \infty$

$R_{O1} = 101 \times 25 = 2525\Omega$

$$\Rightarrow A_1 = G_{m1} R_{O1} = 4\frac{mA}{V} \times 2525\Omega$$
$$= 10.1 \text{ V/V}$$

For the common-emitter:

$$A_5 = -g_{m5} \cdot R_{C5}$$
$$\simeq -\frac{\beta R_L}{r_{e5}} = -\frac{100 \times 10K}{25}$$
$$= -40{,}000 \text{ V/V}$$

For the emitter follower:
$A_6 \simeq 1$

$$A_{2nd\text{-}stage} = A_5 \cdot A_6 = -40{,}000 \text{ V/V}$$

$$A = A_1 \cdot A_{2nd\text{-}stage} = 10.1 \times -40{,}000$$
$$= \underline{\underline{-404{,}000 \text{ V/V}}}$$

(c) Since the dominant low-frequency pole is set by C_C & $r_{\pi 5}$

$$f_P = \frac{1}{2\pi \cdot R_{O1} \underbrace{(A_5 + 1) C_C}_{\text{by Miller effect}}} = 100Hz$$

$$\Rightarrow C \cong 1/(2\pi \times 2525 \times 40K \times 100)$$
$$= \underline{\underline{15.76pF}}$$

7.94

$I_B = 225\ \mu A$

$\mu_n C_{ox} = 180\ \mu A / V^2$

$\mu_p C_{ox} = 60\ \mu A / V^2$

For Q_8 & Q_9 : W/L = 60/0.5

$$\Rightarrow |V_{ov}| = \sqrt{\frac{2I_D}{k_p'(W/L)}}$$

$$|V_{ov}|_{8,9} = \sqrt{\frac{2 \times 225\ \mu}{60\ \mu \times 120}} = 0.25\ V$$

$$\text{then } g_{m8,9} = \frac{2I_D}{|V_{ov}|} = \frac{2 \times 225\ \mu}{0.25\ V}$$

$= 1.8$ mA/V

Since g_m of Q_{10}, Q_{11} & Q_{13} are identical to g_m of Q_8 & Q_9 then $V_{ov13} = 0.25$ V

Thus for Q_{13}

$$(0.25)^2 = \frac{2 \times 225\ \mu}{180\ \mu \times (W/L)_{13}}$$

$\rightarrow (W/L)_{13} = 40$ i.e, (20 / 0.5)

Since Q_{12} is 4 times as wide as Q_{13}, then

$$(W/L)_{12} = \frac{4 \times 20}{0.5} = 80/0.5$$

$$R_B = \frac{2}{\sqrt{2\ k_n'\ (W/L)_{12}\ I_B}} \cdot \left(\sqrt{\frac{(W/L)_{12}}{(W/L)_{13}}} - 1\right)$$

$$= \frac{2}{\sqrt{2 \times 180\ \mu \times \frac{80}{0.5} \times 225\ \mu}} \cdot \left(\frac{\sqrt{\frac{80/0.5}{20/0.5}} - 1}{\sqrt{4} - 1}\right)$$

$\rightarrow R_B = 555.6\ \Omega$

The voltage drop on R_B is :

$555.6 \times 225\mu = 0.125$ V

To obtain the gate voltages:(assume $|V_{tn}| = |V_{tp}| = 0.7$ V)

$$V_{ov12} = \sqrt{\frac{2 \times 225\ \mu}{180\ \mu \times \frac{80}{0.5}}} = 0.125\ V$$

$V_{OV12} = V_{GS12} - V_{tn}$

$\rightarrow V_{GS12} = 0.125 + 0.7 = 0.825$ V

thus,

$V_{G12,13} = V_{GS12} + I_B R_B - V_{SS}$

$= 0.825 + 0.125 - 1.5$

$= -0.55$ V

$V_{OV11} = |V_{OV8}| = 0.25$ V

$\Rightarrow V_{GS11} = 0.25 + 0.7 = 0.95$ V

$V_{G11} = -0.55 + 0.95$

$V_{G11} = V_{G10} = 0.4$ V

$V_{G8} = V_{DD} - V_{SG8} = 1.5 + (-0.25 - 0.7)$

$= +0.55$ V

Finally from the results above:

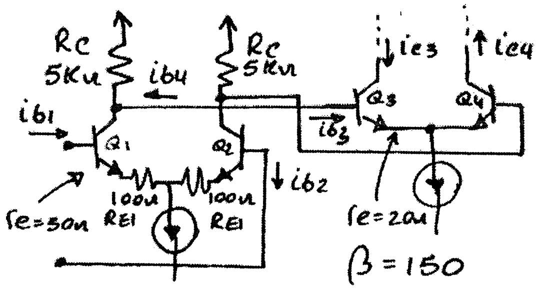
$(W/L)_{10} = 20/0.5$

$(W/L)_{11} = 20/0.5$

$(W/L)_{12} = 80/0.5$

$(W/L)_{13} = 20/0.5$

7.95



$$A_1 = \frac{2R_C \parallel R_{id2}}{2(R_{E1} + r_{e1})}$$

$R_{id2} = (\beta+1)(2r_{e2}) = 6.04\ k\Omega$

$\Rightarrow A_1 = \underline{12.5}$ V/V

$$A_i = \frac{i_{e4}}{i_{b1}} = \beta_1 \cdot \frac{2R_C}{R_{id2} + 2R_C}\ \beta_4$$

$= 1.4 \times 10^4$ A/A

7.96

$$R_O \simeq \frac{R_S}{\beta + 1} + r_{e8} = R_C'$$

Thus R_S affects R_O We want $R_O' \parallel 3\text{ k} = 76$

$\Rightarrow R_O' = 78\ \Omega$

$\Rightarrow R_S = (78 - r_{e8})(\beta + 1)$

$= 7.34\ k\Omega$

$$A_3 = \frac{-R_5 \| R_{i4}}{r_{e4} + R_4} \; ; \; R_{i4} \cong 304 \text{ k}\Omega$$

and $A_3 = -3.09$ V/V

and $A = 8513 \cdot \frac{3.09}{6.42} = 4104$ V / V

The gain has been reduced by a factor of 2.07 and can be restored by reducing R_4 by this same factor to increase A_3. Thus $R_4 = 1.11$ kΩ

(Note that this is a first order approximation).

7.97

(a) $A_3 = \frac{-R_{i4}}{2.325 \text{ k}\Omega} = \frac{-303.5}{2.325}$

$= -130.5$ V/V

i.e A_3 is increased by $\frac{130.5}{6.42}$

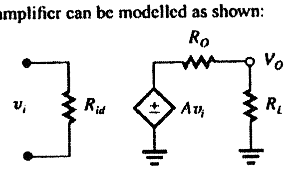
$= 20.33$

$\Rightarrow A = 8513 \times 20.33$

$= 173.1 \times 10^3$ V / V

(b) Let the output resistance of the current source be $R \to \infty$ $R_O = 3\text{ k} \| \left(\frac{R}{\beta + 1} + r_e\right) = 3 \text{ k}\Omega$

The amplifier can be modelled as shown:



Thus,

$$A_{LOAD} = \frac{A \cdot R_L}{R_L + R_O}$$

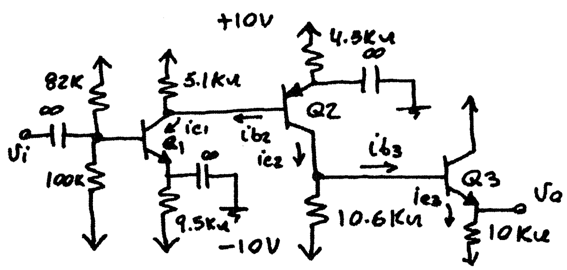
$$= 173.1 \times 10^3 \frac{100}{100 + 3000}$$

$= 5583$ V/V

For the original amplifier:

$$A_{LOAD} = 8513 \times \frac{100}{100 + 152} = 3378 \frac{\text{V}}{\text{V}}$$

7.98



(a) $I_{E1} = \dfrac{\dfrac{20\text{V} \times 100\text{K}}{82\text{K} + 100\text{K}} - 0.7}{9.5\text{K} + \dfrac{(82\text{K} \| 100\text{K})}{\beta+1}}$

