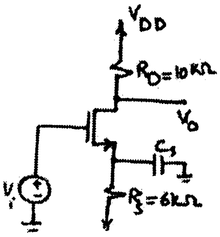
8.1

$I_D = 1\,mA$, $g_m = 1\,mA/V$

using eq. 4.89 we have:

$$A_M = \frac{-g_m R_D}{1+g_m R_S} = -\frac{1\times 10}{1+1\times 6}$$

$A_M = 1.43\,V/V$



$$f_L = \frac{1}{2\pi(\frac{1}{g_m}\|R_S)C_S} = 10\,Hz$$

$$C_S = \frac{1}{2\pi\times 10(1^K\|6^K)} = 18.57\,\mu F$$

8.2

$$f_{C_{C2}} = \frac{1}{2\pi C_{C2}(R_L + R_D\|r_o)} \leq 10^{Hz}$$

$$\Rightarrow C_{C2} \geq \frac{1}{10\times 2\pi\times(10^K + 15^K\|150^K)} \Rightarrow C_{C2} \geq 0.67\,\mu F$$

$\Rightarrow C_{C2} = 0.7\,\mu F \Rightarrow f_{C_{C2}} = 9.62\,Hz$

If I_D is doubled with both r_o and R_D halved:

$$f_{C_{C2}} = \frac{1}{2\pi\times 0.7^{\mu}(10^K + \frac{15^K}{2}\|\frac{150^K}{2})} = 13.5^{Hz}$$

8.3

$g_m = 1\,mA/V$

$$A_M = -\frac{R_G}{R_G+R_{sig}} g_m (R_D\|R_L) \quad \text{where } R_G = 10^M\|47^M, \; R_G = 8.25^{M\Omega}$$

$$A_M = -\frac{8.25}{8.25+0.1}\times 1\times(4.7^K\|10^K) = -3.16\,V/V$$

$$f_{P1} = \frac{1}{2\pi C_{C1}(R_G+R_{sig})} \quad \text{(Eq. 4.134)}$$

$$f_{P1} = \frac{1}{2\pi\times 0.01\times 10^{-6}\times(8.25+0.1)\times 10^6} = 1.9^{Hz}$$

$$f_{P2} = \frac{1}{2\pi C_S(R_S\|\frac{1}{g_m})} = \frac{1}{2\pi\times 10\times 10^{-6}\times(2^K\|\frac{1}{1}^K)} = 23.9^{Hz}$$

$$f_{P3} = \frac{1}{2\pi C_{C2}(R_D+R_L)} = \frac{1}{2\pi\times 0.1\times 10^{-6}\times(4.7+10)\times 10^3} = 108.3^{Hz}$$

$f_L \simeq 108.3\,Hz$

8.4

$$C_{TOT} = C_S + C_{C1} + C_{C2} = 3\,\mu F$$

$R_G = 10\,M\Omega$, $R_{sig} = 100\,k\Omega$, $g_m = 2\,\frac{mA}{V}$,

$R_D = R_L = 10\,k\Omega$

$$f_{P2} = \frac{g_m}{2\pi\cdot C_S} \Rightarrow C_S = \frac{2\times 10^{-3}}{2\pi}\cdot\frac{1}{f_{P2}} = \frac{3.18\times 10^{-4}}{f_{P2}}$$

$$f_{P1} = \frac{1}{2\pi\cdot C_{C1}\cdot(R_G+R_{sig})}$$

$$\Rightarrow C_{C1} = \frac{1.57\times 10^{-8}}{f_{P1}}$$

$$f_{P3} = \frac{1}{2\pi\cdot C_{C2}\cdot(R_D+R_L)}$$

$$\Rightarrow C_{C2} = \frac{7.95\times 10^{-6}}{f_{P3}}$$

If we choose: $f_{P2} = f_L, f_{P1} = f_L/25, f_{P3} = f_L/5$

$C_{TOT} = 3\,\mu F = C_S + C_{C1} + C_{C2}$

$$3\,\mu F = \frac{3.18\times 10^{-4}}{f_L} + \frac{1.57\times 10^{-8}}{f_L/25} + \frac{7.95\times 10^{-6}}{f_L/5}$$

$\Rightarrow f_L = 120$ Hz and $C_s = 2.65\,\mu F$,

$C_{C1} = 3.3$ nF, $C_{C2} = 0.33\,\mu F$

If we choose: $f_{P2} = f_L$, $f_{P1} = \frac{f_L}{5}$, $f_{P3} = \frac{f_L}{25}$

$C_{TOT} = 3\,\mu F = C_S + C_{C1} + C_{C2}$

$$3\,\mu F = \frac{3.18\times 10^{-4}}{f_L} + \frac{1.57\times 10^{-8}}{f_L/5} + \frac{7.95\times 10^{-6}}{f_L/25}$$

$\Rightarrow f_L = 172.3$ Hz, $C_S = 1.8\,\mu F$,

$C_{C1} = 455$ pF, $C_{C2} = 1.15\,\mu F$

8.5

$R_E = 72\,\Omega$,

$R_{C1} = 7.44\,k\Omega$, $R_{C2} = 13\,k\Omega$

If $C_E = 50\,\mu F$, $C_{C1} = C_{C2} = 2\,\mu F$

$$f_{P1} = \frac{1}{2\pi\cdot C_{C1} R_{C1}} = \frac{1}{2\pi\times 2\times 10^{-6}\times 7.44\times 10^3} = 10.7\,Hz$$

$$f_{P2} = \frac{1}{2\pi\cdot C_E\cdot R_E}$$

8.6

$$= \frac{1}{2\pi \cdot 50 \times 10^{-6} \times 72} = 44.2 \text{ Hz}$$

$$f_{P3} = \frac{1}{2\pi C_{C2} R_{C2}}$$

$$= \frac{1}{2\pi \times 2 \times 10^{-6} \times 13 \times 10^3} = 6.1 \text{ Hz}$$

From Eq 9.19

$$f_L = f_{P1} + f_{P2} + f_{P3} = 61 \text{ Hz}$$

$g_m = 40 \text{ mA/V}$, $r_\pi = 2.5 \text{ k}\Omega$ and

$r_e = 25\ \Omega$

If I_C is reduced by half, since

$$g_m = \frac{I_C}{V_T} \rightarrow g_m = 20 \frac{\text{mA}}{\text{V}}$$

$$r_\pi = \frac{\beta}{g_m} \rightarrow r_\pi = 5 \text{ k}\Omega \text{ and}$$

$$r_e = \frac{\alpha}{g_m} \rightarrow r_e = 50\ \Omega$$

Then:

$$R_{C1} = (R_B \parallel r_\pi) + R_{sig}$$

$$= (100 \text{ K} \parallel 5 \text{ K}) + 5 \text{ K} = 9.76 \text{ k}\Omega$$

$$R_E = r_e + \frac{R_B \parallel R_{sig}}{\beta + 1} = 50 + \frac{100 \text{ K} \parallel 5 \text{ K}}{101}$$

$$= 97\ \Omega$$

$$R_{C2} = R_C + R_L = 13 \text{ k}\Omega$$

For C_E to contribute 80% of f_L

$$0.8 \times 2\pi \times 100 = \frac{1}{C_E \cdot 97} \rightarrow C_E = 20.51\ \mu\text{F}$$

For C_{C1} and C_{C2} to contribute 10% of f_L each

$$0.1 \times 2\pi \times 100 = \frac{1}{C_{C1} \cdot 9.76 \times 10^3}$$

$$\rightarrow C_{C1} = 1.64\ \mu\text{F}$$

$$0.1 \times 2\pi \times 100 = \frac{1}{C_{C2} \cdot 13 \times 10^3}$$

$$\rightarrow C_{C2} = 1.23\ \mu\text{F}$$

To verify the value of f_L that results.

$$f_L = \frac{1}{2\pi}\left(\frac{1}{97 \times 20.51\ \mu} + \frac{1}{9.76 \text{ K} \times 1.64\ \mu} + \frac{1}{13 \text{ K} \times 1.23\ \mu}\right)$$

$f_L = 99.89$ Hz

8.7

$R_{sig} = 20 \text{ k}\Omega$, $R_C = 20 \text{ k}\Omega$,

$R_B = 200 \text{ k}\Omega$, $R_C = 10 \text{ k}\Omega$,

$\beta = 100$, $I_C \approx 100\ \mu\text{A}$

$$g_m = \frac{I_C}{V_T} \approx \frac{100\ \mu\text{A}}{25 \text{ mV}} = 4 \frac{\text{mA}}{\text{V}}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{4 \times 10^{-3}} = 25 \text{ k}\Omega$$

$$r_e = \frac{\alpha}{g_m} = \frac{0.99}{4 \times 10^{-3}} = 247.5\ \Omega$$

Then,

$$R_{C1} = (R_B \parallel r_\pi) + R_{sig}$$

$$= (200 \text{ K} \parallel 25 \text{ K}) + 20 \text{ K}$$

$$R_{C1} = 42.22 \text{ k}\Omega$$

$$R_E = r_e + \frac{R_B \parallel R_{sig}}{\beta + 1}$$

$$= 247.5 + \frac{(200 \text{ K} \parallel 20 \text{ K})}{101} = 427.52\ \Omega$$

$$R_{C2} = R_C + R_L = 20 \text{ K} + 10 \text{ K} = 30 \text{ K}$$

If we choose $f_L = 100$ Hz and

$$f_{P2} = 0.9 \times f_L$$

$$C_E = \frac{1}{2\pi(0.9 \times f_L) \times R_E}$$

$$= \frac{1}{2\pi \times 90 \times 427.52} = 4.2\ \mu\text{F}$$

Selecting C_{C1} and C_{C2} such as they contribute 5% of f_L each we have:

$$C_{C1} = \frac{1}{2\pi(0.05 \times 100) \times (42.22 \text{ K})} = 0.8\ \mu\text{F}$$

$$C_{C2} = \frac{1}{2\pi(0.05 \times 100) \times (30 \text{ K})} = 1\ \mu\text{F}$$

The resulting f_L is:

$$f_L = \frac{1}{2\pi}\left\{\frac{1}{4.2\ \mu \times (427.52)} + \frac{1}{0.8\ \mu \times 42.22 \text{ K}} + \frac{1}{1\ \mu \times 30 \text{ K}}\right\}$$

$f_L = 98.65$ Hz

The total capacitance is:

$$C_t = 0.8\ \mu + 1\ \mu + 4.2\ \mu = 6\ \mu\text{F}$$

8.8

$R_{in} = R_1 \| R_2 \| (r_x + r_\pi)$

where $R_1 = 33k\Omega$, $R_2 = 22K\Omega$

$r_x = 50$ and,

$r_\pi = \frac{\beta_o}{g_m} = \frac{120}{0.3 \times 40} = \frac{120}{12} = 10K\Omega$

$R_{in} = 33 \| 22 \| 10.05 = \underline{5.7K\Omega}$

$A_M = -\frac{R_{in}}{R_{in}+R_S} \cdot \frac{r_\pi}{r_\pi + r_x} \cdot g_m (R_C \| R_L \| r_o)$

$= -\frac{5.7}{5.7+5} \cdot \frac{10}{10+0.05} \cdot 12\,(4.7 \| 5.6 \| 300)$

$= \underline{-16.11}$ V/V

$R'_{sig} = r_\pi \| [r_x + (R_1 \| R_2 \| R_{sig})]$

$= 10K \| [50 + (33 \| 22 \| 5)K]$

$= \underline{2.69}\,K\Omega$

$R'_L = r_o \| R_C \| R_L = 300 \| 4.7 \| 5.6 (K\Omega)$

$= 2.53K\Omega$

$C_\pi + C_\mu = \frac{g_m}{2\pi \cdot f_T} = \frac{12 \cdot 10^{-3}}{2\pi \times 700 \cdot 10^6} =$

$= 2.73\,pF$

$C_\pi = (2.73 - 1)\,pF$

$= 1.73\,pF$

$C_{in} = C_\pi + C_\mu (1 + g_m R'_L)$

$= 1.73p + 1p(1 + 12 \times 2.53)$

$= 33pF$

$f_H = 1/2\pi C_{in} \cdot R'_{sig}$

$= 1/2\pi \times 33 \cdot 10^{-12} \times 2.69 \cdot 10^3$

$= \underline{1.79\,MHz}$

8.9

To select C_E so that it contributes 90% of the value of f_L

$$\frac{1}{2\pi\, C_E \cdot R_E'} = 0.9 \times 100 \quad R_E' = 110.8\ \Omega$$

(From problem 9.11)

$\Rightarrow C_E = 15.9\ \mu F$

To select C_{c1} so that it contributes 5% of f_L: $R_{C1} = 10.7\ k\Omega$

$$\Rightarrow C_1 = \frac{1}{2\pi \cdot 10.7 \times 10^3 \times 0.05 \times 100} = 2.97\ \mu F$$

To select C_{C2} so that it contributes 5% of f_L: $R_{C2} = 10.3\ k\Omega$

$$\Rightarrow C_{C2} = \frac{1}{2\pi \cdot 10.3 \times 10^3 \times 0.05 \times 100} = 3.1\ \mu F$$

8.10

$R_{C1} = R_S + [R_B \| (r_x + r_\pi)]$

$= 10 + [10 \| (0.1 + 1)]$

$= 10.99K\Omega$

$R_E' = R_E \| \frac{r_\pi + r_x + (R_B \| R_S)}{\beta_o + 1}$

$\simeq 1 \| \frac{1 + 0.1 + (10 \| 10)}{100+1}$

$\simeq 57\Omega$

For C_E and C_{C1} to contribute equally to the determination of f_L,

$C_E R_E' = C_{C1} \cdot R_{C1}$

$\Rightarrow \frac{C_E}{C_{C1}} = \frac{R_{C1}}{R_E'} = \frac{10.99}{0.057} = \underline{193}$

8.11

a) $I_b = \dfrac{V_s}{R_s + r_\pi}$ $\quad I_C = \beta \cdot I_b = \dfrac{\beta \cdot V_s}{R_s + r_\pi}$

$$V_o = -I_C(R_C \parallel R_L) = -\beta\frac{(R_C \parallel R_L)}{R_s + r_\pi}\cdot V_s$$

$$A_M = \frac{V_O}{V_S} = -\beta\frac{(R_C \parallel R_L)}{R_S + r_\pi}$$

b) Pole due to C_E: $\omega_{PE} = \dfrac{1}{C_E\left(r_e + \dfrac{R_S}{\beta + 1}\right)}$

Pole due to C_C: $\omega_{PC} = \dfrac{1}{C_C(R_C + R_L)}$

zeros are both at $s = 0$

c) $A(s) = A_M \cdot \dfrac{s^2}{(s + \omega_{PE})(s + \omega_{PC})}$

$$A(s) = \frac{-\beta(R_C \parallel R_L)}{R_S + r_\pi}\;\frac{s^2}{\left[s + \dfrac{1}{C_E\left(R_E + \dfrac{R_S}{\beta + 1}\right)}\right] \times \left[s + \dfrac{1}{C_C(R_C + R_L)}\right]}$$

d) $A_M = \dfrac{-100(10 \parallel 10)}{10 + \dfrac{100}{40}} = -40$ V/V

e) Since the resistance that forms the pole ω_{PE} is very small, we choose to make ω_{PE} the dominant pole, thus :

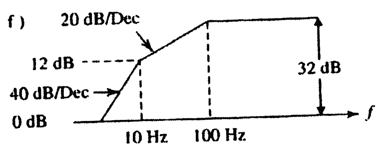
$$f_{PE} = f_L = 100 = \frac{1}{2\pi C_E\left(25 + \dfrac{10\text{ K}}{101}\right)}$$

$$\Rightarrow C_E = \frac{1}{2\pi \cdot 100 \cdot (0.025 + 0.100) \times 10^3} = 12.7\ \mu\text{F}$$

$$f_{PC} = 10\text{ Hz} \Rightarrow 10\text{ Hz} = \frac{1}{2\pi C_C(R_L + R_C)}$$

$$\Rightarrow C_C = \frac{1}{2\pi \times 10(10 + 10)\cdot 10^3} = 0.8\ \mu\text{F}$$

f)



Unity-gain frequency must be an octave lower than 10 Hz i.e. at 5 Hz

g) $A(j\omega) = -A_M \cdot \dfrac{\omega^2}{(\omega_{PE} + j\omega)(\omega_{PC} + j\omega)}$

$$= +40 \cdot \frac{\omega^2}{(\omega_{PE} + j\omega)(\omega_{PC} + j\omega)}$$

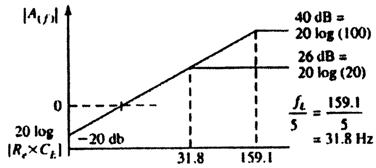
Thus $\phi = \tan^{-1}\left(\dfrac{\omega}{\omega_{PE}}\right) - \tan^{-1}\left(\dfrac{\omega}{\omega_{PC}}\right)$

$$= -\left[\tan^{-1}\frac{f}{f_{PE}} + \tan^{-1}\frac{f}{f_{PC}}\right]$$

$$= -\left[\tan^{-1}\frac{f}{100} + \tan^{-1}\frac{f}{10}\right]$$

Thus at $f = 100$ Hz $\quad \phi = -[\tan^{-1}1 + \tan^{-1}10]$

$\simeq 129.3°$



8.12

(a) $I_e = \dfrac{V_s}{r_e + R_e + \dfrac{1}{sC_E}}$

$I_c \simeq I_e$

$V_o = -R_c I_c = \dfrac{-R_c}{r_e + R_e + \dfrac{1}{sC_E}} \cdot V_s$

$A(s) \equiv \dfrac{V_o}{V_s} = \dfrac{-R_c}{r_e + R_e + \dfrac{1}{sC_E}}$

$= \dfrac{-R_c}{r_e + R_e} \cdot \dfrac{s}{s + \dfrac{1}{C_E(r_e + R_e)}}$

Thus, $A_M = \dfrac{-R_c}{r_e + R_e}$

$\omega_L = \dfrac{1}{C_E \cdot (r_e + R_e)}$

(b) A_M is reduced by the factor $\frac{r_e + R_e}{r_e}$

$$= 1 + \frac{R_e}{r_e}$$

(c) ω_L is reduced by the factor $\left(1 + \frac{R_e}{r_e}\right)$ which is the same as the gain reduction factor. Thus, the value of R_e can be used as the parameter for exercising the gain-bandwidth trade off.

(d) $R_e = 0$:

$$|A_M| = \frac{R_C}{r_e} = \frac{10{,}000}{25} = 400 \text{ V/V}$$

$$f_L = \frac{1}{2\pi C_E r_e} = \frac{1}{2\pi \times 100 \times 10^{-6} \times 25}$$

$$= 63.7 \text{ Hz}$$

To lower f_L by a factor of 5 use:

$R_e = 4r_e = 100\,\Omega$. The gain is also lowered by a factor of 5 to 80 V/V

8.13

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{3.45 \times 10^{-11}}{8 \times 10^{-9}}$$

$$= 4.3 \times 10^{-3} \text{ F/m}^2 = 4.3 \text{ fF}/\mu\text{m}^2$$

$$k_n' = \mu_n C_{ox} = 450 \times 10^{-4} \times 4.3 \times 10^{-3}$$

$$= 193.5 \ \mu\text{A}/\text{V}^2$$

$$I_D = 100 \ \mu\text{A} = \frac{1}{2} \times 193.5 \times \frac{20}{1} V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.23 \text{ V}$$

$$V_{DS} = 1.5 \text{ V} > V_{OV} \Rightarrow \text{Saturation}$$

$$g_m = \frac{2I_D}{V_{OV}} = 880 \ \mu\text{A/V},$$

$$r_O = \frac{1}{\lambda I_D} = \frac{1}{0.05 \times 0.1} = 200 \text{ k}\Omega$$

$$X = \frac{r}{2\sqrt{2\mu_f + V_{SB}}} = \frac{0.5}{2\sqrt{0.65 + 1}} = 0.19$$

$$g_m b = X g_m = 167.2 \ \mu\text{A/V}$$

$$C_{OV} = W L_{OV} C_{ox} = 20 \times 0.05 \times 4.3$$

$$= 4.3 \text{ fF}$$

$$C_{gs} = \frac{2}{3} W L C_{ox} + C_{OV}$$

$$= \frac{2}{3} \times 20 \times 1 \times 4.3 + 4.3 = 61.6 \text{ fF}$$

$$C_{gd} = C_{OV} = 4.3 \text{ fF}$$

$$C_{sb} = \frac{C_{sbo}}{\sqrt{1 + \frac{V_{SB}}{V_O}}} = \frac{15}{\sqrt{1 + \frac{1}{0.7}}} = 9.6 \text{ fF}$$

$$C_{db} = \frac{C_{dbo}}{\sqrt{1 + \frac{V_{db}}{V_O}}} = \frac{15}{\sqrt{1 + \frac{(1 + 1.5)}{0.7}}} = 7 \text{ fF}$$

8.14

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.1}{0.25} = 0.8 \text{ mA/V}$$

$$f_T = \frac{g_m}{2\pi(C_{gs} + C_{gd})} = \frac{0.8 \times 10^{-3}}{2\pi(20 + 5) \times 10^{-15}} = 5.1 \text{ GHz}$$

8.15

$$f_T = \frac{g_m}{2\pi\,(C_{gs} + C_{gd})}$$

$$g_m = \sqrt{2 \cdot \mu_n \cdot C_{ox}\frac{W}{L} \cdot I_D}$$

Also $C_{gs} \simeq \frac{2}{3}WL \cdot C_{ox}$, if $C_{gs} >> C_{gd}$ then we can ignore C_{gd}. If we replace for g_m and C_{gs} then we have :

$$f_T = \frac{\sqrt{2\mu_n \cdot C_{ox}(W/L)I_D}}{2 \cdot \pi \cdot \frac{2}{3}W \cdot L \cdot C_{ox}}$$

$$= \frac{1.5}{\pi \cdot L}\sqrt{\frac{\mu_n \cdot I_D}{2C_{ox} \cdot WL}}$$

Therefore we can see that the higher the current I_D then the higher is f_T. Also the frequency is inversely proportional to the size of the device, i.e. higher frequencies are achievable for smaller devices.

8.16

$$f_T = \frac{g_m}{2\pi(C_{gs} + C_{gd})} \quad (1)$$

For $C_{gs} >> C_{gd}$ and the over lap capacitance of C_{gs} negligibly small : $C_{gs} \simeq \frac{2}{3}WLC_{ox}$

Also $g_m = \frac{2I_D}{V_{OV}} = k_n'\frac{W}{L}V_{OV}$

If we substitude g_m and C_{gs} in (1) from the above formulas : $f_T = k_n'\frac{W}{L}V_{OV}\frac{1}{2\pi \times \frac{2}{3}WLC_{ox}}$

$$\Rightarrow f_T = \frac{3\mu_n V_{OV}}{4\pi L^2}$$

Therefore, for a given device f_T is proportional to $V_{OV} \cdot f_T \propto V_{OV}$

For $L = 1\ \mu m$, $V_{OV} = 0.25$:

$$f_T = \frac{3 \times 450 \times 10^{-4} \times 0.25}{4 \times \pi \times 1 \times 10^{-12}} = 2.7\ \text{GHz}$$

For $V_{OV} = 0.5$ V : $\frac{f_{T1}}{f_{T2}} = \frac{V_{OV1}}{V_{OV2}}$

$$\Rightarrow f_{T2} = 2.7 \times \frac{0.5}{0.25}$$

$f_T = 5.4$ GHz

8.17

The intrinsic gain A_O is

$$= g_m \cdot r_O = \left(\frac{2I_D}{V_{OV}}\right)\cdot\left(\frac{V_A}{I_D}\right) = \frac{2V_A}{V_{OV}} \text{ and}$$

$$V_A = V_A' \cdot L = 5\ \text{V}/\mu\text{m} \times L$$

$$A_O = \frac{2 \times 5[\text{V}/\mu\text{m}] \cdot L}{0.2[\text{V}]} = 50 \times L\ \text{V/V with}$$

L in mm.