$\beta = 100 \Rightarrow I_{E1} = 1.03\text{mA}$

$\alpha = \frac{100}{101} \Rightarrow I_{C1} = 1.02\text{mA}$

$V_{C1} \simeq 10\text{V} - 1.02\text{mA} \times 5.1\text{K}\Omega = 4.8\text{V}$

$I_{E2} = \frac{(10 - 0.7 - 4.8)\text{V}}{4.5\text{K}\Omega} = 1\text{mA}$

$\rightarrow I_{C2} = 0.99\text{mA}$

$V_{C2} \simeq 0.99\text{m} \times 10.6\text{K} - 10 = 0.5\text{V}$

$\Rightarrow v_{O\,DC} = 0.5 - 0.7 = -0.2\text{V}$

$I_{E3} = \frac{-0.2 - (-10)}{10\text{K}} = 0.98\text{mA}$

$\rightarrow I_{C3} = 0.97\text{mA}$

Thus all transistors are operating at $I_C \simeq 1\text{mA}$

(b) $R_{in} = 82K \,||\, 100K \,||\, r_{\pi 1}$

where $r_{\pi 1} = \frac{\beta}{g_{m1}} = \frac{100}{40m} = 2.5K\Omega$

$\Rightarrow R_{in} = (82 \,||\, 100 \,||\, 2.5)K = \underline{\underline{2.37K\Omega}}$

$R_{out} = 10K \,||\, \left[r_{e3} + \frac{10.6K}{\beta+1}\right]$

$= 10K \,||\, \left[25 + \frac{10.6K}{101}\right]$

$= \underline{\underline{128\,\Omega}}$

(c) $\frac{i_{c1}}{v_i} = g_{m1} = 40mA/V$

$\frac{i_{b2}}{i_{c1}} = \frac{5.1K}{5.1K + r_{\pi 2}} = \frac{5.1}{5.1 + 2.5} = 0.671\ \frac{A}{A}$

$\frac{i_{e2}}{i_{b2}} = \beta_2 = 100\ A/A$

$\frac{i_{b3}}{i_{c2}} = \frac{10.6K}{10.6K + (\beta+1)(r_{e3} + 10K)}$

$= 0.01036\ A/A$

$\frac{i_{e3}}{i_{b3}} = \beta_3 + 1 = 101$

$v_o = i_{e3} \times 10K$

Thus,

$\frac{v_o}{v_i} = 10 \times 101 \times 0.01036 \times 100 \times 0.671 \times 40$

$= \underline{\underline{2.81 \times 10^4\ V/V}}$

(d) $f_{P2} = 1/(2\pi C_2 \cdot R_2)$

where: $R_2 = 5.1K \,//\, r_{\pi 2}$

$= 5.1K \,||\, 2.5K = 1.68K\Omega$

$C_2 = C_{\pi 2} + C_{\mu 2}(1 + g_{m2} R_{L2})$

with:

$R_{L2} = 10.6K \,||\, ((\beta+1)(r_{e3} + 10K)$

$= 10.6K \,||\, 101 \times (25 + 10K)$

$= 10.5K\Omega.$

$\Rightarrow C_2 = 10p + 2p\,(1 + 40m \times 10.5K)$

$= 852pF$

$\Rightarrow f_{P2} = \frac{1}{2\pi \times 852p \times 10.5K}$

$= \underline{\underline{17.8\,KHz}}$

7.99

(a) $I_{D1-5,7} = \frac{I}{2}$

$I_{D6,8} = 2\left(\frac{I}{2}\right) = I$

$g_m = \frac{|I_D|}{\frac{|V_{OV}|}{2}}$ So that

$g_{m1-5,7} = \frac{I/2}{V_{OV}/2} = \frac{I}{|V_{OV}|}$

$g_{m6,8} = \frac{I}{\frac{|V_{OV}|}{2}} = \frac{2I}{|V_{OV}|}$

$r_O = \frac{|V_A|}{|I_D|}$ So that

$r_{O1-5,7} = \frac{2|V_A|}{I}$

$r_{O6,8} = \frac{|V_A|}{I}$

In Summary,

	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_8
I_D	$I/2$	$I/2$	$I/2$	$I/2$	$I/2$	I	$I/2$	I
g_m	$\frac{I}{\lvert V_{OV}\rvert}$	$\frac{I}{\lvert V_{OV}\rvert}$	$\frac{I}{\lvert V_{OV}\rvert}$	$\frac{I}{\lvert V_{OV}\rvert}$	$\frac{I}{\lvert V_{OV}\rvert}$	$\frac{2I}{\lvert V_{OV}\rvert}$	$\frac{I}{\lvert V_{OV}\rvert}$	$\frac{2I}{\lvert V_{OV}\rvert}$
r_D	$\frac{2\lvert V_A\rvert}{I}$	$\frac{2\lvert V_A\rvert}{I}$	$\frac{2\lvert V_A\rvert}{I}$	$\frac{2\lvert V_A\rvert}{I}$	$\frac{2\lvert V_A\rvert}{I}$	$\frac{\lvert V_A\rvert}{I}$	$\frac{2\lvert V_A\rvert}{I}$	$\frac{\lvert V_A\rvert}{I}$

(b)To find the differential gain, apply $-\frac{V_{id}}{2}$ to Q_1

and $V_{id}/2$ to Q_2

$$V_{g5} = g_{m1}\left(r_{o1} \parallel r_{o3} \parallel \frac{1}{g_{m3}}\right) V_{id}/2$$

since $\frac{1}{g_{m3}} << r_{o1} \parallel r_{o3}$.

$$V_{g5} = g_{m1}\left(\frac{1}{g_{m3}}\right) \cdot \frac{V_{id}}{2}$$

$$i_{d5} = -V_{g5}\, g_{m5} = -g_{m1}\left(\frac{1}{g_{m3}}\right)(g_{m5})\frac{V_{id}}{2}$$

$$V_{g8} = V_{g7} = i_{d5}\left(r_{o5} \parallel r_{o7} \parallel \frac{1}{g_{m7}}\right) = i_{d5}\left(\frac{1}{g_{m7}}\right)$$

since $g_{m8} = g_{m7}$,

$$V_{g8} = -g_{m1}\left(\frac{1}{g_{m7}}\right)\frac{V_{id}}{2}$$

since $g_{m8} = 2\, g_{m7}$

$$i_{d8} = +g_{m1}\left(\frac{1}{g_{m7}}\right)(2g_{m7}) \cdot \frac{V_{id}}{2} = +g_{m1}V_{id}$$

with $+\frac{V_{id}}{2}$ applied to Q_2,

$$V_{g4} = -g_{m2}\left(r_{o2} \parallel r_{o4} \parallel \frac{1}{g_{m4}}\right),$$

$$V_{g6} = V_{g4} = -g_{m1}\left(\frac{1}{g_{m4}}\right) \cdot \frac{V_{id}}{2}$$

since $g_{m6} = g_{m4} \times 2$,

$$i_{d6} = -g_{m4}(2)(-g_{m1})\left(\frac{1}{g_{m4}}\right)\frac{V_{id}}{2}$$

$$i_{d6} = g_{m1}\, V_{id}$$

$$i_O = g_{m1}\, V_{id} + g_{m1}\, V_{id} = 2\, g_{m1}\, V_{id}$$

$$\frac{A_d}{V_{id}} = \frac{i_O\, R_O}{V_{id}} = 2\, g_{m1}\,(r_{o6} \parallel r_{o8})$$

$$g_{m1} = \frac{I}{\lvert V_{OV}\rvert} \quad r_{o6} = r_{o8} = \frac{\lvert V_A\rvert}{I}$$

$$\frac{A_d}{V_{id}} = 2\frac{I}{\lvert V_{OV}\rvert}\left(\frac{1}{2}\right)\frac{\lvert V_A\rvert}{I} = \frac{V_A}{V_{OV}}$$

(c) If each input transistor (Q_1 and Q_2) is replaced with a current source of $\frac{V_{icm}}{2R_{SS}}$,