* From problem 9.19

$$f_T = \frac{3\mu_n V_{OV}}{4\pi \cdot L^2} = \frac{3 \times 450\left[\frac{\text{cm}^2}{\text{V}\cdot\text{S}}\right] \cdot 0.2[\text{V}]}{4\pi L^2}$$

$$= \frac{2.15 \times 10^{-3}}{L^2}$$

$$L_{min} = 0.18 \times 10^{-6}\ \text{m}$$

	$1\,L_{min}$	$2\,L_{min}$	$3\,L_{min}$	$4\,L_{min}$	$5\,L_{min}$
A_O [V/V]	9	18	27	36	45
f_T[GHz]	66.35	16.59	7.37	4.14	2.65

8.18

$$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)}$$

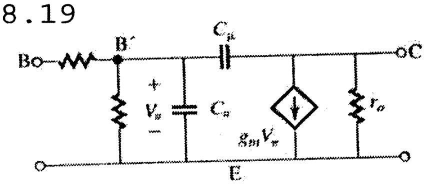
$$= \frac{80 \times 10^{-3}}{2\pi(10 + 1) \times 10^{-12}}$$

$$= \underline{4.24\,\text{GHz}}$$

$$f_\beta = f_T/\beta_0 = (4.24/150) \times 10^9$$

$$= \underline{28.26\,\text{MHz}}$$

8.19



$r_x = 100\ \Omega$

$g_m = \frac{I_C}{V_T} = \frac{0.5\text{ mA}}{25\text{ mV}} = 20\text{ mA/V}$

$r_\pi = \frac{\beta_O}{g_m} = \frac{100}{20} = 5\text{ k}\Omega$

$r_O = \frac{V_A}{I_C} = \frac{50\text{ V}}{0.5\text{ mA}} = 100\text{ k}\Omega$

$C_\mu = \frac{C_{\mu o}}{\left(1+\frac{V_{cB}}{V_{OC}}\right)^{0.5}} = \frac{30}{\left(1+\frac{2}{0.75}\right)^{0.5}}$

$= 15.7\text{ fF}$

$C_{je} \simeq 2C_{jeo} = 2 \times 20 = 40\text{ fF}$

$C_{de} = \tau_F g_m = 30 \times 10^{-12} \times 20 \times 10^{-3}$

$= 600\text{ fF}$

$C_\pi = C_{je} + C_{de} = 0.640\text{ pF}$

$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)}$

$= \frac{20 \times 10^{-3}}{2\pi(0.64 + 0.016) \times 10^{-12}} = 4.85\text{ GHz}$

8.20

$|h_{fe}| \simeq f_T/f$

- At $I_C = 0.2$ mA, $|h_{fe}| = 2.5$ at $f = 500$ MHz, thus:

$f_T = 2.5 \times 500 = \underline{1.25}$ GHz

- At $I_C = 1.0$ mA, $|h_{fe}| = 11.6$ at $f = 500$ MHz, thus:

$f_T = 11.6 \times 500 = \underline{5.8}$ GHz

$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)}$

$C_\pi + C_\mu = \frac{g_m}{2\pi f_T} \rightarrow C_\pi = \frac{g_m}{2\pi f_T} - C_\mu$

$C_\pi (I_C = 0.2\text{mA}) = \frac{8 \times 10^{-3}}{2\pi \times 1.25 \times 10^9} - 0.05 \cdot 10^{-12}$

$= 0.9686$ pF

$C_\pi (I_C = 40\text{mA}) = \frac{40 \times 10^{-3}}{2\pi \times 5.8 \times 10^9} - 0.05 \times 10^{-12}$

$= 1.0476$ pF

Since $C_\pi = C_{je} + \tau_F g_m$,

$C_{je} + 8 \times 10^{-3} \tau_F = 0.9686 \times 10^{-12}$ (1)

$C_{je} + 40 \times 10^{-3} \tau_F = 1.0476 \times 10^{-12}$ (2)

Solving Eqn. (1) and (2) together yields,

$C_{je} = \underline{0.95}$ pF, $\tau_F = \underline{2.47\text{ps}}$

8.21

$\omega_T = g_m / (C_\pi + C_\mu)$

$2\pi \times 5 \times 10^9 = \frac{20 \times 10^{-3}}{(C_\pi + 0.1) \times 10^{-12}}$

$C_\pi + 0.1 = \frac{20}{10\pi} = 0.64$ pF

$C_\pi = \underline{0.54}$ pF

$g_m = \underline{20\text{mA/V}}$

$r_\pi = \beta/g_m = 150/20 = \underline{7.5\text{K}\Omega}$

$f_\beta = f_T/\beta = \frac{5 \times 10^9}{150} = \underline{33.3}$ MHz

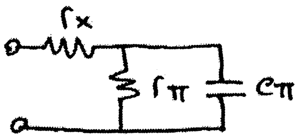
8.22

$|h_{fe}|$ becomes 20 at:

$$f_T/20 = \frac{1\times10^9}{20} = \underline{\underline{50\ MHz}}$$

$$f_\beta = f_T/\beta_0 = \frac{1000\,MHz}{200} = \underline{\underline{5\,MHz}}$$

8.23



$$Z = r_x + \frac{1}{\frac{1}{r_\pi} + j\omega C_\pi}$$

$$= r_x + \frac{r_\pi}{1 + j\omega C_\pi r_\pi}$$

$$Z = r_x + \frac{r_\pi}{1 + j(\omega/\omega_\beta)}$$

$$= r_x + \frac{r_\pi\,(1 - j\,\omega/\omega_\beta)}{1 + \left(\frac{\omega}{\omega_\beta}\right)^2}$$

$$= r_x + \frac{r_\pi}{1 + \left(\frac{\omega}{\omega_\beta}\right)^2} - j\cdot\frac{r_\pi\,(\omega/\omega_\beta)}{1 + \left(\frac{\omega}{\omega_\beta}\right)^2}$$

$$\mathrm{Re}[Z] = r_x + \frac{r_\pi}{1 + \left(\frac{\omega}{\omega_\beta}\right)^2}$$

For $\mathrm{Re}[Z]$ to be an estimate of r_x good to within 10% we must keep

$$\frac{r_\pi}{1 + \left(\frac{\omega}{\omega_\beta}\right)^2} \le \frac{r_x}{10}$$

But $r_x \le r_\pi/10$

Thus,

$$\frac{r_\pi}{1 + \left(\frac{\omega}{\omega_\beta}\right)^2} \le \frac{r_\pi}{100}$$

$$1 + \left(\frac{\omega}{\omega_\beta}\right)^2 \ge 100$$

or $\omega \ge 10\,\omega_\beta$ (approx.)

8.24

	I_E (mA)	r_e (Ω)	g_m (mA/V)	r_π (kΩ)	β_o
(a)	1	25	40	2.5	100
(b)	1	25	40	3.13	125.3
(c)	0.99	25.3	39.6	2.525	100
(d)	10	2.5	400	0.25	100
(e)	0.1	250	4	25	100
(f)	1.0	25	40	0.25	10
(g)	1.25	20	50	0.20	10

CONT.	f_T (MHz)	C_π (pF)	C_m (pF)	f_β (MHz)
(a)	400	2	13.9	4
(b)	501.3	2	10.7	4
(c)	400	2	13.8	4
(d)	400	2	157	4
(e)	100	2	4.4	1
(f)	400	2	13.9	40
(g)	800	1	9	80

8.25

$$C_{in} = C_{gs} + C_{eq}$$

$$= C_{gs} + C_{gd}\,(1 + g_m R_L')$$

$$= 0.5 + 0.1(1 + 29) = 3.5\ \text{pF}$$

Neglecting R_G:

$$f_H = \frac{1}{2\pi \cdot C_{in} \cdot R_{sig}} \quad \text{i.e } R_G \text{ is very large if}$$

$$f_H > 10\ \text{MHz} \Rightarrow \frac{1}{2\pi \cdot 3.5\times10^{-12}\times10^6} > R_{sig}$$

$$\Rightarrow R_{sig} < 4.55\ \text{k}\Omega$$

8.26

Since R_G is very large:

$$f_H = \frac{1}{2\pi \cdot C_{in} \cdot R_{sig}}$$

if $f_H \geq 10$ MHz $\Rightarrow 10 \times 10^6 \leq \frac{1}{2\pi \cdot C_{in} \cdot R_{sig}}$

$$C_{in} \leq \frac{1}{2\pi \times 10 \times 10^6 \times 1 \times 10^3}$$

$C_{in} \leq 15.91$ pF

$\Rightarrow C_{gs} + C_{gd}(1 + g_m R_L') \leq 15.91$ pF

$5 \times 10^{-12} + 1 \times 10^{-12}(1 + 5 \times 10^{-3} \cdot R_L')$

≤ 15.91 pF $\Rightarrow R_L' \leq 1982\ \Omega$

Since R_G is very large: $A_M = -g_m \cdot R_L'$

$A_M \geq -5 \times 10^{-3} \cdot 1982 \Rightarrow A_M \geq -9.91$ V/V

Gain - bandwidth product : $GB \equiv |A_M| \cdot BW$

$GB \geq 9.1 \times 10 \times 10^6$ $GB \geq 91$ MHz

If $f_H \geq \frac{10}{3}$ MHz

then : $R_L' \leq 8349\ \Omega$

$A_M \geq -47.75$ V/V

$GB \geq 139.2$ MHz

8.27

$g_m = 1\ \frac{\text{mA}}{\text{V}}$; $C_{gs} = 1$ pF; $C_{gd} = 0.4$ pF;

$C_{in} = 4.26$ pF; $A_M = -7\frac{\text{V}}{\text{V}}$; $f_H = 382$ KHz

$\Rightarrow GB = 7 \times 382 \cdot 10^3 = 2.67$ MHz

We also Know that:

$$C_{gs} = \frac{2}{3} W \cdot C_{OX} + WL_{OV} \cdot C_{OX}$$

$$= WC_{OX}\left(\frac{2}{3} + L_{OV}\right)$$

$C_{gd} = WL_{OV} C_{ox}$

$\Rightarrow$ if W is reduced by half so are C_{gs} and C_{gd}.

$\Rightarrow C_{gs2} = 0.5$ pF $\quad C_{gd2} = 0.2$ pF

In saturation:

$I_D = \frac{1}{2} k_n' \frac{W}{L}(V_{OV})^2 \Rightarrow$ For I_D to remain unchanged while ω is halved

$\Rightarrow (V_{OV})^2$ is doubled

thus $V_{OV2} = \sqrt{2}\ V_{OV}$

but $g_m = \frac{2I_D}{V_{OV}} = g_{m2} = \frac{1}{\sqrt{2}} g_m$

$= 0.707$ mA / V

we can now calculate the new values of A_M, C_{in}, f_H and GB

$$A_{M2} = \frac{-R_G}{R_G + R_{sig}} \cdot g_m R_L' = -\frac{7}{\sqrt{2}} \cdot \text{V/V}$$

$= -4.9$ V / V

$C_{eq2} = (1 + 0.707 + 7.14) \cdot 0.2 \times 10^{-12}$

$= 1.21$ pF

$\Rightarrow C_{in2} = 0.5 + 1.21 = 1.71$ pF

$$f_{H2} = \frac{1}{2\pi \cdot 1.71 \times 10^{-12} \cdot (0.1 \parallel 4.7) \times 10^6}$$

$= 950$ KHz

$GB_2 = 4.9 \times 950 \cdot 10^3 = 4.65$ MHz

The ratios of new vs old values are:

$\frac{W_2}{W}$	$\frac{V_{OV2}}{V_{OV}}$	$\frac{g_{m2}}{g_m}$	$\frac{C_{gs2}}{C_{gs}}$	$\frac{C_{gd2}}{C_{gd}}$	$\frac{C_{in2}}{C_{in}}$	$\frac{A_{M2}}{A_M}$
$\frac{1}{2}$	$\sqrt{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	$\frac{1}{2}$	0.4	0.7

$\frac{f_{H2}}{f_H}$	$\frac{GB_2}{GB}$
2.49	1.74

8.28

$R_{sig} = 100^{K\Omega}$, $R_{in} = 100K\Omega$, $C_{gs} = 1pF$, $C_{gd} = 0.2pF$

$$A_M = -\frac{R_G}{R_G + R_{sig}} g_m (r_o \| R_D \| R_L)$$

Also $R_{in} = 100K\Omega = R_G$

$$A_M = \frac{-100}{100+100} 3(50^k \| 8K \| 10^k) = \underline{-6.1 V/V}$$

$$f_H = \frac{1}{2\pi C_{in} R'_{sig}} \quad \text{(Eq. 4.132)}$$

$R'_{sig} = R_{sig} \| R_G = 100 \| 100 = 50^{K\Omega}$

$C_{in} = C_{gs} + C_{gd}(1 + g_m R'_L)$

$R'_L = r_o \| R_D \| R_L = 4.1K\Omega$

$C_{in} = 1 + 0.2(1 + 3 \times 4.1) = 3.66pF$

Now we can calculate f_H:

$$f_H = \frac{1}{2\pi \times 3.66 \times 10^{-12} \times 50 \times 10^3} = \underline{870^{KHZ}}$$

In order to double f_H, we have to either decrease C_{in} (by reducing R_{out}) or reduce R'_{sig} by reducing R_{in}.

If we reduce $R_{out} = R_D \| r_o$:

$$\frac{f_{H2}}{f_{H1}} = \frac{C_{in1}}{C_{in2}} \Rightarrow 2 = \frac{3.66\,pF}{1 + 0.2(1 + 3 \times R'_L)}$$

$\Rightarrow R'_L = 1.27K\Omega \qquad R'_L = R_{out} \| R_L = R_{out} \| 10^k$

$\Rightarrow \underline{R_{out} = 1.45K\Omega}$

Therefore in order to double f_H to $870 \times 2 = 1.74^{MHZ}$, we have to reduce $R_{out} = r_o \| R_D$ to $1.45^{K\Omega}$ or equivalently reducing R_D to $1.5^{K\Omega}$. The new midband gain would be:

$$\frac{A_{M2}}{A_{M1}} = \frac{R'_{L2}}{R'_{L1}} \Rightarrow A_{M2} = -6.1 \times \frac{1.27}{4.1} = \underline{-1.9 V/V}$$

Gain is almost reduced by a Factor of $\underline{3}$.

If we reduce $R_{in} = R_G$:

$$\frac{f_{H2}}{f_{H1}} = \frac{R'_{sig1}}{R'_{sig2}} \Rightarrow 2 = \frac{50^K}{R'_{sig2}} \Rightarrow R'_{sig2} = 25^{K\Omega}$$

$\Rightarrow 25K\Omega = 100K \| R_G \Rightarrow \underline{R_G = 33^{K\Omega} = R_{in}}$

Therefore in order to double f_H, R_{in} is reduced by a Factor of 3, from $100K\Omega$ to $33K\Omega$.

The new midband gain would be:

$$\frac{A_{M2}}{A_{M1}} = \frac{R_{G2}}{R_{G1}} \cdot \frac{R_{G1} + R_{sig}}{R_{G2} + R_{sig}} \Rightarrow A_{M2} = -6.1 \times \frac{1}{3} \times \frac{100+100}{33+100}$$

$\underline{A_{M2} = -3.06 V/V}$

Gain is almost reduced by a factor of $\underline{2}$.

8.29

$R_{in} = 2M\Omega$, $g_m = 4mA/V$, $r_o = 100K\Omega$, $R_D = 10K\Omega$

$C_{gs} = 2PF$, $C_{gd} = 0.5pF$, $R_{sig} = 500K\Omega$, $R_L = 10K\Omega$

noting that $R_G = R_{in}$, we have:

$$A_M = -\frac{R_G}{R_G + R_{sig}} g_m (r_o \| R_D \| R_L) = -\frac{2 \times 4}{2 + 0.5}(100^K \| 10^K \| 10^K)$$

$\underline{A_M = -15.2 V/V}$

$$\therefore f_H = \frac{1}{2\pi C_{in} R'_{sig}} \quad \text{where}$$

$C_{in} = C_{gs} + C_{gd}(1 + g_m (r_o \| R_D \| R_L))$, $R'_{sig} = R_{sig} \| R_G$

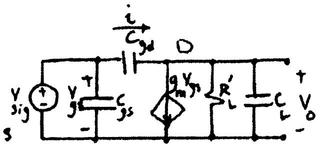
$C_{in} = 2 + 0.5(1 + 4 \times (100^K \| 10^K \| 10^K)) = \underline{12.02pF}$

$R'_{sig} = 0.5^M \| 2^{M\Omega} = 0.4M\Omega = 400K\Omega$

$$f_H = \frac{1}{2\pi \times 12.02 \times 10^{-12} \times 400 \times 10^3} = \underline{33.1^{KHZ}}$$

8.30

If we write KCL at node D:



$i = g_m V_{gs} + \frac{V_o}{R'_L} + V_o C_L s$

then: $V_{sig} = i \times \frac{1}{C_{gd}s} + v_o$

$V_{sig} = (g_m V_{gs} + \frac{V_o}{R'_L} + V_o C_L s)\frac{1}{C_{gd}s} + v_o$, $V_{sig} = V_{gs}$

$v_{sig}(1 - \frac{g_m}{C_{gd}s}) = v_o(1 + \frac{1}{R'_L C_{gd}s} + \frac{C_L s}{C_{gd}s})$

$\frac{v_o}{v_{sig}} = -g_m R'_L \frac{(1 - s(C_{gd}/g_m))}{R'_L C_{gd} s + 1 + R'_L C_L s}$

$\frac{v_o}{v_{sig}} = -g_m R'_L \frac{1 - s\, C_{gd}/g_m}{1 + s(C_L + C_{gd})R'_L}$

If $(g_m/C_{gd}) \gg \omega \Rightarrow \frac{v_o}{v_{sig}} = \frac{-g_m R'_L}{1 + s(C_L + C_{gd})R'_L}$

For $C_{gd} = 0.5\,pF$, $C_L = 2\,pF$, $g_m = 4\,mA/V$, $R'_L = 5\,k\Omega$

$\frac{V_o}{V_{sig}} = \frac{A_M}{1 + s/\omega_H} \Rightarrow \begin{cases} A_M = -g_m R'_L = -4 \times 5 = -20\,V/V \\ f_H = \frac{1}{2\pi(C_L + C_{gd})R'_L} \end{cases}$

$\Rightarrow f_H = \frac{10^{12}}{2\pi \times (2 + 0.5) \times 5 \times 10^3}$

$f_H = 12.7\,MHz$ $\qquad g_m/C_{gd} = \frac{4}{0.5} = 8\,^{GRad/s} \gg \omega_H$

8.31

If $g_m = 1\,\frac{mA}{V}$ and $r_o = 100\,k\Omega$:

$A_M = \frac{R_G}{R_G + R_{sig}} g_m (r_o \parallel R_D \parallel R_L)$ where

$R_G = 10\,M \parallel 47\,M$

$R_G = 8.25\,M\Omega$

$A_M = -\frac{8.25}{8.25 + 0.1} 1(100\,K \parallel 4.7\,K \parallel 10\,K)$

$= -3.06\,V/V$

$f_H = \frac{1}{2\pi C_{in} R'_{sig}}$ where

$R'_{sig} = R_{sig} \parallel R_G = 0.1\,M \parallel 8.25\,M\Omega$

$R'_{sig} \cong 0.1\,M\Omega$

$C_{in} = C_{gs} + C_{gd}(1 + g_m (r_o \parallel R_D \parallel R_L))$

$C_{in} = 1 + 0.2(1 + 1(100\,K \parallel 4.7\,K \parallel 10\,K))$

$= 1.82\,pF$

$f_H = \frac{1}{2\pi \times 1.82 \times 10^{-12} \times 0.1 \times 10^6}$

$= 875\,KHz$

8.32

$I = 2\,mA, \beta = 100, f_T = 800\,MHz$

$R_B = 50\,k\Omega, R_C = 4\,k\Omega, r_x = 50\,\Omega$

$V_A = 100, C_\mu = 1\,pF, R_{sig} = 5\,k\Omega$

$R_L = 5\,k\Omega$

$g_m = \frac{2\,mA}{25\,mV} = 80\,\frac{mA}{V}$

$r_\pi = \frac{\beta_O}{g_m} = \frac{100}{80\,m} = 1250\,\Omega$

$r_o = \frac{V_A}{I_C} = \frac{100\,V}{2\,mA} = 50\,k\Omega$

$C_\pi + C_\mu = \frac{g_m}{\omega_T} = \frac{80 \times 10^{-3}}{2\pi \times 800 \times 10^6} = 16\,pF$

$C_\mu = 1\,pF \Rightarrow C_\pi = 15\,pF$

$A_M = \frac{-R_B}{R_B + R_{sig}} \cdot \frac{r_\pi \times g_m R_L'}{(r_\pi + r_x + (R_B \parallel R_{sig}))}$

where $R'_L = r_o \parallel R_C \parallel R_L$

$= (50 \parallel 4 \parallel 5)\,k\Omega$

$= 2.1\,k\Omega$

$A_M = -\frac{50}{50 + 5} \cdot \frac{1250 \times 168}{1250 + 50 + (50 \parallel 5)\,k\Omega}$

where $168 = g_m \times R'_L$

$= 80 \times 10^{-3} \times 2.1(10^3)$

Then: $A_M = -32.6$

$20 \log|A_M| = 30.3\,dB$

$C_{in} = C_\pi + C_\mu(1 + g_m R'_L)$

$= 15 + 1(1 + 168) = 184\,pF$

$R'_{sig} = r_\pi \parallel [r_x + (R_B \parallel R_{sig})]$

$= 1250 \parallel [50 + (50\,K \parallel 5\,K)] = 983\,\Omega$

$f_H = \frac{1}{2\pi C_{in} R'_{sig}} = \frac{1}{2\pi \times 184 \times 10^{-12} \times 983}$

$= 880\,KHz$

Gain-bandwidth product

$GB = |A_M| \times f_H = 32.6 \times 880 \times 10^3$

$= 29 \times 10^6$

Previously,

$GB = 39 \times 754 \times 10^3 = 29 \times 10^6$

Thus, the designer traded gain for bandwidth by increasing I. However, by doubling I the dissipation increased by a factor of 2, since:

$\text{Power} = \frac{I}{2I'} \times \text{Vsupply}$

8.33

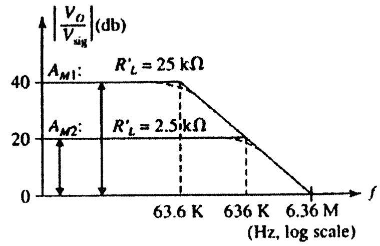
$R_B >> R_{sig}$, $r_X << R_{sig}$

$R_{sig} >> r_\pi$, $g_m R_L' >> 1$,

$g_m R_L' C_\mu >> C\pi$

$$f_H = \frac{1}{2\pi \times 10^{-12} \times 100 \times 2.5(10^3)}$$

f_H = 636 KHz



$GP = 6.36 \times 10^6 = A_M \times f_H$

when $A_M = 1 \Rightarrow f_H = 6.36 \cdot 10^6$ Hz

$$R_L' = \frac{1}{2\pi(6.36 \times 10^6)C_\mu \times \beta}$$

$C_\mu = 1 \times 10^{-12}$

$\beta = 100$

$R_L' = 250\ \Omega$

8.34

$R_{in} = R_1 \parallel R_2 \parallel r_\pi$

where $r_\pi = \frac{\beta}{g_m}$ and $g_m = \frac{I_C}{V_T}$

$g_m = \frac{0.8}{0.025} = 32$ mA / V

$r_\pi = \frac{200}{32} = 6.25$ kΩ

$R_{in} = 68 \parallel 27 \parallel 6.25 = 4.72$ kΩ

$R_L' = R_C \parallel R_L = 4.7 \parallel 10 = 3.2$ kΩ

$$A_M = \frac{R_{in}}{R_S + R_{in}} \times -g_m R_L'$$

$$= \frac{-4.72}{10 + 4.72} \times 32 \times 3.2$$

$= -32.8$ V / V

$C_T = C\pi + C\mu\ (1 + g_m R_L')$

where $C\pi + C\mu = \frac{g_m}{2\pi f_T} = \frac{32 \times 10^{-3}}{2\pi \times 10^9}$

$= 5.1$ pF

$C\pi = 5.1 - 0.8 = 4.3$ pF

$C_T = 4.3 + 0.8\ (1 + 32 \times 3.2)$

$= 87$ pF

The resistance seen by C_T is R_T.

$R_T = r_\pi \parallel R_1 \parallel R_2 \parallel R_3$

$= 6.25 \parallel 68 \parallel 27 \parallel 10 = 3.2$ kΩ

Thus

$$f_H = \frac{1}{2\pi C_T R_T}$$

$$= \frac{1}{2\pi \times 87 \times 10^{-12} \times 3.2 \times 10^3}$$

$= 572$ KHz

8.35

$I_C \simeq 2$ mA, $f_T = 2$ GHz, $C\mu = 1$ pF,

$r_x = 100\ \Omega$, $\beta_O = 120$ $R_{sig} \simeq 0$

$$g_m = \frac{I_C}{V_T} = \frac{2}{25} = 80\ \frac{\text{mA}}{\text{V}}$$

$$r_\pi = \frac{\beta_O}{g_m} = \frac{120}{0.08} = 1.5\ \text{k}\Omega$$

$$f_\pi = \frac{g_m}{2\pi(C_\pi + C\mu)} \Rightarrow 2 \times 10^9$$

$$= \frac{0.08 \times 10^{12}}{2\pi(C_\pi + 1)} \Rightarrow C_\pi = 5.4\ \text{pF}$$

a) If $A_M = -10$ V / V

If $R_{sig} \simeq 0$: $A_M = \frac{-r_\pi}{r_\pi + r_x} \cdot g_m \cdot R_L'$

$-10 = \frac{-1.5}{1.5 + 0.1} \cdot 0.08 \times R_L' = 133.3\ \Omega.$

$C_{in} = C_\pi + C\mu(1 + g_m R_L')$

$= 5.4 + 1(1 + 0.08 \times 133.3)$

$= 17$ pF

Thus $f_H = \frac{1}{2\pi \cdot C_{in}\ (R_L')}$

$$= \frac{1}{2\pi \cdot 17 \times 10^{-12} \times 133.3} = 70.2\ \text{MHz}$$

b) $A_M = (-1$ V / V)

$\Rightarrow R_L' = 13.3\ \Omega$

$C_{in} = 7.5$ pF

$f_H = 1.6$ GHz

8.36

$$g_m = \frac{0.5\ \text{mA}}{25\ \text{mV}} = 20\ \text{mA / V}$$

$$r_\pi = \frac{\beta_O}{g_m} = \frac{100}{20\ \text{mA / V}} = 5\ \text{k}\Omega$$

$$\frac{V_A}{I_C} = \frac{100\ \text{V}}{0.5\ \text{mA}} = 200\ \text{k}\Omega$$

$$C\pi + C\mu = \frac{g_m}{\omega_T} = \frac{20 \times 10^{-3}}{2\pi \times 800 \times 10^6} = 4\ \text{pF}$$

Since $C\mu = 1$ pF $\rightarrow C\pi = 3$ pF

The midband voltage gain is:

$$A_M = \frac{-R_B}{R_B + R_{sig}} \cdot \frac{r_\pi}{r_\pi + r_x + (R_B \| R_{sig})} \cdot g_m \cdot R_L'$$

with $R_L' = r_O \| R_c \| R_L$

$= (200 \| 8 \| 5)$ kΩ $= 3$ kΩ

Then $g_m R_L' = 20 \times 3 = 60$ V/V

$$A_M = \frac{-100}{100+5} \cdot \frac{5}{5 + 0.05 + (100 \| 5)} \times 60$$

$= -29.1$ V/V or 29.3 dB

To determine f_H:

$C_{in} = C_\pi + C\mu\,(1 + g_m R_L')$

$= 4 + 1 \cdot (1 + 60) = 65$ pF

$R_{sig}' = r_\pi \| [r_x + (R_B \| R_{sig})]$

$= 5 \| [0.05 + (100 \| 5)] = 2.45$ kΩ

Thus, $f_H = \dfrac{1}{2\pi \cdot C_{in} \cdot R_{sig}'}$

$$= \frac{1}{2\pi \times 65 \times 10^{-12} \times 2.45 \times 10^3} = 999 \text{ KHz}$$

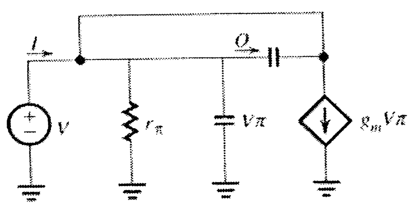
$GB = 29.1 \times 999 \times 10^3 = 29.1$ MHz

In example 9.4: $A_M = -39$ V/V,

$f_H = 754$ KHz $\rightarrow$ GB $= 29.4$ MHz

Notice how operation at lower supply voltage, thus I_C reduced the mid-band gain, increased the f_H while keeping the gain-band width product constant.

8.37



$$I = \frac{V}{r_\pi} + sC\pi\,V\pi + g_m\,V\pi$$

$$Y_{in} = \left(g_m + \frac{1}{r_\pi}\right) + sC\pi$$

$$Z_i = \frac{1}{\left(g_m + \frac{1}{r_\pi}\right) + sC\pi}$$

$$= \frac{1}{\frac{1}{re} + sC\pi} = \frac{re}{1 + sC\pi r_e}$$

$$f_T = \frac{g_m}{2\pi(C\pi + C\mu)}$$

Since $C\pi$ contains a component that is proportional to the bias current, it follows that at high currents $C\pi >> C\mu$ and

$$f_T \simeq \frac{g_m}{2\pi C\pi} \simeq \frac{1}{2\pi \cdot C\pi\, r_e}$$

Thus,

$Z_i = \dfrac{re}{1 + s/\omega_T}$ (at high currents)

The phase angle will be –45° at $\omega = \omega_T$, or

$f = f_T = 400$ MHz

For a lower bias current so that $C\pi = C\mu$,

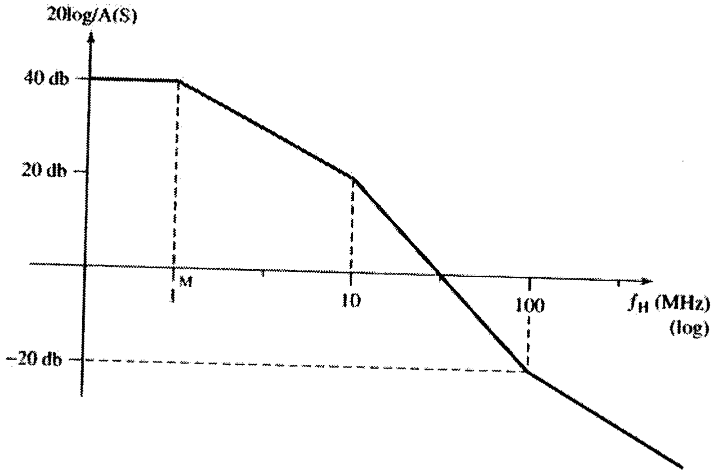
$f_T = \dfrac{1}{4\pi C\pi r_e}$ and $Z_i = \dfrac{r_e}{1 + \dfrac{s}{2\omega_T}}$

-45° angle is obtained at $\omega = 2\omega_T$ or

$f = 2f_T = 800$ MHz

(Assuming f_T remains constant which is not necessarily true)

This figure is for 8.38



8.38

$40 \text{ db} = 20\log A_O \Rightarrow A_O = 100 \text{ V/V}$

$$A(s) = +100\frac{(1 + s/100 \times 10^6 \times 2\pi)}{\left(1 + \frac{s}{2\pi \times 10^7}\right)\left(1 + \frac{s}{2\pi \times 10^6}\right)}$$

$$A(s) = +100\frac{\left(1 + \frac{s}{2\pi \times 10^8}\right)}{\left(1 + \frac{s}{2\pi \times 10^7}\right)\left(1 + \frac{s}{2\pi \times 10^6}\right)}$$

$$\omega_H = \frac{1}{\sqrt{\left(\left(\frac{1}{2\pi \times 10^7}\right)^2 + \left(\frac{1}{2\pi \times 10^6}\right)^2 - 2\left(\frac{1}{2\pi \times 10^8}\right)^2\right)}}$$

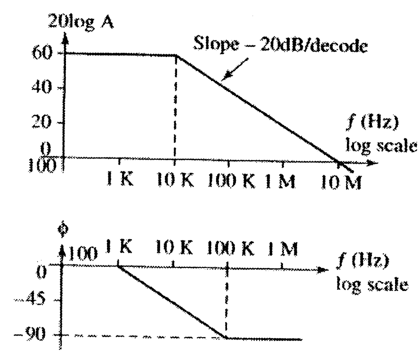
$f_H = 0.995 \text{ MHz}$

8.39

a) Gain $A = 60 \text{ dB} = 1000$

$$A(s) = \frac{1000}{\left(1 + \frac{1}{2\pi \times 10 \times 10^3}\right)} = \frac{1000}{\left(1 + \frac{1}{2\pi \times 10^4}\right)}$$

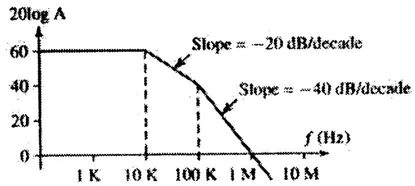
(b)



(c) Gain-bandwidth product = 1000 × 10 K
= 10 MHz

(d) From the gain plot, unity gain frequency = 10 MHz

(e) From the plot unity gain frequency = 1 MHz



8.40

$$f_H(s) = \frac{1}{\left(1+\frac{s}{\omega_{P1}}\right)\left(1+\frac{s}{\omega_{P2}}\right)}$$

$\omega_{P1} < \omega_{P2}$

using dominant-pole approximator: $\omega_H \simeq \omega_{P1}$

using the root sum of squares formula:

$$\omega_H \simeq \frac{1}{\sqrt{\frac{1}{\omega_{P1}^2}+\frac{1}{\omega_{P2}^2}}} = \frac{\omega_{P1}}{\sqrt{1+\left(\frac{\omega_{P1}}{\omega_{P2}}\right)^2}}$$

The difference between the two estimates for ω_H is:

$$\Delta\omega_H = \omega_{P1} - \frac{\omega_{P1}}{\sqrt{1+\left(\frac{\omega_{P1}}{\omega_{P2}}\right)^2}}$$

$$\text{If } n = \frac{\omega_{P2}}{\omega_{P1}};\ \frac{\Delta\omega_H}{\omega_{P1}} = 1 - \frac{1}{\sqrt{1+\frac{1}{n^2}}}$$

for $\frac{\Delta\omega_H}{\omega_{P1}} = 10\% = 0.1 \Rightarrow n = 2.07$

for $\frac{\Delta\omega_H}{\omega_{P1}} = 1\% = 0.01 \Rightarrow n = 7.02$

8.41

$$A(s) = -100\,\frac{1+s/10^6}{(1+s/10^5)(1+s/10^7)}$$

a) $\omega_H \simeq 10^5$ rad/s

b) $\omega_H \simeq \frac{1}{\sqrt{\left(\frac{1}{10^5}\right)^2+\left(\frac{1}{10^7}\right)^2-2\left(\frac{1}{10^6}\right)^2}} = 101$ Krad/s

IF the pole at 10^6 rad/s is lowered to 10^5 rad/s, the transfer function becomes:

$$A(s) = \frac{-100}{1+s/10^7} \Rightarrow f_H = \frac{10^7}{2\pi}\ \text{Hz}$$

8.42

$$30^\circ = 3\tan^{-1}\frac{\omega}{\omega_p} = 3\tan^{-1}\frac{10^6}{\omega_p} \Rightarrow \omega_p = 5.67\times10^6\ \text{rad/s}$$

8.43

$$\omega_H \simeq \frac{1}{\tau_{gs}+\tau_{gd}} = \frac{1}{C_{gs}R_{gs}+C_{gd}R_{gd}}$$

$$\omega_H \simeq \frac{1}{C_{gs}R' + C_{gd}(R'+R_L'+g_m R_L' R')}$$ (From Example 6.6)

For $C_{gs}=C_{gd}=1$ PF, $R_L' = 3.33$ KΩ, $g_m = 4$ mA/V

$$\omega_H = \frac{1}{10^{-12}R' + 10^{-12}(R' + 3.33\times10^3 + 4\times3.33\times R')}$$

To obtain $\omega_H = 2\pi\times150\times10^3$

$$2\pi\times150\times10^3 = \frac{10^{12}}{3.33\times10^3 + 15.32R'} \Rightarrow R' = 69.04\,\text{K}\Omega$$

$R' = R \,//\, R_{in} = R \,//\, 420^K = 69.04 \Rightarrow \underline{R = 82.6\,\text{K}\Omega}$

8.44

$\tau_H = \tau_{gs} + \tau_{gd}$

where: $\tau_{gs} = C_{gs}\cdot R_{gs} = C_{gs}(R_G \| R_{sig})$

$\tau_{gd} = C_{gd}\cdot R_{gd} = C_{gd}(R'_{sig} + R'_L + g_m R'_L \cdot R'_{sig})$

since $R'_{sig} = R_G \| R_{sig}$

$$\tau_{gd} = C_{gd}\cdot(R_G \| R_{sig})\left[1 + \frac{R'_L}{(R_G \| R_{sig})} + g_m R'_L\right]$$

$$\Rightarrow \tau_H = (R_G \| R_{sig})\left[C_{gs} + C_{gd}\left[1 + g_m R'_L + \frac{R'_L}{R_G \| R_{sig}}\right]\right]$$

$$\tau_H = \frac{R_G\cdot R_{sig}}{R_G + R_{sig}}\cdot\left[C_{gs} + C_{gd}\left[1 + g_m R'_L + R'_L\frac{(R_G + R_{sig})}{R_G\cdot R_{sig}}\right]\right]$$

b) $\tau = C_{in} \cdot R'_{sig}$ where $R'_{sig} = R_G \| R_{sig}$ and $C_{in} = C_{gs} + C_{eq}$ where C_{eq} is the result of applying miller's thearem to reflect C_{gd} to the gate-ground nodes.

From Eq 9.76

$$Z_{eq} = \frac{Z}{1-K} = \frac{1/s\,C_{gd}}{1-(-g_m R'_L)}$$

$$= \frac{1}{sC_{gd}(1 + g_m R'_L)}$$

$$\Rightarrow C_{eq} = C_{gd}(1 + g_m R'_L)$$

Thus $\tau = (C_{gs} + C_{gd}(1 + g_m R'_L)) \cdot \dfrac{R_G \cdot R_{sig}}{R_G + R_{sig}}$

Evalvating for: $R_G = 420\ \text{k}\Omega$

$C_{gs} = C_{gd} = 1\ \text{pF}$ $R'_L = 3.33\ \text{k}\Omega$

$R_{sig} = 100\ \text{k}\Omega$ $g_m = 4\ \text{mA/V}$

(1) For the complete expression found in part a)

$\tau_H = 1.230\ \mu\text{S} \rightarrow \omega_H = 813\ \text{K rad/s}$

(2) For the approximate expression found in part b) $\tau = 1.228\ \mu\text{S} \rightarrow \omega_H = 814\ \text{K rad/S}$

$(\tau_H - \tau) \times 100/\tau = 0.163\ \%$

8.45

If a capacitor C_L is connected in parallel with R'_L then $\omega_H \cong \dfrac{1}{\tau_{gs} + \tau_{gd} + R'_L \cdot C_L}$

the values of τ_{gs} and τ_{gd} remain unaffected since each is derived by setting the other capacitors to zero. When considering the resistances seen by C_L, $C_{gs} = C_{gd} = 0$ and $V_{gs} = 0 \Rightarrow$ the open-circuit time constant of C_L is: $R'_L \cdot C_L$

$$\Rightarrow \omega_H = \frac{1}{(80.8 + 1160) \times 10^{-9} + 3.33 \times 10^{3} \cdot 20 \times 10^{-12}}$$

$\omega_H = 765\ \text{K rad/s}$

$f_H = \omega_H/2\pi = 121.7\ \text{MHz}$

8.46

$$A_M = \frac{V_o}{V_i} = -\frac{R_{in}}{R_{in}+R}\, g_m R'_L = -\frac{1.2}{1.2+0.1}(2\times 12)$$

$A_M = -22.2\ \text{V/V}$

$R_{gs} = R_{in} \| R = 1.2\text{M} \| 0.1\text{M} = 92.3\ \text{k}\Omega$

$\tau_{gs} = R_{gs} C_{gs} = 1\times 10^{-12} \times 92.3 \times 10^{3} = 92.3\ \text{ns}$

$R_{gd} = R' + R'_L + g_m R'_L R'$ where $R' = R_{in} \| R = 92.3\ \text{k}\Omega$

$R_{gd} = 92.3 + 12 + 2\times 12 \times 92.3 = 2.32\ \text{M}\Omega$

$\tau_{gd} = C_{gd} R_{gd} = 1\times 10^{-12} \times 2.32 \times 10^{6} = 2320\ \text{ns}$

$$\omega_H = \frac{1}{\tau_{gs} + \tau_{gd}} = \frac{1}{(92.3 + 2320)\times 10^{-9}} = 414.5\ \text{krad/s}$$

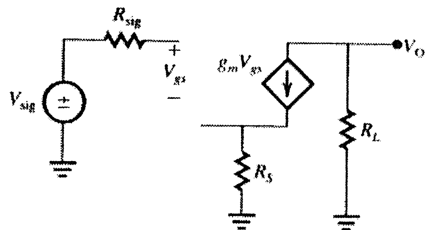
$f_H = 66\ \text{KHz}$

8.47

a) $V_O = -g_m R_L V_{gs}$ (1)

$V_{gs} = V_{sig} - R_S \times g_m V_{gs}$

$V_{gs}(1 + g_m R_S) = V_{sig}$



$$(1) \Rightarrow v_O = \frac{-g_m R_L}{1 + g_m R_S} v_{sig} \Rightarrow \frac{v_O}{v_{sig}} \Rightarrow \frac{-g_m R_L}{1 + g_m R_S}$$

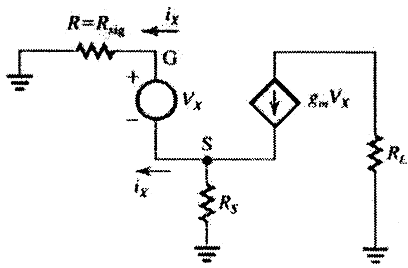
b) $V_S = (g_m V_X - i_X) R_S$

$V_G = V_S + V_X = (g_m V_X - i_X) R_S + V_X$

$$\Rightarrow i_X = \frac{V_G}{R} = \frac{(1 + g_m R_S)}{R} V_X - i_X \frac{R_S}{R}$$

$$R_{gs} = \frac{v_X}{i_X} = \frac{1 + R_S/R}{\dfrac{1 + g_m R_S}{R}} = \frac{R + R_S}{1 + g_m R_S}$$

(R is R_{sig})

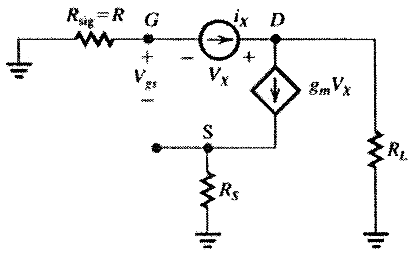


to calculate R_{gd}:

$V_G = -Ri_X$

$\left.\begin{aligned} V_S &= -Ri_X - V_{gs} \\ V_S &= R_S \times g_m V_{gs} \end{aligned}\right\}$

$$R_S g_m V_{gs} = -Ri_X - V_{gs} \Rightarrow V_{gs} = \frac{-Ri_X}{1 + g_m R_S}$$



At D: $i_X = g_m V_{gs} + \dfrac{V_X - R_X i_X}{R_L}$

substitute V_{gs} : $i_X = -\dfrac{g_m R i_X}{1 + g_m R_S} + \dfrac{V_X}{R_L} - \dfrac{R i_X}{R_L}$

$$i_X\left[1 + \frac{g_m R}{1 + g_m R_S} + \frac{R}{R_L}\right] = \frac{V_X}{R_L}$$

$$R_{gd} = \frac{V_X}{i_X} = R_L + R + \frac{g_m R R_L}{1 + g_m R_S} \quad (R \text{ is } R_{sig})$$

c) $R_S = 0$:

$$\frac{v_O}{v_{sig}} = \frac{-4 \times 5\text{ K}}{1 + 4 \times O} = -20\text{V/V}$$

$R_{gs} = R_{sig} = 100\text{ k}\Omega$

$R_{gs} = 5\text{ K} + 100\text{ K} + 4 \times 5 \times 100 = 2105\text{ k}\Omega$

$$\omega_H \simeq \frac{1}{C_{gs}R_{gs} + C_{gd}R_{gd}}$$

$$= \frac{1}{10^{-12} \times 100 \times 10^3 + 10^{-12} \times 2105 \times 10^3}$$

$\omega_H \simeq 453.5$ K rad/s

$|\text{Gain}| \times \text{Bandwidth} = 20 \times 453.5$

$= 9.07$ M rad/s

$R_S = 100\ \Omega$:

$$\frac{v_O}{v_{sig}} = \frac{-4 \times 5}{1 + 4 \times 0.1} = -14.3\text{ V/V}$$

$$R_{gs} = \frac{100 + 0.1}{1 + 4 \times 0.1} = 71.5\text{ k}\Omega$$

$$R_{gd} = 5 + 100 + \frac{4 \times 5 \times 100}{1 + 4 \times 0.1} = 1533.6\text{ k}\Omega$$

$$\omega_H = \frac{1}{10^{-12} \times 71.5 \times 10^3 + 10^{-12} \times 1533.6 \times 10^3}$$

$= 623$ K rad/s

$|\text{Gain}| \times \text{Bandwidth}$

$= 14.3 \times 623\text{ K} = 8.91$ M rad/s

$R_S = 250\ \Omega$:

$$\frac{v_O}{v_{sig}} = \frac{-4 \times 5}{1 + 4 \times 0.25} = -10\text{ V/V}$$

$$R_{gs} = \frac{100 + 0.25}{1 + 4 \times 0.25} = 50.1\text{ k}\Omega$$

$$R_{gd} = 5 + 100 + \frac{4 \times 5 \times 100}{1 + 4 \times 0.25} = 1105\text{ k}\Omega$$

$$\omega_H = \frac{1}{10^{-12} \times 50.1 \times 10^3 + 10^{-12} \times 1105 \times 10^3}$$

$= 865.7$ K rad/s

$|\text{gain}| \times \text{Bandwidth} = 10 \times 865.7\text{ K}$

$= 8.66$ M rad/s

Summary table:

$R_S(\Omega)$	Gain (V/υ)	W (K rad/s)	Gain.BW product (M rad/s)
0	–20	453.5	9.07
100	–14.3	623.0	8.91
250	–10	865.7	8.66

The Gain × Bandwidth is approximately constant.

8.48

$A_M = \frac{V_o}{V_{sig}} = -\frac{R_{in}}{R_{in}+R_{sig}}(g_m R_L') = -\frac{5}{5+1}(0.3\times 100^k)$

$A_M = -25\,V/V$ Now refer to Example 6.6.