From Q_3, with a transfer ratio of $\left(1-\frac{1}{g_m r_O}\right)$,

$$i_{D6} = -\frac{V_{icm}}{2R_{SS}}\left(1-\frac{1}{g_{m4}r_{O4}}\right)(2)$$

From

$$Q_1, i_{D8} = \frac{V_{iCM}}{2R_{SS}}\left(1-\frac{1}{g_{m3}r_{O3}}\right)\left(1-\frac{1}{g_{m7}r_{O7}}\right)(2)$$

$$i_O = i_{D8}+i_{D6}$$

$$i_O = \frac{V_{icm}}{R_{SS}}\left[-1+\frac{1}{g_{m4}r_{O4}}+\left(1-\frac{1}{g_{m3}r_{O3}}\right)\left(1-\frac{1}{g_{m7}r_{O7}}\right)\right]$$

Since $g_{m3} = g_{m4}$ and $r_{O3} = r_{O4}$,

$$i_O = \frac{V_{iCM}}{R_{SS}}\left[\left(1-\frac{1}{g_{m3}r_{O3}}\right)\left(1-\frac{1}{g_{m7}r_{O7}}\right)\right]$$

$V_O = i_O(r_{o6}\| r_{O8})$ and Since $\frac{1}{g_{m3}r_{O3}} >> 1$

$$v_O \simeq \frac{-v_{icm}}{R_{SS}}\left(\frac{1}{g_{m7}r_{O7}}\right)(r_{o6}\| r_{O8})$$

$$|A_{CM}| = \left|\frac{V_O}{V_{iCM}}\right| = \frac{(r_{o6}\| r_{O8})}{R_{SS}} = \frac{1}{g_{m7}r_{O7}}$$

(d) If the current source is fabricated as a simple current mirror, $R_{SS} = \frac{V_A}{I}$

$r_{O6} = r_{O8} = \frac{V_A}{I}$ so, $r_{o6}\| r_{O8} = \frac{V_A}{2I}$

$$g_{m7} = \frac{I}{V_{OV}/2} = \frac{2I}{V_{OV}}$$

$$CMRR = \frac{|A_d|}{|A_{CM}|} = \frac{V_A/V_{OV}}{\frac{r_{o6}\| r_{O8}}{R_{SS}}\cdot\frac{1}{g_{m7}r_{O7}}}$$

$$= \frac{V_A/V_{OV}}{\frac{V_A/2I}{V_A/I}\cdot\frac{1}{\frac{2I}{V_{OV}}\cdot\frac{V_A}{I}}}$$

$$CMRR = \frac{2V_A/V_{OV}}{\frac{1}{2}\left(\frac{V_{OV}}{V_A}\right)} = 4(V_A/V_{OV})^2$$

(e) To find the input common-mode range, consider both upper and lower limits: Lower limit is when the current source begins to leave the saturation region at $V_{DS} = V_{OV}$

So,

$V_{I(min)} = V_{GS1}+V_{OV}+V_{SS} = V_t+2V_{OV}-V_{DD}$

The maximum limit occurs when Q_1 or Q_2 begins to leave the saturated region: For example, when

$V_{S1} = V_{DD}-V_{SG3}-V_{OV1}$

$V_{I(max)} = V_t+V_{OV}+V_{S1}$

$V_{I(max)} = V_t+V_{OV}+V_{DD}-V_t+2V_{OV}$

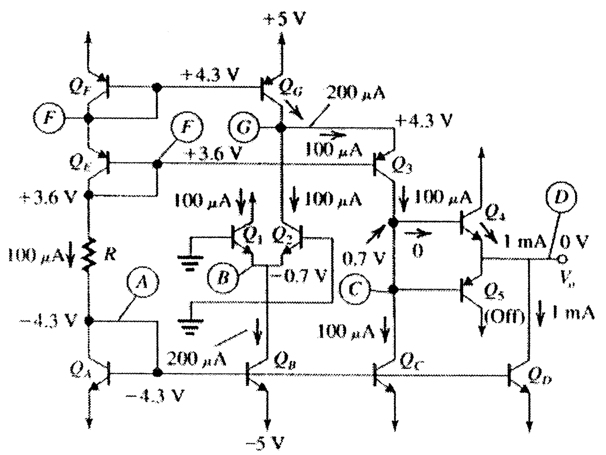
$V_I(\max) = V_{DD}-V_{OV}$

So the range is

$(-V_{DD}+V_t+2V_{OV} \le V_{ICM} \le V_{DD}-V_{OV})$

7.100

(a)