$R_{gs} = R_{in} \| R_{sig} = 5^{M\Omega} \| 1^{M\Omega} = 0.83^{M\Omega}$

$\tau_{gs} = R_{gs} C_{gs} = 0.2\times 10^{-12} \times 0.83\times 10^{6} = 166.7\,ns$

$R_{gd} = R' + R_L' + g_m R_L' R'$

$R' = R_{in} \| R_{sig} = 0.83\,M\Omega$ $\Big\} \Rightarrow R_{gd} = 0.83 + 0.1 + 0.83 \times 0.3 \times 100$

$R_{gd} = 25.92\,M\Omega$

$\tau_{gd} = C_{gd} R_{gd} = 25.92\times 10^{6} \times 0.1\times 10^{-12} = 2592\,ns$

$\omega_H \simeq \frac{1}{\tau_{gs} + \tau_{gd}} = \frac{1}{(166.7 + 2592)\times 10^{-9}} = 362.5^{Krad/s}$

$f_H = 57.7^{KHz}$

8.49

If we assume that capacitors are perfect open circuits for midband, then:

$A_M = \frac{V_o}{V_{sig}} = \frac{-R_{in}}{R_{in}+R_{sig}}(g_m R_L') = \frac{-650}{650+150}(5\times 10) = 40.6\,V/V$

$\tau_{gs} = C_{gs} R_{gs} = C_{gs}(R_{in} \| R_{sig}) = 2^P \times (150^K \| 650K)$

$\tau_{gs} = 243.75\,ns$

$\tau_{gd} = C_{gd} R_{gd}$, Refer to Example 6.6

$R_{gd} = R' + R_L' + g_m R_L' R'$

$R' = 150^K \| 650K = 121.9\,K\Omega$ $\Rightarrow R_{gd} = 121.9 + 10 + 5\times 10 \times 121.9$

$R_{gd} = 6.2\,M\Omega$

$\tau_{gd} = C_{gd} R_{gd} = 0.5^P \times 6.2^M = 3100\,ns$

$\tau_L = R_L' C_L = 10^k \times 3^P = 30\,ns$

$\omega_H = \frac{1}{\tau_{gs} + \tau_{gd} + \tau_L} = \frac{1}{30^n + 3100^n + 243.75^n} = 296.4\,Krad/s$

$f_H = 47.2\,KHz$

8.50

$R_{in} = \frac{R}{1-Gain} = \frac{100}{1-0.95} = 2000\,K\Omega = 2\,M\Omega$

8.51

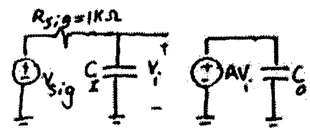
$Z_I = Z/(1-K) \Rightarrow C_I = 0.1\times(1-(-1000))$

$\Rightarrow C_I = 100.1\,pF$

$C_o = 0.1\times\left(\frac{-1}{1000}+1\right)$

$C_o = 99.9\,fF$

(using Miller's Theorem)



$V_o = AV_i = A \times V_{sig}\,\frac{1/C_I s}{R_{sig} + \frac{1}{C_I s}} \Rightarrow \frac{V_o}{V_{sig}} = \frac{A}{1 + C_I R_{sig} s}$

$\omega_H = \frac{1}{C_I R_{sig}} = \frac{1}{100.1^P \times 1^K} = 9.99\,Mrad/s \Rightarrow f_H = 1.59\,MHz$

To calculate unity gain frequency:

$|Gain| = 1$

$\frac{V_o}{V_i} = \frac{A}{1 + C_I R_{sig} s} = \frac{-1000}{1 + 100.1\times 10^{-9} s}$ $(s = j\omega)$

$\frac{1000}{\sqrt{1 + (100.1\times 10^{-9} \times \omega_T)^2}} = 1 \Rightarrow \omega_T \simeq 10\,Grad/s$

$f_T \simeq 1.59\,GHz$

As we can see $f_T \simeq f_H \times A$

8.52

Using Miller's Theorem, in each case the capacitance at the input is C(1–A) and the capacitance at the output is $C\left(1-\frac{1}{A}\right)$.

Thus :

a) $A = -1000$ V/υ and $C = 1$ pF
$C_i = 1.001$ nF and $C_o = 1.001$ pF

b) $A = -10$ V/υ and $C = 10$ pF
$C_i = 110$ pF and $C_o = 11$ pF

c) $A = -1$ V/υ and $C = 10$ pF
$C_i = 20$ pF and $C_o = 20$ pF

d) $A = 1$ V/υ and $C = 10$ pF
$C_i = 0$ pF and $C_o = 0$ pF

e) $A = 10$ V/υ and $C = 10$ pF
$C_i = -90$ pF and $C_o = 9$ pF

In (e) the negative capacitance at the input can be used to cancel the effect of the input capacitance of the amplifier.

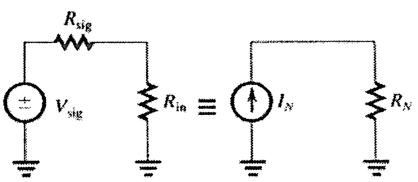
8.53

a) $R_{in} = \frac{R}{1-A} = \frac{R}{1-2} = -R$

(Miller's theorem)

b) $I_N = \frac{V_{sig}}{R_{sig}}$

$R_N = R_{sig} \| R_{in}$

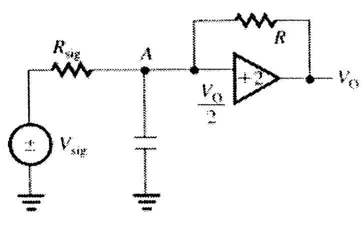


If $R_{sig} = R$ then :

$R_N = R \| (-R) = \infty \Rightarrow I_L = I_N$

$= \frac{V_{sig}}{R_{sig}} = \frac{V_{sig}}{R}$

c)



KCL at A:

$$\frac{\frac{V_O}{2} - V_{sig}}{R_{sig}} + \frac{v_O}{2} \times Cs + \frac{-v_O}{2R} = 0$$

If $R_{sig} = R \Rightarrow \frac{+v_{sig}}{R} = \frac{v_O}{2}Cs \Rightarrow \frac{v_O}{v_{sig}} = \frac{2}{RCs}$

8.54

$A_M = -g_m R'_L = -5 \times 20 = -100$ V/V

$C_{in} = C_{gs} + C_{gd}(1 + g_m R'_L)$ (Eq. 6.55)

$C_{in} = 2 + 0.1(1 + 5 \times 20) = 12.1$ pF

$f_H \simeq \frac{1}{2\pi C_{in} R_{sig}}$ (Eq. 6.54)

$f_H \simeq \frac{1}{2\pi \times 12.1 \times 10^{-12} \times 20^k} = 658$ KHz

8.55

$\tau_H = C_{gs} R_{gs} + C_{gd} R_{gd} + C_L R_{CL}$

$\tau_H = C_{gs} R_{sig} + C_{gd}\left[R_{sig}(1 + g_m R'_L) + R'_L\right] + C_L R'_L$

$\tau_H = 2^p \times 20^k + 0.1^p\left[20^k(1 + 5 \times 20) + 20^k\right] + 1^p \times 20^k$

$\tau_H = 264$ ns

$f_H \simeq \frac{1}{2\pi \tau_H} = 603$ KHz

$A_M = -g_m R'_L = -5 \times 20 = -100$ V/V

τ_{gs} : 15.1%
τ_{gd} : 77.3%
τ_L : 7.6%

Contribution of each time-constant to the overall τ_H.

If we compare f_H to the one obtained in Problem 6.66, we notice that Problem 6.66 has a larger f_H due to neglecting the time constants of C_L and C_{gs}.

8.56

we have: $\omega_z = g_m / C_{gd}$

$\Rightarrow f_z = \frac{g_m}{2\pi C_{gd}} = \frac{5^m}{2\pi \times 0.1^p} = 7.96\,GHz$

f_{p1} and f_{p2} are the poles of the transfer function of equation (6.60), whose denominator is a quadratic polynomial with coefficient of s:

$= [C_{gs} + C_{gd}(1 + g_m R'_L)] R_{sig} + (C_L + C_{gd}) R'_L$
$= [2 + 0.1(1 + 5\times 20)]20 + (1 + 0.1)\times 20$
$= 264\,ns = 264\times 10^{-9}\,sec$

Coefficient of s^2:

$= [(C_L + C_{gd})C_{gs} + C_L C_{gd}] R_{sig} R'_L$
$= [(1+0.1)2 + 1\times 0.1]\, 20^K \times 20^K =$
$= 920\times 10^{-18} (sec)^2$

Therefore the quadratic equation is:

$1 + 264\times 10^{-9} s + 920\times 10^{-18} s^2 = 0$

Denoting the frequencies of the roots of this equation with ω_{p1} and ω_{p2}, we have:

$\omega_{p1} = 3.84\times 10^6\,rad/s \Rightarrow f_{p1} = \frac{\omega_{p1}}{2\pi} = 611.15\,KHz$

$\omega_{p2} = 283.12\times 10^6\,rad/s \Rightarrow f_{p2} = \frac{\omega_{p2}}{2\pi} = 45.06\,MHz$

Since $f_{p1} \ll f_{p2}$ and $f_{p1} \ll f_z$, a good estimate for f_H is f_{p1}:

$f_H \simeq f_{p1} = 611.15\,KHz$

Approximate value of f_{p1} obtained using (Eq. 6.66) is:

$$f_{p1} \cong \frac{1}{2\pi[(C_{gs} + C_{gd}(1 + g_m R'_L))R_{sig} + (C_L + C_{gd})R'_L]}$$

$f_{p1} \simeq 603.16\,KHz$

Approximate value of f_{p2} obtained using (Eq. 6.67) is:

$$f_{p2} = \frac{[C_{gs} + C_{gd}(1 + g_m R'_L)]R_{sig} + (C_L + C_{gd})R'_L}{2\pi[(C_L + C_{gd})C_{gs} + C_L C_{gd}] R'_L R_{sig}}$$

$f_{p2} = 45.67\,MHz$

The estimate of f_{p1} using Eq. 6.66 is 1.3% lower than the exact value, while the estimate of f_{p2} is about 1.3% higher than its exact value.

8.57

$R'_L = 5K\Omega$:

$A_M = -g_m R'_L = -5\times 5 = -25\,V/V$

$$f_{p1} \cong \frac{1}{2\pi[(C_{gs} + C_{gd}(1 + g_m R'_L))R_{sig} + (C_L + C_{gd})R'_L]}$$

$$f_{p1} = \frac{1}{2\pi[(2 + 0.1\times(1 + 5\times 5))20 + (1 + 0.1)\times 5]}$$

$f_{p1} = 1.63\,MHz$

$$f_{p2} = \frac{[C_{gs} + C_{gd}(1 + g_m R'_L)]R_{sig} + (C_L + C_{gd})R'_L}{[(C_L + C_{gd})C_{gs} + C_L C_{gd}] R'_L R_{sig} \times 2\pi}$$

$$f_{p2} = \frac{(2 + 0.1(1 + 5\times 5))20 + (1 + 0.1)5}{((1 + 0.1)\times 2 + 1\times 0.1)\,5\times 20\times 2\pi}$$

$f_{p2} = 67.5\,MHz$

$S_z = \frac{g_m}{C_{gd}} \Rightarrow f_z = \frac{g_m}{2\pi C_{gd}} = \frac{5^m}{2\pi 0.1^p} = 7.96\,GHz$

$f_{p1} \ll f_{p2}$ and $f_{p1} \ll f_z \Rightarrow f_{p1}$ is the dominant pole.

$f_H \simeq f_{P_1} = 1.63\ MHz$

$|Gain| \times Bandwidth = 25 \times 1.63 = 40.75\ MHz$

$f_t = |A_M| f_H = 40.75 MHz$

Since $f_{P_1} \ll f_{P_2}$ and $f_{P_1} \ll f_Z$, a dominant pole exists.

$R_L' = 10 K\Omega$:

$A_M = -5 \times 10 = -50 V/V$

$$f_{P1} = \frac{1}{2\pi[(2+0.1(1+5\times10))20 + (1+0.1)\times10]} = 1.04 MHz$$

$$f_{P2} = \frac{(2+0.1(1+5\times10))20 + (1+0.1)\times10}{[(1+0.1)2 + 1\times0.1]\,10\times20\times2\pi} = 52.96 MHz$$

$$f_Z = \frac{5}{2\pi\times0.1} = 7.96 GHz$$

$f_{P_1} \ll f_{P2}$, $f_{P_1} \ll f_Z$ $\Rightarrow f_{P_1}$ is the dominant pole and therefore $f_H \simeq f_{P_1} = 1.04 MHz$

$|A_M| \cdot f_H = 50 \times 1.04 = 52 MHz$

Since f_{P_2} is still slightly greater than $|A_M| \cdot f_H$, therfore: $f_T \simeq 52 MHz$

$R_L' = 20 K\Omega$:

$A_M = -5 \times 20 = -100 V/V$, from Problem 6.68 we have:

$f_{P_1} = 603.16\ KHz$

$f_{P_2} = 45.67\ MHz$

$f_Z = 7.96\ GHz$

Again $f_{P_1} \ll f_{P_2}$ and $f_{P_1} \ll f_Z$, therefore f_{P_1} is the dominant pole and f_H can be approximated by f_{P_1}. $f_H \simeq f_{P_1} = 603.16 KHz$

$|A_M| \cdot f_H = 60.32\ MHz$

Since $f_{P2} < |A_M| \cdot f_H$, therefore f_T is smaller than $|A_M| \cdot f_H$

The results are summarized in this table:

R_L'	$5^{K\Omega}$	$10^{K\Omega}$	$20^{K\Omega}$
$A_M (V/V)$	-25	-50	-100
$f_{P_1} (MHz)$	1.63	1.04	0.60
$\lvert A_M \rvert \cdot f_H (MHz)$	40.75	52.00	60.32

8.58

$$A_M = -\frac{r_\pi}{R_{sig} + r_x + r_\pi}(g_m R_L')$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{20} = 5K\Omega \Rightarrow A_M = -\frac{5}{1+0.2+5}(20\times5)$$

$A_M = 80.65 V/V$

Using Miller's Theorem and Eq. 6.71:

$C_{in} = C_\pi + C_\mu(1 + g_m R_L') = 10 + 0.5(1+20\times5) = 60.5\ pF$

Eq. 6.69: $R_{sig}' = r_\pi \| (R_{sig} + r_x) = 5^K \| (1^K + 0.2)$

$R_{sig}' = 0.97 K\Omega$

Eq. 6.72: $f_H \simeq \dfrac{1}{2\pi C_{in} R_{sig}'} = \dfrac{1}{2\pi \times 60.5 \times 0.97^K} \Rightarrow$

$f_H = 2.71 MHz$

8.59 $A_M = -139$ V/V

Using the method of open-circuit time constants, from equation 9.100:

$$\tau_H = C_\pi \cdot R_{sig}' + C_\mu[(1 + g_m R_L') \cdot R_{sig}' + R_L'] + C_L \cdot R_L'$$

$$R_{sig}' = r_\pi \| (R_{sig} + r_x)$$

$= 2.5\ k\Omega \| (1\ k\Omega + 0.1\ k\Omega) = 764\ \Omega$

$\tau_H = (10\ p \times 764) + 0.3\ p[(1 + 40 \times 5) \cdot 764 + 5\ K] + 3\ p \times 5\ K$

$\tau_H = 7.64$ ns + 47.57 ns + 15 ns = 70.21 ns

$$f_H = \frac{1}{2\pi\tau_H} \simeq 2.27\ MHz$$

The % contributions to τ_H of each capacitance are: $C\pi$: 10.8%, $C\mu$: 67.8%, C_L : 21.4%

f_H is 10.6% higher than the f_H obtained in this problem

8.60

we have: $r_\pi = 5\,k\Omega$ and

$A_M = -80.65\ V/V$, $R'_{sig} = 0.97\,k\Omega$

$$f_Z = \frac{g_m}{2\pi C_\mu} = \frac{20m}{2\pi \times 0.5p} = 6.37\,GHz$$

$$f_{P1} \simeq \frac{1}{2\pi[(C_\pi + C_\mu(1+g_m R'_L))R'_{sig} + (C_L + C_\mu)R'_L]}$$

$$f_{P1} \simeq \frac{1}{2\pi[(10 + 0.5(1+20\times5))0.97 + 2.5\times5]}$$

$$f_{P1} = 2.24\,MHz$$

$$f_{P2} = \frac{(C_\pi + C_\mu(1+g_m R'_L))R'_{sig} + (C_L + C_\mu)R'_L}{2\pi\,[C_\pi(C_L + C_\mu) + C_L C_\mu]\,R'_{sig} R'_L}$$

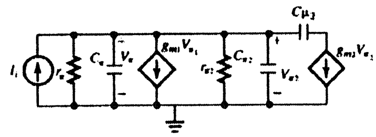
$$f_{P2} = \frac{(10 + 0.5(1+20\times5))0.97 + 2.5\times5}{2\pi\,(10(2+0.5) + 2\times0.5)0.97\times5}$$

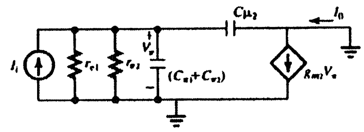
$$f_{P2} = 89.89\,MHz$$

Since $f_{P1} \ll f_{P2}$ and $f_{P1} \ll f_Z$, we can approximate f_H by f_{P1}: $f_H \simeq f_{P1} = 2.24\,MHz$

If we compare f_H to the results obtained from applying Miller's Theorem then our results are 17% lower.

8.61





$$V_\pi = \frac{I_i}{\left(\frac{1}{r_{e1}} + \frac{1}{r_{\pi2}}\right) + s(C_{\pi1} + C_{\pi2} + C_{\mu2})}$$

$$I_o = g_{m2}V_\pi - C_\mu s V_\pi$$

$$= \frac{(g_{m2} - C_\mu s)I_i}{\left(\frac{1}{r_{e1}} + \frac{1}{r_{\pi2}}\right) + s(C_{\pi1} + C_{\pi2} + C_{\mu2})}$$

$$\frac{I_o}{I_i} = \frac{g_{m2} - C_\mu s}{\left(\frac{1}{r_{e1}} + \frac{1}{r_{\pi2}}\right) + s(C_{\pi1} + C_{\pi2} + C_{\mu2})}$$

$I_{C1} = I_{C2} \Rightarrow r_{\pi1} = r_{\pi2}$,

$g_{m1} = g_{m2}$, $C_{\pi1} = C_{\pi2}$

$$\frac{I_O}{I_i} = \frac{g_m - C_\mu s}{\left(\frac{1}{r_e} + \frac{1}{r_\pi}\right) + (C_\mu + 2C_\pi)s}$$

$$= \frac{1 - \frac{C_\mu}{g_m}s}{\left(\frac{1}{g_m r_e} + \frac{1}{g_m r_\pi}\right) + s\frac{C_\mu + 2C_\pi}{g_m}}$$

$$g_m r_e = \frac{I_C V_T}{V_T I_E} = \alpha = \frac{\beta}{\beta+1}$$

$g_m r_\pi = \beta$

$$\Rightarrow \frac{I_O}{I_i} = \frac{1 - \frac{C_\mu}{g_m}s}{1 + \frac{1}{\beta} + \frac{1}{\beta} + s(2C_\pi + C_\mu)/g_m}$$

$$\frac{I_o}{I_i} = \frac{1}{1 + \frac{2}{\beta}}\ \frac{1 - s\frac{C_\mu}{g_m}}{1 + s\frac{(2C_\pi + C_\mu)}{g_m\left(1 + \frac{2}{\beta}\right)}}$$

If the circuit is biased at 1 mA and $\beta = \infty$, $f_t = 400$ MHz and $C_\mu = 2$ pF:

$$g_m = \frac{1}{0.025} = 40 \text{ mA/}v$$

$$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)} \Rightarrow C_\pi + C_\mu$$

$$= \frac{40 \text{ m}}{2\pi \times 400 \text{ M}} = 15.9 \text{ pF}$$

$C_\pi = 15.9 - 2 = 13.9$ pF

Pole frequency :

$$f_P = \frac{g_m}{2\pi(2C_\pi + C_\mu)} = \frac{40 \times 10^{-3}}{2\pi(2 \times 13.9 + 2)\text{pF}}$$

$f_P = 213.74$ MHz

Zero frequency:

$$f_Z = \frac{g_m}{2\pi C_\mu} = \frac{40 \text{ m}}{2\pi \times 2 \text{ p}} = 3.18 \text{ GHz}$$

8.62

$A_M = -g_m R'_L = -5 \times 20 = \underline{-100 \text{ V/V}}$

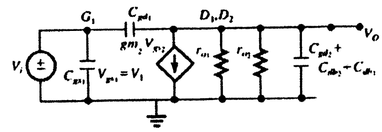
$$f_H = \frac{1}{2\pi(C_L + C_{gd})R'_L} = \frac{1}{2\pi(1+0.1)\times 20} = 7.23 \text{ MHz}.$$

(Note that in this case there is no R_{sig} and we used Eq. 6.79)

$f_{3dB} = f_H = \underline{7.23 \text{ MHz}}$

$f_t = |A_M| \cdot f_H = \underline{723 \text{ MHz}}$

8.63



$$g_m = \sqrt{2\mu_n C_{ox}\frac{W}{L}I_D} = \sqrt{2 \times 90 \times \frac{100}{1.6} \times 100}$$

$= 1060\ \mu$A / v

$g_m = 1.06$ mA/v

$$r_{O1} = \frac{V_{A1}}{I_{D1}} = \frac{12.8}{0.1} = 128 \text{ k}\Omega$$

$$r_{O2} = \frac{|V_{A2}|}{|I_{D2}|} = \frac{19.2}{0.1} = 192 \text{ k}\Omega$$

DC-gain $= -g_m(r_{O1} \parallel r_{O2})$

$= -1.06 \times (128 \parallel 192)$

$= -81.4$ V / V

Total capacitance between output node and ground

$= C_{gd2} + C_{db1} + C_{db2} = 0.015 + 0.020 + 0.036$

$C_L = 0.071$ pF

Write a KCL at output:

$$sC_{gd1}(v_i - v_O) = g_m v_i + \frac{v_O}{r_{O1}} + \frac{v_O}{r_{O2}} + v_O s C_L$$

$$\frac{v_O}{v_i} = \frac{g_m - sC_{gd1}}{\frac{1}{r_{O1}} + \frac{1}{r_{O2}} + (C_L + C_{gd1})s}$$

Thus:

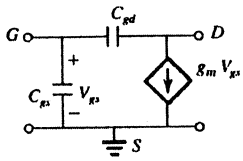
$$f_Z = \frac{g_m}{2\pi C_{gd1}} = \frac{1.06 \text{ m}}{2\pi \times 0.015 \text{ p}} = 11.3 \text{ GHz}$$

$$f_P = \frac{1}{2\pi}\frac{\frac{1}{r_{O1}} + \frac{1}{r_{O2}}}{C_L \times C_{gd1}} = \frac{\frac{1}{128 \text{ k}\Omega} + \frac{1}{192 \text{ k}\Omega}}{2\pi(0.071 + 0.015) \text{ p}}$$

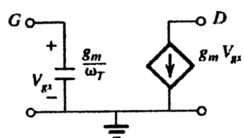
$f_P = 24.1$ MHz

8.64

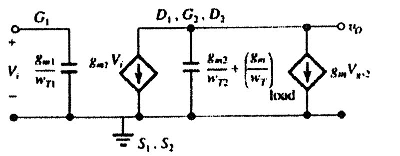
a) For small C_{gd} and low gain from G to D, we can neglect the Miller effect and C_{gd}.



$$\omega_T = \frac{g_m}{C_{gs} + C_{gd}} \simeq \frac{g_m}{C_{gs}} \text{ Thus } C_{gs} \simeq \frac{g_m}{\omega_T}$$



b) replace the controlled source $gm_2 v_{gs2}$ with a resistance $\frac{1}{g_{m2}}$. (source absorbtion theory)



$$V_o = -g_{m1} v_i \frac{1}{g_{m2} + s\left(\frac{g_{m2}}{\omega_{T2}} + \frac{gmload}{\omega_T load}\right)}$$

Since the load device is identical to Q_1,

$g_{mload} = g_{m1}$ and $\omega_{Tload} = \omega_{T1} = \omega_T$

Thus :

$$\frac{v_o}{v_i} = \frac{-g_{m1}/g_{m2}}{1 + \frac{s}{\omega_T}\left(1 - \frac{g_{m1}}{g_{m2}}\right)}$$

$$\frac{g_{m1}}{g_{m2}} = \frac{\mu_n C_{OX}\left(\frac{W}{L}\right)_1 V_{ov}}{\mu_n C_{OX}\left(\frac{W}{L}\right)_2 V_{ov}} = \frac{W_1}{W_2}$$

$$\Rightarrow \frac{v_o}{v_i} = \frac{-A_o}{1 + \frac{S}{\omega_T}(1 + A_o)}$$

where $A_o = \frac{W_1}{W_2} = \frac{g_{m1}}{g_{m2}}$

c) $A_o = 3v/v$, $W_2 = 25\mu m$

$$A_o = \frac{W_1}{W_2} \Rightarrow W_1 = 3 \times 25 = 75\mu m$$

$$I_{D1} = \frac{1}{2}\mu_n C_{OX}\frac{W_1}{L_1}(V_{GS} - V_t)^2 = \frac{1}{2} \times 200^{\mu} \times \frac{75}{0.5}0.3^2$$

$$I_{D1} = 1.35mA$$

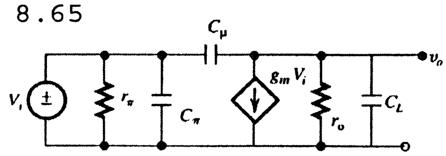
$$I_{D2} = \frac{1}{2} \times 200^{\mu A/v^2} \times \frac{25}{0.5} \times 0.3^2 = 0.45mA$$

Thus:

$$I = I_{D1} + I_{D2} = 1.35 + 0.45 = 1.8mA$$

$$f_{3db} = \frac{f_T}{1 + A_o} = \frac{12 \times 10^9}{1 + 3} = 3GHz$$

8.65



Writing a node equation at the output yields:

$$sC_\mu(v_i - v_O) = g_m \cdot v_i + \frac{v_O}{r_O} + v_O \cdot C_L \cdot s$$

$$\frac{v_O}{v_i} = \frac{C_\mu \cdot s - g_m}{\frac{1}{r_O} + (C_L + C_\mu)s}$$

$$= -g_m \cdot r_O\left[\frac{1 - sC_\mu/g_m}{1 + s(C_L + C_\mu)r_O}\right]$$

For $I_C = 200\ \mu A$, $V_A = 100\ V$:

$$g_m = \frac{200\ \mu}{0.025} = \frac{8\ mA}{V} \text{ and}$$

$$r_O = \frac{100}{200\ \mu} = 0.5\ M\Omega$$

Thus the DC-gain $= -g_m \cdot r_O$

$= -8 \times 0.5 \times 10^3 = -4000$ V/V

For $C_L = 1$ pF, $C_\mu = 0.2$ pF

$$\omega_{3dB} = \frac{1}{(C_L + C_\mu)r_O} = \frac{1}{(1 + 0.2)p \times 0.5\ M}$$

$= 1.67$ M rad/s

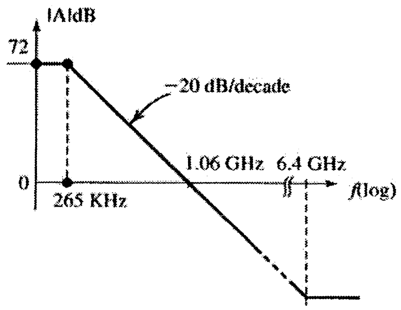
$f_{3dB} = f_H = 265.4$ KHz

$$f_Z = \frac{g_m}{2\pi \cdot C_\mu} = \frac{8\text{ m}}{2\pi \times 0.2\text{ p}} = 6.4\text{ GHz}$$

$$f_T = |A_0| \cdot f_H = 4000 \times 265.4 = 1.06\text{ GHz}$$

Bode plot for $|A|$:

4000 V/V = 720 dB



8.66

$$f_T = \frac{g_m}{2\pi(C_L + C_{gd})}, \quad g_m = 1\text{ mA/V}, \quad f_T = 2\text{ GHz}$$

$$\Rightarrow C_L + C_{gd} = \frac{1\times10^{-3}}{2\pi \times 2\times10^9} = 79.61\text{ fF}$$

To have $f_{T2} = 1$ GHz, we need:

$$C_L + C_{gd} = \frac{1\times10^{-3}}{2\pi\times1\times10^9} = 159.23\text{ fF}$$

Thus we need an additional capacitance of

$159.23 - 79.61 = \underline{79.61\text{ fF}}$

8.67

$$f_H \simeq f_{Pi} = \frac{1}{2\pi C_{in} R'_{sig}}, \text{ where}$$

$$R'_{sig} = \frac{r_O}{2} = \frac{20\text{ k}\Omega}{2} = 10\text{ k}\Omega$$

$$C_{in} = C_{gs} + C_{gd}(1 + g_m R'_L) \text{ where}$$

$$R'_L = r_O \| r_O = \frac{r_O}{2} = 10\text{ k}\Omega$$

$$\Rightarrow C_{in} = 0.1\text{ p} + 0.1\text{ p}(1 + 2\times10) = 2.2\text{ pF}$$

$$\Rightarrow f_H = \frac{1}{2\pi \times 2.2\text{ p} \times 10\text{ K}} = 7.23\text{ MHz}$$

i) If the bias current I is reduced by a factor of 4:

Since $I_D \propto V_{OV}^2 \Rightarrow$ For I_D to reduce by 1/4, V_{OV} is reduced by 1/2

$$g_{mi} = \frac{2(I_D/4)}{(V_{OV}/2)} = \frac{1}{2}\left(\frac{2I_D}{V_{OV}}\right) = \frac{1}{2}\cdot 2\text{ mA/V}$$

$$= 1\ \frac{\text{mA}}{\text{V}}$$

$$r_{oi} = \frac{V_A}{I_D/4} = 4\times\frac{V_A}{I_D} = 4\times20\text{ k}\Omega = 80\text{ k}\Omega$$

8.68

then: $R'_{sig} = R'_L = \frac{r_O}{2} = \frac{80\text{ k}\Omega}{2} = 40\text{ k}\Omega$

$$C_{in} = 0.1\text{ p} + 0.1\text{ p}(1 + 1\times40) = 4.2\text{ pF}$$

$$\Rightarrow f_H = \frac{1}{2\pi\times4.2\text{ p}\times40\text{ K}} = 0.95\text{ MHz}$$

ii) if the bias current is increased by $\times$ 4:

$\rightarrow V_{OV}$ is increased by a factor of 2.

$$g_{mii} = \frac{2(4\times I_D)}{(2\times V_{OV})} = 2\left(\frac{2I_D}{V_{OV}}\right)$$

$$= 2\times 2\ \frac{\text{mA}}{\text{V}} = 4\ \frac{\text{mA}}{\text{V}}$$

$$r_{O_{ii}} = \frac{V_A}{4\times I_D} = \frac{1}{4}\left(\frac{V_A}{I_D}\right) = \frac{1}{4}\times20\text{ k}\Omega = 5\text{ k}\Omega$$

then $R'_{sig} = R'_L = r_{O_{ii}}/2 = 2.5\text{ k}\Omega$

$$C_{in} = 0.1\text{ p} + 0.1\text{ p}(1 + 4\times2.5) = 1.2\text{ pF}$$

$$f_H = \frac{1}{2\pi\times1.2\text{ p}\times2.5\text{ K}} = 53\text{ MHz}$$

$$\tau_H = C_{gs}R_{gs} + C_{gd}R_{gd} + C_L\cdot R_{CL}$$

$$= C_{gs}\cdot R'_{sig} + C_{gd}\cdot[R'_{sig}(1 + g_m R'_L) + R'_L] + C_L\cdot R'_L$$

Where $R'_{sig} = 100\text{ k}\Omega$

$R'_L = 20\text{ k}\Omega \| 12\text{ k}\Omega = 7.5\text{ k}\Omega$

$$\Rightarrow \tau_H = 0.2\text{ p}\times100\text{ K} + 0.2\text{p}[100\text{ K}(1 + 1.5\times7.5) + 7.5\text{ K}] + C_L\cdot R'_L = 20\text{ ns} + 246\text{ ns} + C_L\cdot(7.5\text{ K})$$

$$= 266\text{ ns} + C_L\cdot(7.5\text{ K})$$

a) If $C_L = 0 \Rightarrow \tau_H = 266 \text{ ns} \rightarrow f_H = \frac{1}{2\pi\tau_H}$

$= 598 \text{ MHz}$

b) $C_L = 10\text{pF} \Rightarrow \tau_H = 341\text{ns} \rightarrow f_H$

$= 467 \text{ MHz}$

c) $C_L = 50 \text{ pF} \Rightarrow \tau_H = 641\text{ns} \rightarrow f_H$

$= 248 \text{ MHz}$

Using the Miller approximation : Eq 9.80, 9.82

$C_{in} = C_{gs} + C_{gd}(1 + g_m R_L')$

$= 0.2 \text{ p} + 0.2 \text{ p}$

$(1 + 1.5 \times 7.5) = 2.65 \text{ pF}$

$$f_H = \frac{1}{2\pi \cdot C_{in} \cdot R_{sig}} = \frac{1}{2\pi \times 2.65 \text{ p} \times 100 \text{ K}}$$

$= 600 \text{ MHz}$

Notice how this result is close to case a) where $C_L = 0$ and diverges further with increasing value of C_L, showing the importance of C_L in determining f_H

8.69 the low-frequency gain $\frac{v_o}{v_{sig}}$ can be written as:

$$\frac{v_o}{v_{sig}} = \frac{\frac{1}{g_m + g_{mb}}}{R_s + \frac{1}{g_m + g_{mb}}} \times (g_m + g_{mb}) \times R_L'$$

$$\frac{v_o}{v_{sig}} = \frac{(g_m + g_{mb}) R_L'}{1 + (g_m + g_{mb}) R_{sig}} = \frac{(5 + 0.2 \times 5) \times 20}{1 + (5 + 0.2 \times 5) \times 1} = 17.14 \text{ V/V}$$

$$\frac{v_o}{v_{sig}} = 17.14 \text{ V/V}$$

From Eq. 6.105 we have $f_{P1} = \frac{1}{2\pi C_{gs}\left(R_{sig} \| \frac{1}{g_m + g_{mb}}\right)}$

$$f_{P1} = \frac{1}{2\pi \times 2\text{pF}\left(1\text{K} \| \frac{1}{5 + 0.2 \times 5}\right)} = 557 \text{ MHz}$$

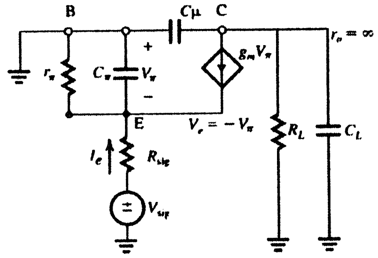
From Eq. 6.106 we have:

$$f_{P2} = \frac{1}{2\pi (C_{gd} + C_L) R_L'} = \frac{1}{2\pi (0.1^p + 2^p) 20^K} = 3.79 \text{ MHz}$$

Since $f_{P2} \ll f_{P1}$, then f_{P2} is the dominant pole and

$f_H \simeq f_{P2} = 3.79 \text{ MHz}$

8.70



We observe that V_e, voltage at the emitter, is equal to $-V_\pi$. We can write a node equation at the emitter:

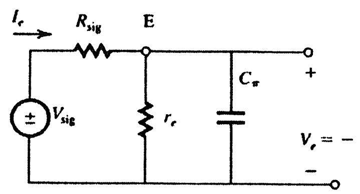
$$I_e = -V_\pi\left(\frac{1}{r_\pi} + sC_\pi\right) - g_m \cdot V_\pi$$

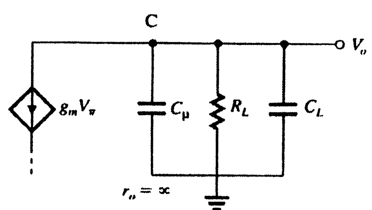
$$= V_e\left(\frac{1}{r_\pi} + g_m + sC_\pi\right)$$

Thus, the input admittance looking into the emitter is:

$$\frac{I_e}{V_e} = \frac{1}{r_\pi} + g_m + sC_\pi = \frac{1}{r_e} + sC_\pi$$

Therefore we can replace the transistor at the input of the circuit by this admittance as shown below





a) As we can see above, the circuit can be separated into two parts, each with its own pole:

$$f_{P1} = \frac{1}{2\pi \cdot C_\pi (R_{sig} \| r_e)} \text{ (input side)}$$

$$f_{P2} = \frac{1}{2\pi (C_\mu + C_L) R_L} \text{ (output side)}$$

If we compare the poles for MOSFETS, we observe that these equations are their bipolar counteparts:

$$f_{P1} = \frac{1}{2\pi \cdot C_{gs}\left(R_{sig} \| \frac{1}{g_m}\right)} \quad 9.108$$

$$f_{P2} = \frac{1}{2\pi(C_{gd} + C_L)R_L} \quad 9.109$$

b) For $C\pi = 14$ pF, $C_\mu = 2$ pF, $C_L = 1$ pF, $I_c = 1$ mA

$$R_{sig} = 1\ \text{k}\Omega,\ R_L = 10\ \text{k}\Omega \Rightarrow g_m = \frac{1}{0.025}$$

$$= 40\ \frac{\text{mA}}{\text{V}}$$

Assuming $\beta = 100$

$$f_{P1} = \frac{1}{2\pi \times 14\ \text{p}\left(1\ \text{K} \| \frac{100}{4}\right)} = 15.9\ \text{MHz}$$

$$f_{P2} = \frac{1}{2\pi \times (2\ \text{p} + 1\ \text{p})10\ \text{K}} = 5.3\ \text{MHz}$$

$$f_T = \frac{g_m}{2\pi(C\pi + C_\mu)} = \frac{40\ \text{m}}{2\pi(14\ \text{p} + 2\ \text{p})}$$

$$= 398.1\ \text{MHz}$$

$f_t >> f_{P1}$ and $f_t >> f_{P2}$

8.71

If $f_H = 300$ MHz $\Rightarrow \tau_H = \frac{1}{2\pi f_H}$

$= 530.5$ ps

$$\tau_H = C_{gs}R_{gs} + (C_{gd} + C_L')R_{gd}$$

From Example 9.12

$C_{gs} = 20$ fF, $R_{gs} = 1.38$ kΩ, $C_{gd} = 5$ fF, $R_{gd} = 18.7$ kΩ

$$\Rightarrow 530.5\ \text{p} = 20\ \text{f} \times 1.38\ \text{K} + (5\ \text{p} + C_L') \times 18.7\ \text{K}$$

Thus $C_L' = 21.9$ fF

Since the original C_L in Eq. 12 was 15 fF

$\Rightarrow$ We must add $(21.9 - 15) = 6.9$ fF at the output to reduce f_H from 396 MHz to 300MHz

8.72

$$R_O = 2r_O + (g_m r_O)r_O = 2 \times 50\ \text{k}\Omega + (1 \times 50) \times 50\ \text{k}\Omega = 2.6\ \text{M}\Omega$$

$$A_V = -g_m(R_O \| R_L) = -1\ \text{m}(2.6\ \text{M} \| 2\ \text{M})$$

$$= -1130\ \frac{\text{V}}{\text{V}}$$

$$A_V = -1130\ \frac{\text{V}}{\text{V}}$$

$$R_{in2} = \frac{r_O + R_L}{g_m r_O} = \frac{50\ \text{k}\Omega + 2\ \text{M}\Omega}{1 \times 50} = 41\ \text{k}\Omega$$

$$R_{d1} = r_O \| R_{in2} = 50\ \text{k}\Omega \| 41\ \text{k}\Omega = 22.5\ \text{k}\Omega$$

$$\tau_H = R_{sig}[C_{gs} + C_{gd}(1 + g_m R_{d1})] + R_{d1}(C_{gd} + C_{db} + C_{gs}) + (R_L \| R_o)(C_L + C_{db} + C_{gd})$$

$$\tau_H = 100\ \text{K} \times [30\ \text{f} + 10\ \text{f}(1 + 1 \times 22.5)] + 22.5\ \text{K} \times [10\ \text{f} + 10\ \text{f} + 30\ \text{f}] + (2\ \text{M} \| 2.6\ \text{M}) \times [40\ \text{f} + 10\ \text{f} + 10\ \text{f}]$$

$$\tau_H = 26.5\ \text{ns} + 1.125\ \text{ns} + 67.8\ \text{ns} = 95.42\ \text{ns}$$

$$f_H = \frac{1}{2\pi\tau_H} = 1.67\ \text{MHz}$$

8.73

a) $A_M = -g_m R_L' = -5 \times (20^K \| 20^K) = \underline{-50\ V/V}$

$R_{gs} = R_{sig} = 20\ \text{K}\Omega$

$R_{gd} = R_{sig}(1 + g_m R_L') + R_L' = 20^K(1 + 5 \times 20^K \| 20^K) + 20^K \| 20^K$

$R_{gd} = 1030\ \text{K}\Omega = 1.03\ \text{M}\Omega$

We use Eq. 6.57 :

$$\tau_H = C_{gs}R_{gs} + C_{gd}R_{gd} + C_L R_L'$$

$$\tau_H = 2^P \times 20^K + 0.2^P \times 1030^K + 1^P \times (20^K \| 20^K) = 256\ \text{ns}$$

$$f_H = \frac{1}{2\pi\tau_H} = \underline{622\ \text{kHz}}$$

$\underline{|A_M| \cdot f_H = 31.1\ \text{MHz}}$

b) For the cascode amplifier:

$A_{o1} = g_{m1} r_{o1} = 5 \times 20 = 100 V/V$

$A_{vo2} = 1 + (g_{m2} + g_{mb2}) r_{o2} = 1 + (5 + 0.2 \times 5) \times 20^K$

$A_{vo2} = 121 V/V$

$R_{out} = r_{o2} + A_{vo2} r_{o1} = 20^K + (121 \times 20^K) = 2.44 M\Omega$

$A_v = A_{vo} \frac{R_L}{R_L + R_{out}} = -121 \times 100 \times \frac{20}{20 + 2440} = \underline{98.4 V/V}$

Using Eq. 6.137,

$\tau_H = R_{sig}[C_{gs1} + C_{gd1}(1 + g_{m1} R_{d1})] + R_{d1}(C_{gd1} + C_{db1} + C_{gs2})$
$+ (R_L \| R_{out})(C_L + C_{gd2})$

$R_{d1} = r_{o1} \| \left(\frac{1}{g_{m2} + g_{mb2}} + \frac{R_L}{A_{vo2}}\right)$

$R_{d1} = 20^K \| \left(\frac{1}{5 + 0.2 \times 5} + \frac{20}{121}\right) = 0.327 k\Omega$

$\tau_H = 20^K[2 + 0.2(1 + 5 \times 0.327)] + 0.327(0.2^P + 0.2^P + 2) +$
$(20^K \| 2.44^M)(1 + 0.2)$

$\tau_H = 75.1 ns$

$f_H = \frac{1}{2\pi \tau_H} = \underline{2.12^{MHz}}$

$\underline{|A_v| \cdot f_H = 208.61 MHz}$

8.74

$A_v = 66dB = 1995 V/V$

$A_v = A_{vo} \frac{R_L}{R_L + R_{out}}$ and $R_L = R_{out} \Rightarrow A_v = A_{vo} \times \frac{1}{2}$

$A_{vo} = (1 + g_{m2} r_{o2}) g_{m1} r_{o1} \simeq g_m^2 r_o^2 = \left(\frac{2I_D}{V_{ov}} \cdot \frac{V_A}{I_D}\right)^2 = \left(\frac{2V_A}{V_{ov}}\right)^2$

$\Rightarrow 1995 = \frac{1}{2} \times \left(\frac{2 \times 10}{V_{ov}}\right)^2 \Rightarrow V_{ov} = 0.317^V$

$\Rightarrow I_D = \frac{1}{2} k_n' \frac{W}{L} V_{ov}^2 = \frac{1}{2} \times 200 \times 10^{-3} \times 10 \times 0.317^2 = 0.1 mA$

Since R_{sig} is small:

$\tau_H \simeq (C_L + C_{gd})(R_L \| R_{out})$

$r_o = \frac{V_A}{I_D} = \frac{10}{0.1} = 100 k\Omega$, $g_m = \frac{2I_D}{V_{ov}} = 0.631 mA/V$

$R_{out} = A_{vo2} r_{o1} + r_{o2} = (1 + g_m r_o) r_o + r_o$

$R_{out} = 6510 k\Omega \simeq 6.5^{M\Omega}$, $R_L = R_{out}$.

$\tau_H \simeq (1pF + 0.1pF)\left(\frac{6510^K}{2}\right) = 3580.5 ns$

$f_H = 44.5 KHz$

$f_t \simeq |A_v| \cdot f_H = 1995 \times 44.5 = 88.8 MHz$

IF the cascode transistor is removed, then we have a common-source configuration.