DC Analysis

$$R = \frac{3.6 - (-4.3)}{100\ \mu A} = 79\ k\Omega$$

Node voltages:

$V_A = -4.3\ V$ $\quad V_B = -0.7\ V$

$V_C = +0.7\ V$ $\quad V_D = 0\ V$

$V_E = +3.6\ V$ $\quad V_F = +4.3\ V$

$V_G = +4.3V$

(b)

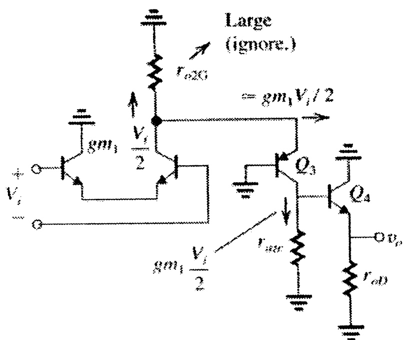
Transistor	I_C(mA)	g_m(mA/V)	r_o(mΩ)
Q_1	0.1	4	2
Q_2	0.1	4	2
Q_3	0.1	4	2
Q_4	1.0	40	0.2
Q_5	0	0	∞
Q_A	0.1		
Q_B	0.2		
Q_C	0.1	....	2
Q_D	1.0	....	0.2
Q_E	0.1		
Q_F	0.1		
Q_G	0.2	....	1

(c) Total resistance at collector Q_3 is

$$\approx \beta_3 r_{o3} \| r_{oC} \| \ (\beta_4 + 1)(r_{o4} \| r_{oD})$$

$$= 100 \times 2 \| 2 \| \ 101(0.2 \| 0.2)$$

$$= 1.65\ M\Omega$$



$$\frac{v_{C3}}{v_i} = +g_{m1} \times \frac{1}{2} \times 1.65 \times 10^3 = 3300 \frac{V}{V}$$

$$\frac{v_O}{v_{C3}} \simeq 1$$

Thus, $\frac{v_O}{v_i} \simeq 3300$ V/V (Polarity correct)

(d) $R_{in} = 2\, r_{\pi 1}$

$$= 2 \times \frac{100}{4} = 50\ k\Omega$$

$$R_{out} = r_{OD} \| r_{O4} \| \left[r_{e4} + \frac{r_{O2C} \| \beta_3 r_{O3}}{\beta + 1}\right]$$

$$= 0.2 \| 0.2 \| \left[25.10^{-6} + \frac{2 \| \ 100 \times 2}{101}\right]$$

$$\simeq 16.4\ k\Omega$$

(e) $v_{ICM[min]} = -4.3 - 0.4 + 0.7$

$= -4\ V$

$v_{ICM[max]} = V_G + 0.4 = +4.7\ V$

(f) The voltage at the base of Q_4 can rise to V_{B3}.

$(V_E) + 0.4 = +4\ V$

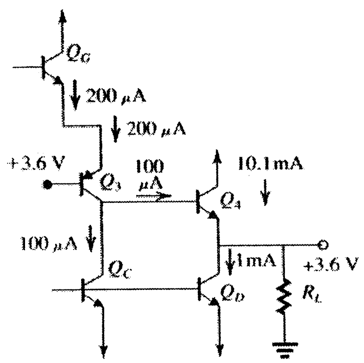
before Q_3 saturating. Thus v_O can go up to + 3.3 V

The voltage at the output can go down to V_{base} of

$Q_D + 0.4 = V_A - 0.4 = -4.3 - 0.4 = -4.7\ V$

Thus the linear range at the output is − 4.7 V to +3.3 V

(g) At the positive limit of v_O



i.e. $v_O = +3.3$ V and Q_2 just cut off

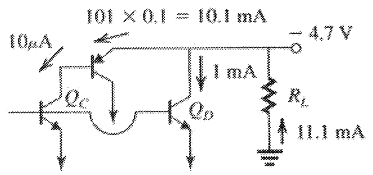
$$R_L = \frac{3.3\ V}{9.1\ mA}$$

$= 363\ \Omega$

(this is the minimum allowed R_L for + 3.3 V output)

At the negative limit of v_O i.e. $v_O = -3.3$ V and Q_1 has cut-off. Q_3 will also be cut-off, and Q_4 will cut-off.