$A_M = -g_m(r_o \| R_L) = -0.637(100^K \| 6510^K)$

$A_M = -62.74 V/V$

$f_H = \frac{1}{2\pi(C_L + C_{gd}) R_L'} = \frac{1}{2\pi(1 + 0.1)(100^K \| 6510^K)} = 1.47^{MHz}$

$f_H = 1.47 MHz$

$|A_M| \cdot f_H = 92.2 MHz \simeq f_T$

Note that the unity-gain stays nearly unchanged. The result is the same as

8.75

$R_{sig} = 4 k\Omega$, $R_L = 2.4 k\Omega$,
$I = 1 mA$, $\beta = 100$, $r_o = 100 k\Omega$

$A_M = -\frac{r_\pi}{r_\pi + r_x + R_{sig}} \times g_m(\beta r_o \| R_L)$

$r_\pi = \frac{\beta}{g_m} = \frac{100}{1/0.025} = 2.5 k\Omega$

$g_m = \frac{I_C}{V_T} = 40 mA/V$

$A_M = \frac{2.5}{2.5 + 0.05 + 4} \times 40 \times (100 \times 100 \| 2.4 K)$

$A_M = -36.6 V/V$

$R_{sig}' = r_\pi \| (r_x + R_{sig}) = 2.5 K \| (0.05 + 4 K)$

$R_{sig}' = 1.55 k\Omega = R_{\pi 1}$

$R_{\mu 1} = R_{sig}' = (1 + g_m R_{e1}) + R_{e1}$

$R_{e1} = r_{o1} \| r_{e2}\left(\frac{r_o + R_L}{r_o + R_L/\beta + 1}\right)$

$= 100 K \| \frac{100 K}{101}\left(\frac{100 + 2.4}{100 + \frac{2.4}{101}}\right)$

$R_{e1} = 1 k\Omega$

$R_{\mu 1} = 1.55(1 + 40 \times 1) + 1 = 64.55 k\Omega$

$R_{out} = \beta \cdot r_o = 10 M\Omega$

$\tau_H = C_{\pi 1} R_{\pi 1} + C_{\mu 1} R_{\mu 1} + (C_{CS1} + C_{\pi 2}) R_{e1}$
$+ (C_L + C_{CS2} + C_{\mu 2})(R_L \| R_{out})$

$\tau_H = 14 \times 1.55 + 2 \times 64.55 + (0 + 14)$
$\times 1 + (0 + 2)(2.4 K \| 10 M)$

$\tau_H = 169.6 ns$ $f_H = 939 KHz$

8.76

a) If we employ Miller's theorem to C_μ:

$$\frac{1}{C_{\mu_1}s}\cdot\frac{1}{1-A}=\frac{1}{C_\mu s}\cdot\frac{1}{1-(-1)}=\frac{1}{2C_\mu s}$$

or $2C_\mu$ appears in parallel with C_π. Thus the time constant due to $(C_\pi+2C_\mu)$ is: $R'_{sig}(C_\pi+2C_\mu)$ which results in:

$$f_{P_1}=\frac{1}{2\pi R'_{sig}(C_\pi+2C_\mu)}$$

If we refer to Fig. 6.42, we'll see that the output pole is: $f_{P2}=\dfrac{1}{2\pi(C_L+C_{cs2}+C_{\mu e})R_L}$

b) $R_{sig}=1\,K\Omega \Rightarrow R'_{sig}=r_\pi \| R_{sig}=\frac{100}{1/0.025}\|1^K$

$\Rightarrow R'_{sig}=0.714\,k\Omega$

$$f_{P_1}=\frac{1}{2\pi\times0.714^K(5+2\times1)^P}=31.85\ MHz$$

$$f_{P_2}=\frac{1}{2\pi(0+0+1)\times10}=15.9\ MHz$$

(Assume $R_L=10\ K\Omega$)

$$f_H=\frac{1}{\sqrt{\left(\frac{1}{f_{P_1}}\right)^2+\left(\frac{1}{f_{P_2}}\right)^2}}=14.2\ MHz$$

If $R_{sig}=10\,K\Omega$:

$R'_{sig}=2.5^K\|10^K=2K\Omega$

$f_{P_1}=\frac{1}{2\pi(5+2)2}=11.4\ MHz$

f_{P_2} is the same: $f_{P2}=15.9\ MHz$

$f_H=9.26\ MHz$

8.77

$$g_m=\frac{I_C}{V_T}=\frac{0.1\text{ mA}}{25\text{ mV}}=\frac{4\text{ mA}}{\text{V}};$$

$$r_\pi=\frac{\beta}{g_m}=\frac{100}{4\text{ m}}=25\text{ k}\Omega$$

$R_{sig}=r_\pi=25\text{ k}\Omega$

$$r_O=\frac{V_A}{I_C}=\frac{100}{0.1\text{ m}}=1\text{ M}\Omega;$$

$R_L=\beta\cdot r_O=100\text{ M}\Omega$

$$f_T=\frac{1}{2\pi}\cdot\frac{g_m}{(C_\mu+C_\pi)}\Rightarrow C_\mu+C_\pi=\frac{g_m}{2\pi\cdot f_T}$$

$$\rightarrow C_\pi=\frac{4\times10^{-3}}{2\pi\times10^9}-0.1\times10^{-12}=0.54\text{ pF}$$

To obtain the DC-gain A_M:

$$A_M=\frac{-r_\pi}{r_\pi+R_{sig}}\cdot g_m(\beta r_O\|R_L)$$

$R_{sig}=r_\pi\ \ R_L=\beta r_O$

assuming $r_x=0$

$$A_M=\frac{-1}{2}\cdot g_m\times\frac{\beta r_O}{2}$$

$$=\frac{-1}{4}\times4\times10^{-3}\times100\times1\times10^6$$

$A_M=-100\text{ KV/V}$

$$R'_{sig}=r_\pi\|R_{sig}=\frac{r_\pi}{2}=12.5\text{ k}\Omega$$

$R_{\pi1}=R'_{sig}=12.5\text{ k}\Omega$

$$r_{e2}=r_e=\frac{r_\pi}{\beta+1}=\frac{25\text{ k}\Omega}{101}=247\ \Omega$$

$$R_{C1}=r_O\|r_e\left(\frac{r_O+R_L}{r_O+\frac{R_L}{\beta+1}}\right)$$

$$=1\text{ M}\|247\times\left(\frac{1\text{ M}+100\text{ M}}{1\text{ M}+\frac{100\text{ M}}{101}}\right)$$

$R_{C1}=12.4\text{ k}\Omega$

$R_{\mu1}=R'_{sig}(1+g_mR_{C1})+R_{C1}$

$=12.5\text{ K}(1+4\times12.4)+12.4\text{ k}\Omega$

$R_{\mu1}=645\text{ k}\Omega$

$\Rightarrow \tau_H=C_{\pi1}R_{\pi1}+C_{\mu1}R_{\mu1}+C_{\pi2}R_{C1}$

$+C_{\mu2}(R_L\|R_O)$

where

$R_O=\beta_2\cdot r_{O2}=100\times1\text{ M}\Omega=100\text{ M}\Omega$

$\tau_H=0.54\text{ p}\times12.5\text{ K}+0.1\text{ p}\times645\text{ K}+0.54\text{ p}$

$\times12.4\text{ K}+0.1\text{ p}(100\text{ M}\|100\text{ M})$

$\tau_H=5.08\ \mu\text{s}$

$$f_H=\frac{1}{2\pi\tau_H}=31.3\text{ KHz}$$

$f_T=|A_M|\times f_H=3.13\text{ GHz}$

8.78

a) $I = \frac{1}{2} k_n' \frac{W}{L} V_{ov}^2 = \frac{1}{2} \times 160^{\mu} \times 100 \times (0.5)^2 = 2\,mA$

b) $g_m = \frac{2I_D}{V_{ov}} = 2 \times \frac{2}{0.5} = 8\,mA/V$

$g_{mb} = \chi g_m = 0.2 \times 8 = 1.6\,mA/V$

$r_o \cong \frac{1}{\lambda I_D} = \frac{1}{0.05 \times 2} = 10\,k\Omega$

$\therefore A_{v_o} = \frac{g_m r_o}{1 + (g_m + g_{mb}) r_o}$

$A_{v_o} = \frac{8 \times 10}{1 + (8 + 1.6) \times 10} = 0.82\,V/V$

IF we use the approximation formula

$A_{v_o} = \frac{1}{1 + \chi} = 0.83\,V/V$

$R_o = \frac{1}{g_m + g_{mb}} \| r_o = \frac{1}{8 + 1.6}^{k} \| 10^k = 103\,\Omega$

d) with $R_L = 1\,k\Omega$

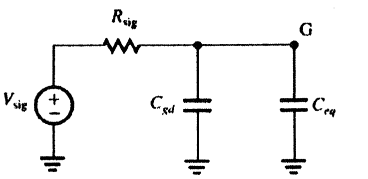
$A_v = \frac{g_m R_L'}{1 + g_m R_L'}$

$R_L' = R_L \| r_o \| \frac{1}{g_{mb}} = 1^k \| 10^k \| \frac{1}{1.6}^k = 370\,\Omega$

$A_v = \frac{8^m \times 370 \times 10^{-3}}{1 + 8 \times 0.370} = 0.75\,V/V$

8.79

Using the Miller approximation, the resulting input equivalent circuit is:



where $C_{eq} = C_{gs}(1 - K)$

and $K = \frac{g_m R_L'}{1 + g_m R_L'}$

$$\Rightarrow C_{eq} = C_{gs}\left[1 - \frac{g_m R_L'}{1 + g_m R_L'}\right]$$

$$= C_{gs}\left[\frac{1}{1 + g m R_L'}\right]$$

$$C_{in} = C_{gd} \| C_{eq} = C_{gd} + C_{gs}\left[\frac{1}{1 + g_m R_L'}\right]$$

$$\tau_H = R_{sig} \cdot C_{in} \Rightarrow f_H$$

$$= \frac{1}{2\pi \cdot R_{sig}\left(C_{gd} + \frac{C_{gs}}{1 + g_m R_L'}\right)}$$

Notice that this is the same result as obtained in problem 9.86. This estimate is higher than that obtained from the method of open-time constants since it neglects the contribution of C_L to τ_H and reduces the contribution of C_{gs} from:

$C_{gs} \cdot \frac{(R_{sig} + R_L')}{1 + g_m R_L'}$ to $\frac{C_{gs} \cdot R_{sig}}{1 + g_m R_L'}$, thus

effectively reducing the value of τ_H, and therefore increasing f_H

8.80

$$g_m = \frac{I_C}{V_t} = \frac{1}{0.025} = 40\,mA/V,$$

$$r_e = \frac{\beta}{(\beta + 1) g_m} = 25\,\Omega$$

$$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)} = 2\,GHz$$

$$\Rightarrow C_\pi + C_\mu = 3.18\,pF$$

$$C_\mu = 0.1\,pF \Rightarrow C_\pi = 3.08\,pF$$

$$r_\pi = \frac{\beta}{g_m} = 2.5\,k\Omega$$

$$r_O = \frac{V_A}{I_C} = \frac{20}{1} = 20\,k\Omega$$

r_o is in effect parallel to R_L, so $R_L' = R_L \| r_O$

$R_L' = 1\,k\Omega \| 20\,k\Omega = 0.95\,k\Omega.$

$$A_M = \frac{R_L'}{\frac{R_{sig} + r_\pi + r_x}{\beta + 1} + R_L'}$$

$$= \frac{0.95}{\frac{1 + 2.5 + 0.1}{101} + 0.95} = 0.964\,V/V$$

$$R_\mu = R_{sig}' \| (r_\pi + (\beta + 1) R_L')$$

$$R_{sig}' = R_{sig} + r_x = 1 + 0.1 = 1.1\,k\Omega$$

$$R_\mu = 1.1\,K \| (2.5\,K + 101 \times 0.95)$$

$$= 1.08\,k\Omega$$

8.81

$K_n'(W/L) = 128\frac{\mu A}{V^2} \times 25 = 3.2 mA/V^2$

(a) $V_{ov} = \sqrt{\frac{I}{K_n' W/L}} = \sqrt{\frac{0.2}{3.2}} = \underline{\underline{0.25V}}$

$g_m = \frac{I}{V_{ov}} = \frac{0.2mA}{0.25V} = \underline{\underline{0.8\frac{mA}{V}}}$

(b) $A_d = g_m(R_D \| r_o)$
where $r_o = \frac{V_A}{I/2} = \frac{20}{0.2/2} = 200K\Omega$

$\Rightarrow A_d = 0.8m \times (20K \| 20K)$
$= \underline{\underline{14.54\ V/V}}$

(c) For a CS amplifier when R_{sig} is low:

$f_H = \frac{1}{2\pi (C_L + C_{gd}) R_L'}$

where $R_L' = R_D \| r_o$
$= 20K \| 200K$
$= 18.18K\Omega$

and $C_L' = C_L + C_{db}$

Since for a grounded source terminal C_{db} is in parallel with the load.

$\rightarrow C_L' = 90 + 5 = 95fF$

thus,

$f_H = \frac{1}{2\pi (95+5)10^{-15} \times 18.18K}$
$= \underline{\underline{87.54 MHz}}$

(d) Using the open-circuit time-constants method for $R_S = 20K\Omega$

$f_H = \frac{1}{2\pi \tau_H}$

where $\tau_H = C_{gs}.R_S$
$+ C_{gd}[R_S(1+g_m R_L') + R_L']$
$+ C_L.R_L'$

thus,

$\tau_H = 30f \times 20K$
$+ 5f[20K(1+0.8 \times 18.18) + 18.18K]$
$+ (90f + 5f) \times 18.18K$

$\tau_H = 0.6ns$
$+ 1.64ns$
$+ 1.72ns = 3.96ns$

$\Rightarrow f_H = \frac{1}{2\pi\, 3.96ns} = \underline{\underline{40.2 MHz}}$

8.82

$f_Z = \frac{1}{2\pi C_{SS} R_{SS}} = \frac{1}{2\pi (0.2p)(100K)} = \underline{\underline{7.95MHz}}$

8.83

$$f_Z = \frac{1}{2\pi \cdot C_{SS} \cdot R_{SS}}$$

R_{ss} is the output resistance of the current source, which for the single-transistor current source is:

$$R_{SS} = r_O = \frac{V_A}{I_D} = \frac{30\text{ V}}{100\ \mu\text{A}} = 300\text{ k}\Omega$$

$$\Rightarrow f_Z = \frac{1}{2\pi \cdot 100\text{ f} \cdot 300\text{ K}} = 5.3\text{ MHz}$$

If V_{ov} is reduced from 0.5 V to 0.2 V while I is unchanged.

For the current-source transistor: when V_{ov} = 0.5 V

$$I = I_D = \frac{1}{2}k_n'\left(\frac{W}{L}\right)V_{OV}^2 \rightarrow 100\ \mu\text{A}$$

$$= \left[\frac{1}{2}k_n'\frac{W_1}{L_1}\right] \times (0.5)^2$$

$$\rightarrow \frac{1}{2}k_n'\frac{W_1}{L_1} = 400\ \frac{\mu\text{A}}{\text{V}^2}$$

when V_{OV} = 0.2:

$$100\ \mu\text{A} = \left[\frac{1}{2}k_n'\frac{W_2}{L_2}\right] \times (0.2)^2$$

$$\rightarrow \frac{1}{2}k_n'\frac{W_2}{L_2} = 2500\ \frac{\mu\text{A}}{\text{V}^2}$$

Assuming that $L_1 = L_2$, (the length of the transistor is unchanged)

$$\Rightarrow \frac{W_2}{W_1} = \frac{2500}{400} \rightarrow W_2 = 6.25\ W_1$$

The width of the current-source transistor is made 6.25 times larger to operate at V_{ov} = 0.2 V, I_n = 100 μA

If C_{SS} is directly proportional to W:

$$C_{SS2} = 6.25 \times 100\text{ f}$$

$$f_{Z2} = \frac{f_{Z1}}{6.25} = 848\text{ KHz}$$

8.84

(a) $V_{OV} = \sqrt{\dfrac{I}{(W/L)k_n'}}$

$= \sqrt{\dfrac{80\ \mu}{100 \times 0.2\ \frac{mA}{V^2}}} = 63.2\ mV$

$g_m = \dfrac{2(I/2)}{V_{OV}} = \dfrac{80\ \mu}{63.2\ m} = 1.27\ \dfrac{mA}{V}$

(b) $A_d = g_m(R_D \| r_O)$;

$r_O = \dfrac{V_A}{(I/2)} = \dfrac{20\ V}{40\ \mu} = 500\ k\Omega$

$\Rightarrow A_d = 1.27\ m(20\ k\Omega \| 500\ k\Omega) = 24.4\ V/V$

(c) $f_H = \dfrac{1}{2\pi(C_L + Cgd)R_L'}$

$R_L' = R_D \| r_O = 20\ k\Omega \| 500\ k\Omega = 19.23\ k\Omega$

$C_L = 100\ fF + Cdb = 110\ fF$

$\Rightarrow f_H = \dfrac{1}{2\pi(110\ f + 10\ f) \times 19.23\ K}$

$= 69\ MHz$

(d) Using the open-circuit time-constants method

for $R_{sig} = 100\ k\Omega\ \left(R_S = \dfrac{100\ k\Omega}{2} = 50\ k\Omega\right)$

$\tau_H = C_{gs} \cdot R_S$

$+ C_{gd}[R_S(1 + g_m R_L') + R_L']$

$+ C_L \cdot R_L'$

$\tau_H = 50\ f \times 50\ K$

$+ 10\ f[50\ K(1 + 1.27 \times 19.23) + 19.23\ K]$

$+ (100\ f + 10\ f) \times 19.23\ K$

$\tau_H = 2.5\ ns$

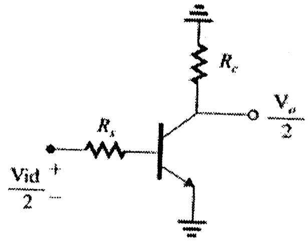
$+ 12.9\ ns$

$+ 2.11\ ns = 17.51\ ns$

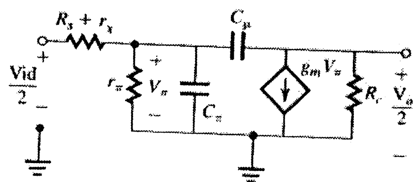
$f_H = \dfrac{1}{2\pi\tau_H} = \dfrac{1}{2\pi \times 17.51\ n} = 9.1\ MHz$

8.85

(a) Differential half-circuit:



High-frequency equivalent circuit (r_o is very large)



(b) $I = 0.5\ mA \rightarrow g_m = \dfrac{0.5}{25} = 20\ \dfrac{mA}{V}$

$r_\pi = \dfrac{\beta}{g_m} = \dfrac{100}{20\ m} = 5\ k\Omega$

$C_\pi + C_\mu = \dfrac{20\ m}{2\pi \times 600} = 5.3\ pF.$

if $C_\mu = 0.5\ pF \Rightarrow C_\pi = 4.8\ pF$

$R_S = 10\ k\Omega,\ R_C = 10\ k\Omega,\ r_x = 100\ \Omega$

. (Notice that $R_B = \infty$,

$r_O = \infty$)

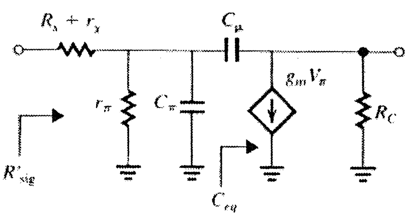
$A_M = \dfrac{-r_\pi}{r_\pi + R_S + r_X} \cdot g_m \cdot R_C$

$= \dfrac{-5\ K}{5\ K + (10\ K + 100)} \cdot 20\ m \times 10\ K$

$A_M = -66.22\ V/V$

(c) Γ

$f_H = \dfrac{1}{2_\pi C_{in} \cdot R_{sig}}$



where:

$C_{in} = C_\pi + C_{eq}$

using Miller's approximation:

$C_{eq} = C_\mu(1 + g_m R_C)$

and $R'_{sig} = (R_S + r_x) \| r_\pi$

Thus,

$$f_H = \frac{1}{2\pi[(R_S + r_x) \| r_\pi] \cdot [C_\pi + C_\mu(1 + g_m R_C)]}$$

$$= \frac{1}{2\pi[(10\text{ K} + 100) \| 5\text{ K}] \cdot [4.8\text{ p} + 0.5\text{ p}(1 + 20 \times 10)]}$$

$= 452$ KHz

$GBW = 66.22 \times 452\text{ K} = 30\text{ MHz}$

8.86

The CMRR will have..

poles at 500KHz and at

$$\frac{1}{2\pi \times 10^6 \times 10 \times 10^{-12}} = 15.9\text{ KHz}$$

8.87

$I = 0.6$mA

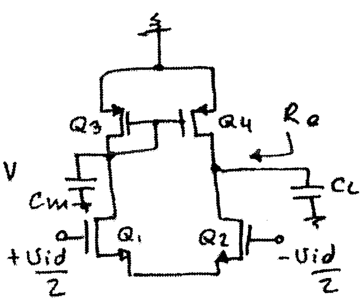
$V_{OVn} = 0.3$V

$V_{OVp} = 0.5$V

$V_{An} = |V_{Ap}| = 9$V

$C_m = 0.1$pF

$C_L = 0.2$pF



All ro's are identical:

$$r_o = \frac{V_A}{I_D} = \frac{9}{0.3m} = 30K\Omega.$$

$$g_m = \frac{2I_D}{V_{OV}} \Rightarrow g_{m1,2} = \frac{0.6m}{0.3} = 2\frac{mA}{V}$$

$$g_{m3,4} = \frac{0.6m}{0.5} = 1.2\frac{mA}{V}$$

The low frequency differential gain is:

$$A_d = g_{m1,2}(r_{o2} \| r_{o4})$$

$$= 2\frac{mA}{V}(30K \| 30K) = 30\frac{V}{V}$$

$$f_{P1} = 1/(2\pi C_L R_o)$$

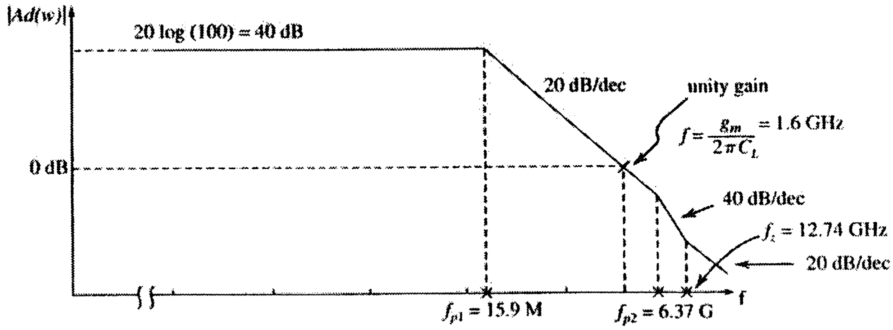
where $R_o = r_{o2} \| r_{o4} = 15K\Omega$

$$f_{P1} = \frac{1}{2\pi \times 0.2p \times 15K} = 53MHz$$

(7.193) $$f_{P2} = \frac{g_{m3}}{2\pi C_m} = \frac{1.2mA/V}{2\pi \times 0.1pF} = 1.9GHz$$

(7.194) $$f_Z = \frac{2g_{m3}}{2\pi C_m} = \frac{2 \times 1.2m}{2\pi \times 0.1p} = 3.8GHz$$

This figure is for 8.88



8.88

All V_{ov} are the same, all V_A are the same

$\Rightarrow$ All r_o's are identical, and all g_m's are identical

$$Ad(s) = g_m R_O \left[\frac{1 + \frac{s \cdot C_m}{2g_{m3}}}{1 + \frac{s \cdot C_m}{g_{m3}}}\right] \cdot \left(\frac{1}{1 + sC_L R_O}\right)$$

where we know that the frequencies of the zero f_z, and the pole f_P occur at very high frequencies. Thus we can assume that the pole $f_{P1} = 1/(2\pi C_L R_O)$ dominates the response of $A_d(s)$ passed the unity gain

Thus:

$$A_d(\omega) \simeq g_m R_O \left(\frac{1}{1 + j\omega C_L R_O}\right)$$

At unity gain: $|A_d(\omega_1)| = 1$

$$\Rightarrow 1 = \frac{g_m R_O}{\sqrt{1 + (\omega_1 C_L R_O)^2}}$$

$$\Rightarrow \omega_1^2 = \frac{(g_m R_O)^2 - 1}{(C_L R_O)^2}$$

Since $g_m R_O = g_m \frac{r_O}{2} >> 1 \Rightarrow \omega_1 \simeq g_m / C_L$

$$\Rightarrow f_1 = \frac{g_m}{2\pi \cdot C_L}$$

For: $V_A = 20$ V, $V_{OV} = 0.2$ V,

$I = 0.2$ mA, $C_L = 100$ fF, $C_m = 25$ fF

All r_o's are identical:

$r_O = \frac{V_A}{I_D} = \frac{20 \text{ V}}{(0.2 \text{ m}/2)} = 200 \text{ k}\Omega$. All g_m's are identical: $g_m = \frac{2I_D}{V_{OV}} = \frac{0.2 \text{ m}}{0.2} = \frac{1 \text{ mA}}{\text{V}}$

The low-frequency differential gain is:

$$A_{dDC} = g_m \times \frac{r_O}{2} = 1 \text{ m} \times 100 \text{ k}\Omega = 100 \text{ V/V}$$

$$f_{P1} = \frac{1}{2\pi C_L \cdot R_O} \qquad R_O = \frac{r_O}{2} = 100 \text{ k}\Omega$$

$$f_{P1} = \frac{1}{2\pi \times 100 \text{ f} \times 100 \text{ K}} = 15.9 \text{ MHz}$$

$$f_{P2} = \frac{g_m}{2\pi C_m} = \frac{1 \text{ m}}{2\pi \times 25 \text{ f}} = 6.37 \text{ GHz}$$

$$f_Z = \frac{2g_m}{2\pi C_m} = 2 \times f_{PZ} = 12.74 \text{ GHz}.$$

Notice in the Bode plot the location of the unity-gain frequency.

8.89

a) $A_M = -A_{vo}\frac{R_L}{R_L+R_{out}} = -g_m r_o \frac{R_L}{R_L+r_o}$

$A_M = -5\times 40\times\frac{40}{40+40} = -100\ V/V$

$R'_L = R_L \| R_{out} = R_L \| r_o = 20\ k\Omega$

$R_{gd} = R_{sig}(1+G_m R'_L)$ where $G_m = g_m$

$\Rightarrow R_{gd} = 20^K(1+5\times 20) = 2020\ k\Omega = 2.02\ M\Omega$

$R_{gd} = 2.02\ M\Omega$

$R_S = 0 \Rightarrow R_{gs} = R_{sig} = 20\ k\Omega$

$R_{C_L} = R'_L = 20\ k\Omega$

$\tau_H = C_{gs}R_{gs} + C_{gd}R_{gd} + C_L R_{C_L}$

$\tau_H = 2^P\times 20^K + 0.1^P\times 2.02^M + 1\times 20^K = 262\ ns$

$f_H = \frac{1}{2\pi\tau_H} = 607.8\ KHz$

$|A_M|\cdot f_H = 100\times 607.8 = 60.78\times 10^3\ KHz = 60.78\ MHz$

b) $R_S = 500\ \Omega$

$R_{out} = r_o[1+(g_m+g_{mb})R_S] = 40[1+(5+1)0.5] = 160\ k\Omega$

$A_M = -g_m r_o\frac{R_L}{R_L+R_{out}} = -5\times 40\times\frac{40}{40+160} = -40\ V/V$

$R'_L = R_L \| R_{out} = 40K \| 160K = 32\ k\Omega$

$R_{gd} = R_{sig}(1+G_m R'_L)$

$G_m = \frac{g_m r_o}{r_o[1+(g_m+g_{mb})R_S]}$

$G_m = \frac{5\times 40}{40[1+(5+1)0.5]} = 1.25\ mA/V$

$R_{gd} = 20K(1+1.25\times 32^K) = 820\ k\Omega$

$R_{gs} = \frac{R_{sig}+R_S}{1+(g_m+g_{mb})R_S\frac{r_o}{r_o+R_L}}$

$R_{gs} = \frac{20^K+0.5^K}{1+(5+1)0.5\frac{40}{40+40}} = 8.2\ k\Omega$

$R_{C_L} = R_L \| R_{out} = R'_L = 32\ k\Omega$

$\tau_H = C_{gs}R_{gs} + C_{gd}R_{gd} + C_L R_{C_L} = 2\times 8.2 + 0.1\times 820 + 32\times 1$

$\tau_H = 130.4\ ns$

$f_H = \frac{1}{2\pi\tau_H} = 1.22\ MHz$

$|A_M|\cdot f_H = 48.8\ MHz$

8.90

$f_T = |A_M|\cdot f_H = \frac{1}{2\pi C_{gd}\times R_{sig}}$

for $C_{gd} = 0.1$ pF and $R_{sig} = 10$ kΩ

$f_T = 159.2$ MHz

(b) If $|A_M| = 20$ V/V $\Rightarrow f_T = 159.2$ MHz

and $f_H = \frac{159.2\ M}{20} = 7.96$ MHz

(c)

$g_m = 5\frac{mA}{V}$, $A_O = 100$ V/V, $R_L = 20$ kΩ

$A_O = g_m\cdot r_O \Rightarrow r_O = \frac{100}{5\ m} = 20\ k\Omega$

$R_L = r_O = 20\ k\Omega$

we can rewrite

$A_M = -G_m(R_O \| R_L)$ as

$A_M = -(g_m r_O)\cdot\frac{R_L}{R_L+R_O}$

$\Rightarrow 20 = 100\times\frac{20}{R_O+20} \Rightarrow R_O = 80\ k\Omega$

Since

$R_O = r_O[1+g_m R_S] = 20\ K\ [1+5\ m\times R_S]$

$= 80\ K \Rightarrow R_S = 600\ \Omega$

8.91

$R_{gs} = \frac{R_{sig}+R_S}{1+\underbrace{(g_m+g_{mb})R_S}_{K}\frac{r_o}{r_o+R_L}}$

If we define:

$K = (g_m+g_{mb})R_S$ and $R_{sig} \gg R_S$

then: $R_{gs} \cong \frac{R_{sig}}{1+K\frac{r_o}{r_o+r_o}} = \frac{R_{sig}}{1+K/2}$

$G_m = \frac{g_m}{1+(g_m+g_{mb})R_S} = g_m/(K+1)$

$R'_L = R_L \| R_{out}$

$R_{out} = r_o[1 + (g_m + g_{mb})R_S] = r_o(1+K)$

$R'_L = r_o \| r_o(1+K) = r_o \frac{(1+K)}{2+K}$

Using Eq. 6.148:

$R_{gd} = R_{sig}(1 + G_m R'_L) + R'_L$

$R_{gd} = R_{sig}\left(1 + \frac{g_m}{1+K} \times r_o \frac{1+K}{2+K}\right) + r_o \frac{1+K}{2+K}$

$R_{gd} = R_{sig}\left(1 + \frac{A_o}{2+K}\right) + r_o \frac{1+K}{2+K}$

$R_{C_L} = R'_L = r_o \frac{1+K}{2+K}$

$\tau_H = R_{gs}C_{gs} + R_{gd}C_{gd} + R_{C_L}C_L$

$\tau_H = \frac{R_{sig}}{1+K/2}C_{gs} + R_{sig}\left(1 + \frac{A_o}{2+K}\right)C_{gd} + (C_L + C_{gd})r_o\frac{(1+K)}{2+K}$

8.92

$R_{out} = r_o[1 + (g_m + g_{mb})R_S] = r_o(1+K)$

$R_{out} = 40(1+K)$

$A_M = -g_m r_o \frac{R_L}{R_L + R_{out}} = -5 \times 40 \times \frac{40}{40 + 40(1+K)}$

$\boxed{A_M = -\frac{200}{2+K}}$

$\tau_H = \frac{C_{gs}R_{sig}}{1+K/2} + C_{gd}R_{sig}\left(1 + \frac{A_o}{2+K}\right) + (C_L + C_{gd})r_o\frac{1+K}{2+K}$

(From problem 6.111)

$\tau_H = \frac{2^p\, 20^k}{1+K/2} + 0.1^p \times 20^k\left(1 + \frac{5 \times 40}{2+K}\right) + (1^p + 0.1)40^k \frac{1+K}{2+K}$

$\tau_H = \frac{80}{2+K} + 2\left(1 + \frac{200}{2+K}\right) + 44\frac{1+K}{2+K}$

$\tau_H = \frac{528 + 46K}{2+K}$ nS

$f_H = \frac{1}{2\pi\tau_H} = \frac{(2+K) \times 10^3}{2\pi(528 + 46K)}$ MHz

$f_T = |A_M| \cdot f_H$.

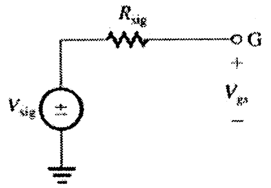
K	A_M (V/V)	f_H (MHz)	$\lvert A_M\rvert \cdot f_H$ (MHz)
0	−100	0.603	60.3
1	−66.67	0.832	55.47
2	−50.00	1.027	51.35
3	−40.00	1.195	47.8
4	−33.33	1.342	44.73
5	−28.57	1.471	42.03
6	−25.00	1.584	39.6
7	−22.22	1.686	37.46
8	−20.00	1.777	35.54
9	−18.18	1.859	33.8
10	−16.67	1.934	32.24
11	−15.38	2.002	30.79
12	−14.28	2.064	29.47
13	−13.33	2.121	28.27
14	−12.5	2.174	26.75
15	−11.76	2.223	26.14

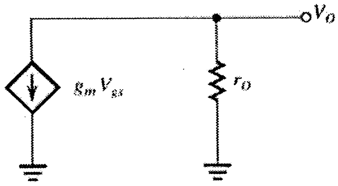
If f_H = 2MHz, then by looking at the table, $K \approx 11$. Therefore: $K = 11 = (g_m + g_{mb})R_S \Rightarrow$

$R_S = \frac{11}{5+1} = 1.83\,k\Omega$

From the table: $A_M = -15.38$

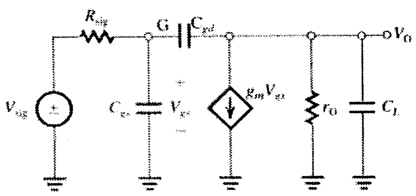
8.93





then $\frac{V_O}{V_{sig}} = A_M = -g_m r_O$

The high-frequency small-signal circuit is:



Using the open-circuit time constants method:

$\tau_H = R_{gs} \times C_{gs} + R_{gd} \times C_{gd} + R_L \times C_L$

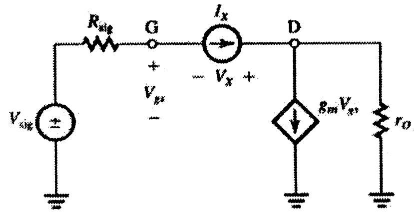
Setting $C_{gd} = C_L = 0$ we can see that

$R_{gs} = R_{sig}$

Setting $C_{gs} = C_{gd} = 0$ we can see that

$R_L = r_O$

To obtain R_{gd} set $C_{gs} = C_L = 0$ and consider the following circuit:



$R_{gd} = R_X = \frac{V_X}{I_X}$

At node G: $V_{gs} = -I_X \cdot R_{sig}$ Eq 1.

At node D: $I_X = g_m V_{gs} + \frac{V_X + V_{gs}}{r_O}$ Eq 2.

Substituting V_{gs} in Eq 2 by Eq 1 and re-arranging:

$\frac{V_X}{I_X} = R_X = R_{sig}[1 + g_m r_O] + r_O$

Thus,

$\tau_H = C_{gs} \cdot R_{sig} + C_{gd}[R_{sig}[1 + g_m r_O] + r_O]$
$+ C_L r_O$

If $g_m = 1\frac{\text{mA}}{\text{V}}$, $r_O = 20\text{ k}\Omega$, $R_{sig} = 20\text{ k}\Omega$

$C_{gs} = 20\text{ fF}$, $C_{gd} = 5\text{ fF}$, $C_L = 10\text{ fF}$

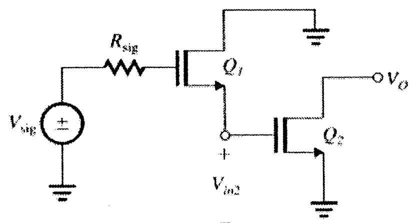
$A_M = -g_m \cdot r_O = -1 \times 20 = -20\text{ V/V}$

$\tau_H = 20\text{ f} \times 20\text{ K} + 5\text{ f}[20\text{ K}(1 + 1 \times 20) + 20\text{ K}]$
$+ 10\text{ f} \times 20\text{ K}$
$= 2.8\text{ ns}$

$f_H = \frac{1}{2\pi\tau_H} = 56.8\text{ MHz}$

$|A_M| \cdot f_H = 20 \times 56.8\text{ M} = 1.14\text{ GHz}$

(b) For the low-frequency analysis of the CD-CS consider the following circuit



or the CS:

$V_O = V_{in2}(-g_{m2}r_{O2})$ (Eq 1)

For a CD amplifier:

$$A_M = \frac{(R_L \parallel r_O)}{(R_L \parallel r_O) + (1/g_m)}$$

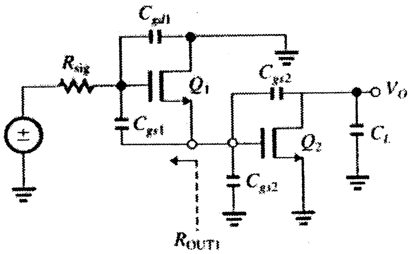
which adapted to our circuit provides:

$$V_{in2} = V_{sig}\frac{r_{O1}}{r_{O1} + 1/g_{m1}} \text{ (Eq. 2)}$$

Combining Eq. 1 and Eq. 2:

$$\frac{V_O}{V_{sig}} = A_M = \frac{-r_{O1}(g_{m2}\,r_{O2})}{\frac{1}{g_{m1}} + r_{O1}}$$

For the high-frequency analysis of the CD-CS consider the following circuit



$$\tau_H = C_{gd1} \times R_{gd1} + C_{gs1} \times R_{gs1} + C_{gs1} \times R_{gs2} + C_{gd2}R_{gd2} + C_L \cdot R_{CL}$$

To obtain the value of the resistors we must determine the corresponding equivalent circuits when all other capacitances are set to zero.

For C_{gd1}: Due to the ground connection on the drain of Q_1 C_{gd1} sees $R_{gd1} = R_{sig}$

For C_{gs1}:

$$I_X = \frac{V_X[1 + g_m r_O]}{R_{sig} + r_O}$$

$$\Rightarrow R_{gs1} = R_X = (R_{sig} + r_{O1})/(1 + g_{m1}r_{O1})$$

For C_{gs2}: C_{gs2} sees R_{OUT1} which for a CD amplifier is :

$$R_{OUT1} = R_{gs2} = \frac{1}{g_{m1}} \parallel r_{O1}$$

For C_{gd2}: Referring to the analysis of the CS amplifier

$$R_{gd} = R_{sig}(1 + g_m R_L') + R_L'$$

which adapted to our circuit becomes:

$R_L' = r_{O2}$ and

$$R_{sig} = R_{OUT1} = \frac{1}{g_{m1}} \parallel r_{O1}$$

$$\Rightarrow R_{gd2} = \left(\frac{1}{g_{m1}} \parallel r_{O1}\right) \cdot (1 + g_{m2} \cdot r_{O2}) + r_{O2}$$

For C_L: C_L sees $r_{O2} \Rightarrow R_{CL} = r_{O2}$.

Therefore:

$$\tau_H = C_{gd1} \cdot R_{sig} + C_{gs1}\left(\frac{R_{sig} + r_{O1}}{1 + g_{m1}r_{O1}}\right) + C_{gs2}\left(\frac{1}{g_{m1}} \parallel r_{O1}\right) + C_{gd2}\left[\left(\frac{1}{g_{m1}} \parallel r_{O1}\right)(1 + g_{m2}\,r_{O2}) + r_{O2}\right] + C_L \cdot r_{O2}$$

For the circuit parameters of part (a): $r_{O1} = r_{O2}$

$g_{m1} = g_{m2}$

$$A_M = -\frac{20\text{ K}}{\frac{1}{1\text{ m}} + 20\text{ K}} \cdot (1 \times 20) = -19 \text{ V/V}$$

$C_{gd1} \cdot R_{sig} = 5\text{ f} \times 20\text{ K} = 100\text{ ps}$

$$C_{gs1} \cdot \frac{R_{sig} + r_{O1}}{1 + g_{m1}r_{O1}} = 20\text{ f} \times \frac{(20\text{ K} + 20\text{ K})}{(1 + 1 \times 20)}$$

$= 38.1$ ps

$$C_{gs2}\left(\frac{1}{g_{m1}} \parallel r_{O1}\right) = 20\text{ f}(1\text{ K} \parallel 20\text{ K}) = 19.0\text{ ps}$$

$$C_{gd2}\left[\left(\frac{1}{g_{m1}} \parallel r_{O1}\right) \cdot (1 + g_{m2}r_{O2}) + r_{O2}\right]$$

$= 5\text{ f} \times \{(1\text{K} \parallel 20\text{ K})(1 + 20) + 20\text{ K}\}$

$= 200$ ps

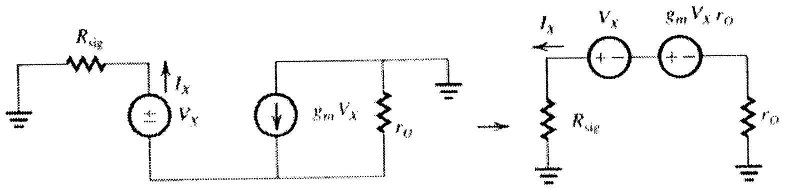
$C_L \cdot r_{O2} = 10\text{ f} \times 20\text{ K} = 200\text{ ps}$

$\tau_H = 100 + 38.1 + 19 + 200 + 200 = 557.1$ ps

$$\Rightarrow f_H = \frac{1}{2\pi\,\tau_H} = 285.7\text{ MHz}$$

$|A_M| \cdot f_H = 19 \times 285.7\text{ M} = 5.4\text{ GHz}$

Comparing with the stand-alone CS amplifier of part (a) we can see how A_M is approx. the same, while f_H and thus the gain-bandwidth product have increased by a factor of 5.



8.94

Each of the transistors is operating at a bias current of approximately 100 μA. Thus:

$$g_m = \frac{0.1}{0.025} = 4 \text{ mA/V},$$

$$r_\pi = \frac{100}{4} = 25 \text{ k}\Omega$$

$$r_e \simeq 250\ \Omega,\ r_O = \frac{100}{0.1} = 1 \text{ M}\Omega$$

$$C_\pi + C_\mu = \frac{g_m}{2\pi f_T} = \frac{4 \text{ m}}{2\pi \times 400 \text{ M}} = 1.59 \text{ pF}$$

$$\Rightarrow C_\pi = 1.39 \text{ pF}$$

a) $R_{in} = (\beta + 1)[r_{e1} + (r_{\pi 2} \parallel r_{O1})]$

$$R_{in} = 101[250 \times 10^{-3} + 25 \text{ k}\Omega \parallel 1 \text{ M}\Omega]$$

$$\simeq 2.5 \text{ M}\Omega$$

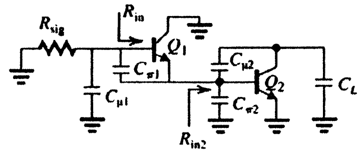
Using Miller's Theorem for $C_{\mu 2}$:

$$A_M = -\frac{R_{in}}{R_{in} + R_{sig}} \times \frac{r_{\pi 2} \parallel r_{O1}}{r_{e1} + (r_{\pi 2} \parallel r_{O1})} \times g_{m2} r_{O2}$$

$$A_M = -\frac{2.5 \text{ M}}{2.5 \text{ M} + 0.01} \times \frac{25 \text{ K} \parallel 1 \text{ M}}{0.25 + (25 \text{ K} \parallel 1 \text{ M})} \times 4 \times 1 \text{ M}$$

$$A_M = -3943.6 \text{ V/V}$$

b) To calculate f_H.



$$R_{\mu 1} = R_{sig} \parallel R_{in} = 10 \text{ k}\Omega \parallel 2.5 \text{ M}\Omega = 10 \text{ k}\Omega$$

$$R_{in2} = r_{\pi 2} \parallel r_{O1}$$

$$R_{in2} = 25 \text{ k}\Omega \parallel 1 \text{ M}\Omega$$

$$R_{in2} = 24.4 \text{ k}\Omega$$

$$R_{\pi 1} = \frac{R_{sig} + R_{in2}}{1 + \frac{R_{sig}}{r_{\pi 1}} + \frac{R_{in2}}{r_{e1}}}$$

$$R_{\pi 1} = \frac{10 + 24.4}{1 + \frac{10}{25} + \frac{24.4}{0.25}} = 0.35 \text{ k}\Omega$$

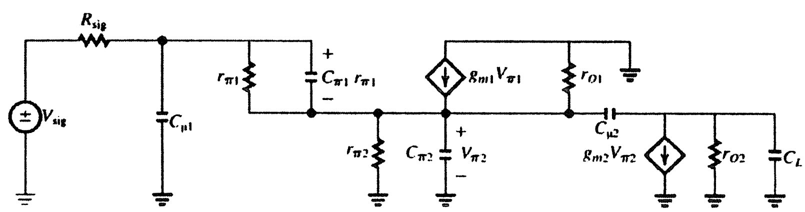
$$C_T = C_{\pi 2} + C_{\mu 2}(1 + g_{m2} r_{O2})$$

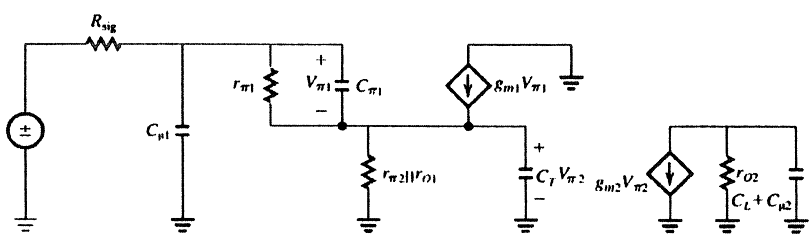
$$C_T = 1.39 + 0.2(1 + 4 \times 1000) = 801.6 \text{ pF}$$

$$R_T = r_{\pi 2} \parallel r_{O1} \parallel \frac{r_{\pi 1} + R_{sig}}{\beta + 1}$$

$$= 25 \text{ K} \parallel 1000 \text{ K} \parallel \frac{25 + 10}{101}$$

$$R_T = 342\ \Omega$$





$R_{\mu 2} = r_{O2} = 1000\ k\Omega$

$\tau_H = C_{\mu 1}R_{\mu 1} + C_{\pi 1}R_{\pi 1} + C_T R_T$
$\quad + (C_{\mu 2} + C_L) + R_{\mu 2}$

$\tau_H = 0.2 \times 10 + 1.39 \times 0.35 + 801.6 \times 0.342$
$\quad + (0.2 + 1) \times 1000$

$\tau_H = 2 + 0.49 + 274.15 + 1200$ ns

Thus $(C_L + C_{\mu 2})R_{\mu 2}$ is the dominating term,
The second most significant term is $C_T R_T$.
So $(C_L + C_{\mu 2})$ dominates and then C_T or equivalently $C_{\mu 2}$.

$$f_H = \frac{1}{2\pi\tau_H} = \frac{1}{2\pi \times 1476.6\ n} = 107.8\ \text{MHz}$$

8.95 Note: Although rather long, this is an excellent problem with considerable educational value.

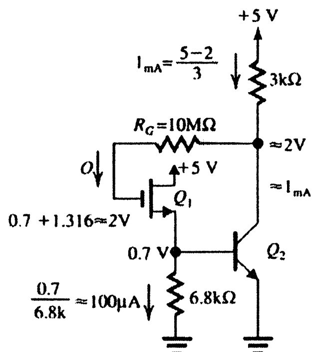
a) DC-Bias

For Q_1: $I_{D1} = \frac{1}{2}k'_n\frac{W}{L}(V_{GS} - V_t)^2$

$0.1 = \frac{1}{2} \times 2(V_{GS} - 1)^2$

$\Rightarrow V_{GS} = 1.316$ V

$I_{D1} \simeq 0.1$ mA
$I_{D1} \simeq 1$ mA
see analysis



b) For Q_1:

$g_{m1} = \sqrt{2\pi k'_n\frac{W}{L}I_D} = \sqrt{2 \times 2 \times 0.1}$
$= 0.63$mA/V

For Q_2: $g_{m2} = 40$ mA/V $r_{\pi 2} = \frac{200}{40} = 5\ k\Omega$

$$C_\pi + C_\mu = \frac{g_m}{2\pi f_T} = \frac{40 \times 10^{-3}}{2\pi \times 600 \times 10^6}$$
$= 10.6$ pF

Since $C_\mu = 0.8$ pF $\Rightarrow C_\pi = 9.8$ pF

c) at mid band :

$$\frac{V_\pi}{V_i} = \frac{6.8 \| 5}{(6.8 \| 5) + \frac{1}{g_{m1}}}$$

$$\frac{V_\pi}{V_i} = \frac{2.88}{2.88 + \frac{1}{0.63}}$$

$\frac{V_\pi}{V_i} = 0.64$ V / V

$\frac{V_o}{V_\pi} = -g_{m2}V_\pi(1\ K \| 3\ K)$ where we have neglected the effect of R_G.