Thus,



$R_L = \dfrac{4.7}{11.1 \text{ mA}} = 423\ \Omega$ This is the minimum allowed R_L for a − 4.7 V output.

7.101

DC analysis

(a) $I_{REF} = 10\ \mu\text{A} = \frac{1}{2} \times 40 \times \frac{5}{5}(V_{GS_A} - V_t)^2$

$\Rightarrow V_{GS_A} = 1.71 \text{ V} \approx 1.7 \text{ V}$

$10 = \frac{1}{2} \times 20 \times \frac{5}{5}(V_{GS_{E\,F}} - 1)^2$

$\Rightarrow V_{GS_{E\,F}} = 2 \text{ V}$

$R = \dfrac{3 - (-3.3)}{10\ \mu\text{A}} = 660 \text{ k}\Omega$

(b) See figure above

$V_{GS1} = V_{GS2} = V_{GSA} \approx 1.7$ V

$V_{GS3} = \sqrt{\dfrac{2 \times 10}{20 \times \frac{10}{3}}} + 1 = 1.71 \text{ V} \approx 1.7 \text{ V}$

$V_{GS5} = V_{GS3} = 1.7$ V

For Q_6 : $50 = \frac{1}{2} \times 40 \times \frac{50}{5}(V_{GS6} - V_t)^2$

$\Rightarrow V_{GS6} = 1.50$ V

$V_A = -3.3$ V $\quad V_B = -1.7$ V

$V_C = +1.5$ V $\quad V_D = 0$ V

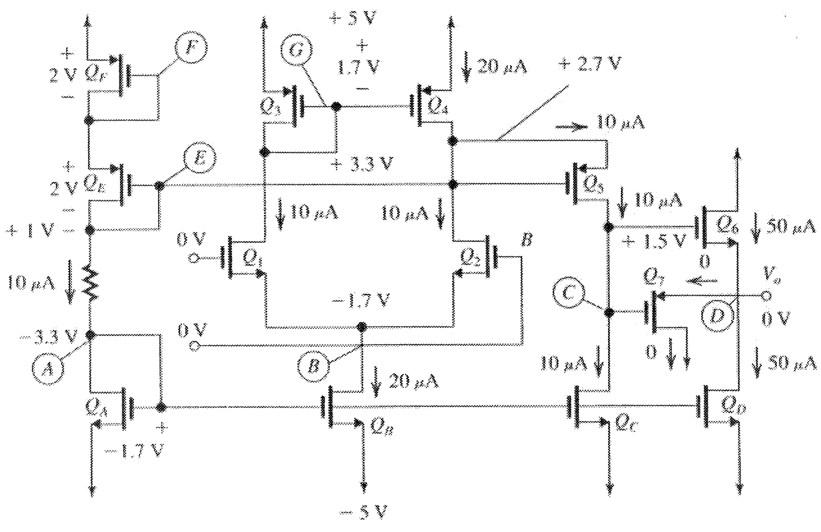
$V_E = +1$ V $\quad V_F = +3$ V

$V_G = +3.3$ V $\quad V_H = +2.7$ V

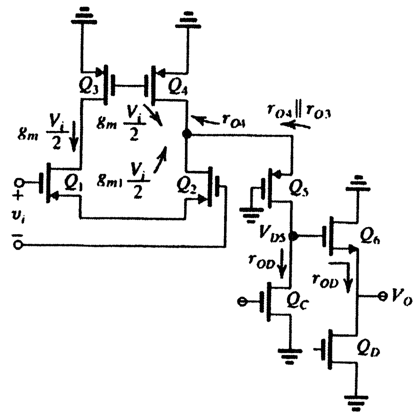
(c)

Transistor	I_D (μA)	V_{GS} (V)	gm (mA/V)	r_O (MΩ)
Q_1	10	1.7	28.3	5
Q_2	10	1.7	28.3	5
Q_3	10	1.7	28.3	5
Q_4	20	1.7	56.6	2.5
Q_5	10	1.7	28.3	5
Q_6	50	1.5	200	1
Q_7	0	−1.5*	0	∞
Q_A	10	1.7	28.3	5
Q_B	20	1.7	56.6	2.5
Q_C	10	1.7	28.3	5
Q_D	50	1.7	141.4	1
Q_E	10	2	20	5
Q_F	10	2	20	5

* Cut-off.



(d)



Total resistance at the drain of Q_5, R is:

$R = (g_{m5}\, r_{O5})(r_{O4} \| r_{O2}) \| r_{OC}$

$= [(28.3 \times 5)(2.5 \| 2)] \| 5$

$= 4.9\ \text{M}\Omega$

Thus, $\frac{v_{d5}}{v_i} = g_{m1}R$

$= 28.3 \times 4.9 = 138.7\ \text{V/V}$

and $\frac{v_O}{v_{d5}} = \frac{(r_{OD} \| r_{O6})}{(r_{OD} \| r_{O6}) + \frac{1}{g_{m6}}}$

$= \frac{(1 \| 1)}{(1 \| 1) + \frac{1}{200}} \approx 1$

$\frac{v_O}{v_i} = 138.7\ \text{V/V}$

$R_{in} = \infty$

$R_{out} = r_{OD} \| r_{O6} \| 1/g_{m6}$

$= 1 \| 1 \| 1/200\ \text{M}\Omega$

$\approx 5\ \text{k}\Omega$

(c) $v_{ICM\max} = V_G + V_t$

$= +4.3\ \text{V}$

$v_{ICM\min} = V_{GS1} + V_B\min$

$= V_{GS1} + V_A - V_t$

$= 1.7 - 3.3 - 1 = -2.6\ \text{V}$

(f) $V_{O\max} = V_{C\max} - V_{GS6}$

$= V_E + |V_t| - V_{GS6}$

$= +1 + 1 - 1.5 = +0.5\ \text{V}$

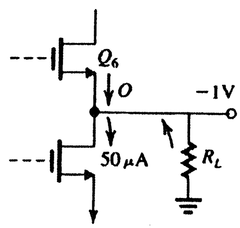
$v_{O\min} = V_A - V_t = -3.3 - 1 = -4.3\ \text{V}$

(g) $Q6$ cuts off

thus,

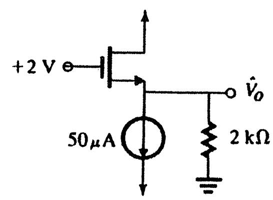
$\frac{1V}{R_L} = 50\ \mu\text{A}$

$R_L = \frac{1\ V}{50\ \mu\text{A}} = 20\ \text{k}\Omega$



(h)Maximum possible voltage at drain of Q_5 is +2V. At theis value we have:

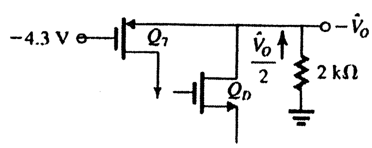
$I_D = 50\mu\text{A} + \frac{\hat{V}_o}{2}\ \text{mA}$



$I_D = \frac{1}{2}\mu_n C_{OX}\frac{W}{L}(2 - V_o - V_t)^2$

$\Rightarrow V_o \approx 0.17\text{V}$

For the lowest posible output, the circuit becomes



Where:

Q_6 cuts off and Q_7 conducts

$I_D = \frac{V_O}{2} - 0.05\ \text{mA}$

$= \frac{1}{2}\mu_P C_{OX}\left(\frac{100}{5}\right)(-\hat{V}_O + 4.3 - 1)^2$

$\Rightarrow \hat{V}_O = 1.45\ \text{V}$

That is, the range of v_o is

$-1.45\ \text{V}$ to $+0.17\ \text{V}$