$\frac{V_o}{V_\pi} = -40 \times \frac{3}{4}V_\pi = -30V_\pi \Rightarrow \frac{V_o}{V_\pi}$
$= 0.64 \times (-30) = -19.2$ V/V

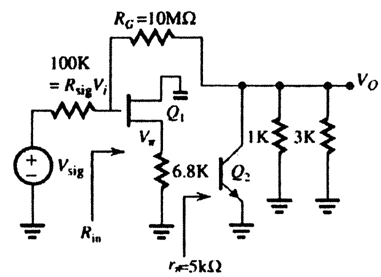
$\frac{V_o}{V_\pi} = -19.2$ V / V

$$R_{in} = \frac{R_G}{1 - \frac{v_o}{v_i}} = \frac{10^M}{1 - (-19.2)} = 495\ k\Omega$$

$$\frac{V_o}{V_{sig}} = \frac{R_{in}}{R_{sig} + R_{in}} \times \frac{v_o}{v_i} = \frac{495}{495 + 100} \times -19.2$$
$= -16$ V/V

d) At low frequencies :

$$C_1 \rightarrow f_{P1} = \frac{1}{2\pi \times 0.1 \times 10^{-6} \times (100 + 495) \times 10^3}$$
$= 2.7$ Hz



$$C_2 \rightarrow f_{P2} = \frac{1}{2\pi \times 1 \times 10^{-6} \times (3 + 1) \times 10^{-3}}$$
$= 40$ Hz

Thus $f_L \simeq 40$ Hz

e) At high frequencies :
The high frequency equivalent circuit is as follows:

$C_T = C_\pi + C_\mu(1 + g_{m2}R'_L)$

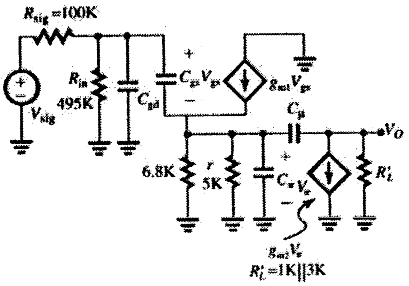
$= 9.8 + 0.8\left(1 + 40 \times \frac{3}{4}\right) = 34.6$ pF

$R_{gd} = R'_{sig} = 100\text{ K} \| 495\text{ K} = 83.2\text{ k}\Omega$

$$R_{gs} = \frac{R'_{sig} + (6.8\text{ K} \| 5\text{ K})}{1 + g_{m1}(6.8\text{ K} \| 5\text{ K})} = \frac{83.2 + 2.88}{1 + 0.63 \times 2.88}$$

$= 30.6\text{ k}\Omega$

$R_T = 6.8 \| 5 \| \frac{1}{g_m} = 1\text{ k}\Omega$



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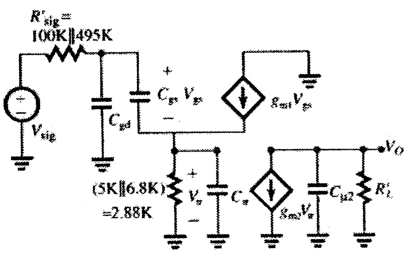


$R'_L = 0.75\text{ k}\Omega$

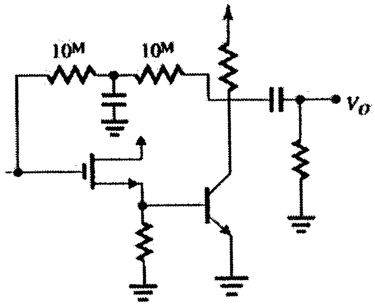
$\tau_H = C_{gd}R_{gd} + C_{gs}R_{gs} + C_T R_T + C_\mu R'_L$

$\tau_H = 1 \times 83.2 + 1 \times 30.6 + 34.6 \times 1 + 0.8 \times 0.75 = 149\text{ nS}$

$f_H \simeq \frac{1}{2\pi\tau_H} = 1.07\text{ MHz}$

f) There will no longer be a signal feedback. The lefthand side 10 MΩ Resistor will in effect appear between the input terminal and ground. Thus: $R_{in} = 10\text{ M}\Omega$ (a factor of 20 increase) and correspondingly A_M becomes:

$$A_M = \frac{10}{10.1} \times (-19.2) = -19\text{ V/V}$$



(an increase from −16 V/V) Now R'_{sig} becomes approximately 100 kΩ, as compared to 83.2 kΩ, and correspondingly R_{gd} becomes 100 kΩ, and R_{gs} becomes 36.6 kΩ while R_T and R'_L remain practically unchanged. Thus τ_H becomes 172.5 nS and f_H decreases from 1.07 MHz to 0.92 MHz.

8.96

$$V_{G1} = V_S \cdot \frac{2/g_m}{2/g_m + R_S} \qquad I = \frac{V_{G1}}{2/g_m}$$

$$V_O = I R_D = \frac{V_{G1}}{2/g_m} \times R_D$$

$$= V_S \times \frac{2/g_m}{2/g_m + R_S} \cdot \frac{R_D}{2/g_m}$$

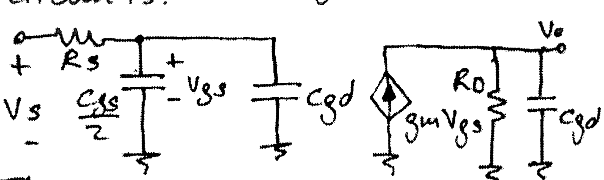
$$= \frac{V_S \cdot R_D}{2/g_m + R_S}$$

$$\Rightarrow A_O = \frac{V_O}{V_S} = \frac{g_m R_D}{2 + g_m R_S}$$

$$g_m = \frac{200\,\mu A}{0.25V} = 0.8\,\frac{mA}{V}$$

$$\Rightarrow A_O = \frac{0.8 \times 50}{2 + 0.8 \times 200} = 0.24\text{ V/V}$$

The high-frequency equivalent circuit is:



Thus, the pole at the input has a frequency f_{P1}:

CONT.

$$f_{p1} = \frac{1}{2\pi R_s \times \left(\frac{C_{gs}}{2} + C_{gd}\right)}$$

$$= \frac{1}{2\pi \times 200K \times (1/2 + 1)p}$$

$$= 530\ KHz$$

and the pole at the output has a frequency f_{p2}:

$$f_{p2} = \frac{1}{2\pi R_o C_{gd}} = \frac{1}{2\pi \times 50K \times 1p}$$

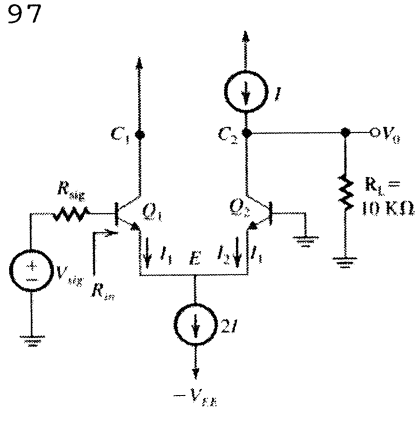
$$= 3.18\ MHz$$

Thus $f_H \simeq \dfrac{1}{\sqrt{\left(\frac{1}{530K}\right)^2 + \left(\frac{1}{3.18M}\right)^2}}$

$$= 523\ KHz$$

Notice that this low value of f_H is due to the large value of R_s.

8.97



$I_1 = I_2 = I = 1$ mA

$g_m = \dfrac{I}{V_T} = \dfrac{1\text{ mA}}{25\text{ mV}} = 40$ mA/V

$r_e = \dfrac{V_T}{I_E} = 25\ \Omega$

$r_\pi = (\beta + 1)r_e \simeq 3\ k\Omega$

$2\pi f_T = \dfrac{g_m}{C_\pi + C_\mu}$

$C_\pi + C_\mu = \dfrac{g_m}{2\pi f_T} = \dfrac{40 \times 10^{-3}}{2\pi \times 700 \times 10^6}$

$\simeq 9.1$ pF

$C_\pi = 9.1 - C_\mu = 8.6$ pF

$R_{in} = (\beta + 1)(2r_e) = 2r_\pi$

$= 6\ k\Omega$

$A_M = \dfrac{1}{2}\left(\dfrac{R_{in}}{R_{in} + R_{sig}}\right) \times g_m R_L$

$= \dfrac{1}{2}\left(\dfrac{6}{6 + 20}\right) \times 40 \times 10$

$A_M = 46.15$ V/V

The pole at the input side is

$f_{p1} = \dfrac{1}{2\pi\left(\frac{C_\pi}{2} + c_\mu\right)(R_{sig} \| 2r_\pi)}$

$= \dfrac{1}{2\pi\left(\frac{8.6}{2} + 0.5\right) \times 10^{-12} \times (20\text{ K} \| 6\text{ K})}$

$= 7.18$ MHz

$f_{p2} = \dfrac{1}{2\pi C_\mu R_L} = \dfrac{1}{2\pi \times 0.5 \times 10^{-12} \times 10\text{ K}}$

$= 31.8$ MHz

$f_H = \dfrac{1}{\sqrt{\left(\frac{1}{f_{p1}}\right)^2 + \left(\frac{1}{f_{p2}}\right)^2}}$

$\simeq 7$ MHz

8.98

$I_1 = I_2 = I = 1mA \Rightarrow g_m = 40\ mA/V$, $r_\pi = \frac{120}{40} = 3K\Omega$

$r_e = \frac{3}{121} \cong 25\Omega$, $C_\pi + C_\mu = \frac{g_m}{2\pi f_T} = \frac{40^m}{2\pi \times 700^M} = 9.1pF$

$\Downarrow$

$C_\pi = 8.6pF$

Using Eq. 6.185:

$$A_M = \frac{v_o}{v_{sig}} = \frac{1}{2}\left(\frac{R_{in}}{R_{in} + R_{sig}}\right) g_m R_L$$

$$R_{in} = 2r_\pi = 2 \times 3K\Omega = 6K\Omega$$

$$A_M = \frac{1}{2} \times \frac{6}{6+20} \times 40 \times 10^K = 46.15\ V/V$$

$$f_{p1} = \frac{1}{2\pi\left(\frac{C_\pi}{2} + C_\mu\right)(R_{sig} \| 2r_\pi)} = \frac{1}{2\pi\left(\frac{8.6}{2} + 0.5\right)(20^K \| 6^K)}$$

$$f_{p1} = 7.19MHz$$

$$f_{p2} = \frac{1}{2\pi C_\mu R_L} = \frac{1}{2\pi \times 0.5^P \times 10^K} = 31.8\ MHz$$

$$f_H = \frac{1}{\sqrt{\left(\frac{1}{f_{p1}}\right)^2 + \left(\frac{1}{f_{p2}}\right)^2}} = 7.01MHz$$

8.99

All the transistors in this problem are operating at a bias current of 0.5 mA and thus have :

$r_e = 50\Omega,\ g_m = 20\ \text{mA/V},\ r_\pi = 5\ \text{k}\Omega$

$$C_\pi + C_\mu = \frac{20\ \text{m}}{2\pi \times 400\ \text{m}} = 8\ \text{pF}$$

since $c_\mu = 2\ \text{pF} \Rightarrow C_\pi = 6\ \text{pF},\ r_o = \infty,$

$r_x = 0$

a) Common-Emitter amplifier:

$R_{sig} = 10\ \text{k}\Omega,\ R_C = 10\ \text{k}\Omega$

$$A_M = -\frac{r_\pi}{R_{sig} + r_\pi} g_m R_C = -\frac{5}{10+5} 20 \times 10$$

$= -66.7\text{V/V}$

$$f_H = \frac{1}{2\pi(R_{sig} \| r_\pi)[C_\pi + (1 + g_m R_c)C_\mu]} \Rightarrow$$

$$f_H = \frac{1}{2_\pi(10^k \| 5^k)[6^p + (1 + 20 \times 10)^2]}$$

$= 117\ \text{KHz}$

b) Cascode :

$$A_M = -\frac{\beta_1 \alpha_2 R_c}{R_{sig} + r_\pi} = -\frac{100 \times 0.99 \times 10}{10 + 5}$$

$= -66\ \text{V/V}$

Input pole

$$= f_{p1} = \frac{1}{2\pi(R_{sig} \| r_{M1})(C_{\pi 1} + 2C_{\mu 1})}$$

$$f_{p1} = \frac{1}{2_\pi(10^k \| 5^k)(6 + 4)^p} = 4.77\ \text{MHz}$$

output pole :

$$f_{p3} = \frac{1}{2\pi C_{\mu 2} R_C} = \frac{1}{2\pi \times 2^p \times 10^k} = 7.96\ \text{MHz}$$

pole at midband node :

$$f_{p2} = \frac{1}{2\pi C_{\pi 2} r_{e2}} = \frac{1}{2\pi \times 6^p \times 50} = 530.5\text{MHz}$$

very high

$$f_H = \sqrt{\frac{1}{\left(\frac{1}{f_{p1}}\right)^2 + \left(\frac{1}{f_{p2}}\right)^2}} = 4.1\text{MHz}$$

c) CC−CB Cascade (modified diff. amplifier)

$$A_M = \frac{\beta R_C}{R_{sig} + 2r_\pi} = \frac{100 \times 10}{10 + 10} = 50\ \text{V/V}$$

Input pole

$$:f_{p1} = \frac{1}{2\pi(R_{sig} \| 2r_\pi)(C_{\pi/2}/2 + C_\mu)}$$

$$f_{p1} = \frac{1}{2\pi(10^k \| 10^k)(3 + 2)^p} = 6.4\ \text{MHz}$$

Output pole:

$$f_{p2} = \frac{1}{2\pi C_{\mu 2} R_C} = \frac{1}{2\pi \times 2^p \times 10^k} = 7.96\ \text{MHz}$$

$$\text{Thus: } f_H = \frac{1}{\sqrt{\left(\frac{1}{f_{p1}}\right)^2 + \left(\frac{1}{f_{p2}}\right)^2}} = 5\ \text{MHz}$$

d) CC−CE Cascade :

$$A_M = -\frac{(\beta_1 + 1)\beta_2 R_C}{R_{sig} + r_{\pi 1} + (\beta_1 + 1)r_{\pi 2}}$$

$$= -\frac{101 \times 100 \times 10}{10 + 5 + 101 \times 5} = -194\ \text{V/V}$$

Refer to Example 6.13 in :

$R_{\mu 1} = (R_{sig} \| R_{in}) = 10^k \| (\beta + 1)[r_{e1} + r_{\pi 2}]$

$Ri_{\mu 1} = 10^k \| 101 \times [0.05 + 5] = 9.81\ \text{k}\Omega$

$$R_{\pi 1} = r_{\pi 1} \| \frac{R_s + r_{\pi 2}}{1 + g_{m1} r_{\pi 2}} = 5 \| \frac{10 + 5}{1 + 20 \times 5} = 144\ \Omega$$

$$R_T = r_{\pi 2} \| \frac{r_{\pi 1} + R_{sig}}{\beta + 1} = 5^k \| \frac{5 + 10}{101} = 144\ \Omega$$

where

$C_T = C_{\pi 2} + C_{\mu 2}(1 + g_{m2} R_C) = 6 + 2(1 + 200)$

$C_T l = 408\ \text{pF}$

$R_{\mu 2} = R_C = 10\ \text{k}\Omega$

$\tau_H = C_{\mu 1} R_{\mu 1} + C_{\pi 1} R_{\pi 1} + C_T R_T + C_{\mu 2} R_{\mu 2}$

$\tau_H = 2 \times 9.81 + 6 \times 0.144 + 408 \times 0.144 + 2 \times 10$

$\tau_H = 19.62 + 0.86 + 58.75 + 20 = 99.2\text{ns}$

$$f_H = \frac{1}{2\pi\tau_H} = \frac{1}{2\pi \times 99.2^n} = 1.6\ \text{MHz}$$

e) Folded Cascode :

$$A_M = -\frac{\beta_1 \alpha_2 R_C}{R_{sig} + r_{\pi 1}} = -\frac{100 \times 0.99 \times 10}{10 + 5}$$

$= -66(\text{V})/\text{V}$

Input pole :

$$f_{p1} = \frac{1}{2\pi(R_{sig} \| r_{\pi 1})(C_{\pi 1} + 2C_{\mu 1})}$$

$$= \frac{1}{2\pi(10 \| 5)(6 + 4)}$$

$f_{p1} = 4.77\text{MHz}$

At middle:

$$f_{p2} = \frac{1}{2\pi C_{\pi 2} r_{e2}} = \frac{1}{2\pi \times 6^p \times 0.05}$$

$= 530\text{MHz}$ very high!

At output:

$$f_{p3} = \frac{1}{2\pi C_{\mu 2} R_C} = \frac{1}{2\pi \times 2 \times 10}$$

$f_{p3} = 7.96\text{MHz}$

$$\text{Thus : } f_H \simeq \frac{1}{\sqrt{\frac{1}{4.77^2} + \frac{1}{7.96^2}}} = 4.1\ \text{MHz}$$

f) CC−CB Cascade :

$$A_M = \frac{(\beta_1 + 1)\alpha_2 R_C}{R_{sig} + (\beta_1 + 1)2r_e} = \frac{101 \times 0.99 \times 10}{10 + 101 \times 0.1}$$

$\simeq 50$ V/V

$$\text{Input pole}: f_{P1} = \frac{1}{2\pi(R_{sig} \| 2r_\pi)(C_{\pi/2} + C_\mu)}$$

$$f_{P1} = \frac{1}{2\pi(10^k \| 10^k)(3^p + 2^p)} = 6.4 \text{ MHz}$$

Output pole:

$$f_{P2} = \frac{1}{2\pi R_C C_\mu} = \frac{1}{2\pi \times 10^k \times 2^p} = 7.96 \text{ MHz}$$

$$f_H \simeq \frac{1}{\sqrt{\frac{1}{6.4^2} + \frac{1}{7.96^2}}} = 5 \text{ MHz}$$

Summary of results :

ConFiguration	A_M (V/V)	f_H (MHz)	G.B (MHz)
a)CE	−66.7	0.117	7.8
b) Cascode	−66	4.1	271
c) CC_CB Cascade	+50	5.0	250
d)CC_CE Cascade	−194	1.6	310
e) Folded Cascode	−66	4.1	271
f) CC_CB Cascade	+50	5.0	250

DC-gain = $(G_{m1}R_1) \times (G_{m2}R_2)$

$= (1 \times 100) \times (2 \times 50) = 10$ K V/V → 80 dB

(b) From Eq (9.175) if C_C is not connected:

$$\omega_{P1} = \frac{1}{C_1 \cdot R_1 + C_2 \cdot R_2}$$

$$= \frac{1}{0.1\text{ p} \times 100\text{ K} + 2\text{ p} \times 50\text{ K}}$$

$$= 9.1\text{ M}\frac{\text{rad}}{\text{s}}$$

$$\Rightarrow f_{P1} = 1.45 \text{ MHz}$$

To obtain ω_{P2} we equate the coefficients of s^2 in Eq (9.171) to $1/(\omega_{P1} \cdot \omega_P)$

Thus, for C_C not connected.

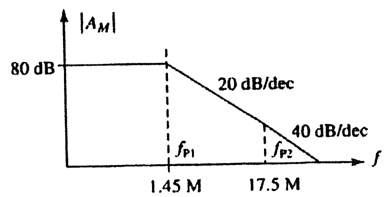
$$C_1 C_2 R_1 R_2 = \frac{1}{\omega_{P1} \cdot \omega_{P2}}$$

$$\Rightarrow \omega_{P2} = \frac{C_1 R_1 + C_2 R_2}{C_1 C_2 \cdot R_2 \cdot R_2}$$

$$\omega_{P2} = \frac{0.1\text{ p} \times 100\text{ K} + 2\text{ p} \times 50\text{ K}}{0.1\text{ p} \times 2\text{ p} \times 100\text{ K} \times 50\text{ K}}$$

$= 110$ MHz $\Rightarrow f_{P2} = 17.5$ MHz

The Bode plot for the gain magnitude is



(c) Since (C_1 = 0.1 pF) << (C_2 = 2 pF) and if $C_1 << C_C$ then from Eq (9.177)

$$\omega_{P2} \simeq \frac{G_{m2}}{C_2} \rightarrow \omega_{P2} \simeq \frac{2\text{ m}}{2\text{ p}} \simeq 1 \text{ G rad/s}$$

$$\rightarrow f_{P2} = 159 \text{ MHz}$$

two octaves below are = $\omega_{P2}/4$ = 250 M rad/s → 40 MHz then, from Eq 9.178:

$$250\text{ M} \le \frac{G_{m1}}{C_C} \Rightarrow C_C \le \frac{1\text{ m}}{250\text{ M}} \Rightarrow C_C \le 4 \text{ pF}$$

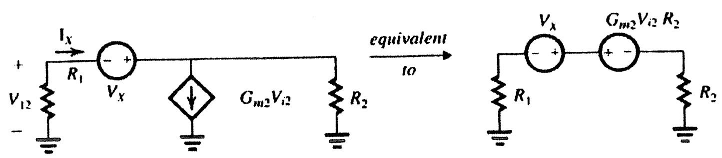
For C_C = 4 pF and Eq (9.176)

$$\omega_{P1} \simeq \frac{1}{R_1 C_C G_{m2} R_2}$$

$$\simeq \frac{1}{100\text{ K} \times 4\text{ p} \times 2\text{ m} \times 50\text{ K}} = 25\text{ K}\frac{\text{rad}}{\text{s}}$$

8.100

(a) To obtain the DC-gain: for s = 0 in



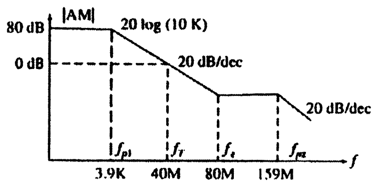
$\Rightarrow f_{P1} = 3.9$ KHz

From Eq (9.173):

$$\omega_Z = \frac{G_{m2}}{C_C} = \frac{2\text{ m}}{4\text{ p}} = 500\text{ M}\frac{\text{rad}}{\text{s}}$$

$\Rightarrow f_Z = 79.6\text{ MHz} \sim 80\text{ MHz}$

The Bode plot for the gain magnitude is



8.101

$G_{m1} = g_{m1} = g_{m2}$; transconductance of input stage.

$G_{m2} = g_{m6}$; transconductance of second stage.

C_1 = C at node D_2 = 0.2 pF

C_2 = C at node D_6 = 3 pF

For $f_T = 50\text{ MHz} = \dfrac{G_{m1}}{2\pi \times C_C}$

$$\Rightarrow C_C = \frac{1\text{ m}}{2\pi \times 50\ \mu} = 3.2\text{ pF}$$

$$f_Z = \frac{G_{m2}}{2\pi C_C} = \frac{3\text{ m}}{2\pi \times 3.2\text{ P}} \cong 149\text{ MHz}$$

$$f_2 = \frac{G_{m2}}{2\pi C_2} = \frac{3\text{ m}}{2\pi \times 3\text{ P}} \cong 159\text{ MHz}$$

$f_T(50\text{ MHz}) < f_Z\ (149\text{ MHz}) < f_2(159\text{ MHz})$

8.102

For both transistors:

$V_{ov} = 0.2$ V $C_{gs} = 20$ fF

$I = 0.1$ mA $C_{gd} = 5$ fF

$|V_A| = 10$ V $C_{db} = 5$ fF

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.1\text{ mA}}{0.2\text{ V}} = 1\ \frac{\text{mA}}{\text{V}}$$

$$r_O = \frac{V_A}{I_D} = \frac{10}{0.1\text{ mA}} = 100\text{ k}\Omega$$

(a) DC - Voltage gain

$V_O = -g_{m2}r_{O2} \times V_{gs2}$ and

$$V_{gs2} = -g_{m1}r_{O1}V_{\text{sig}} \Rightarrow A_M = \frac{V_O}{V_{\text{sig}}}$$

$$= (g_m r_O)^2 = (1 \times 100)^2 = 10\text{ KV/V}$$

(b) Using the Miller approximation at node $G1$

$C_{eq} = C_{gd}(1 + g_m R_L')$ Eq (9.46)

$\Rightarrow C_{eq1} = C_{gd1}(1 + g_{m1}r_{O1})$

$= 5\text{ f}\ (1 + 100) = 50\text{ fF}$

$C_{\text{in1}} = C_{gs1} + C_{eq1} = 20\text{ f} + 505\text{ f} = 525\text{ fF}$

(c) $R_{\text{sig}} = 10\text{ k}\Omega$

The pole caused by C_{in1} at node G_i is

$$f_{P1} = \frac{1}{2\pi.\ R_{\text{sig}}.C_{\text{in1}}} = \frac{1}{2\pi \times 10\text{ K} \times 525\text{ f}}$$

$= 30.3$ MHz

(d) Using the Miller approximation at node G_2

$C_{eq2} = C_{gd2}(1 + g_{m2}r_{O2}) = 505\text{ fF}$

$C_{in2} = C_{db_1} + C_{gs2} + C_{eq2} = 5\text{ f} + 20\text{ f} + 505\text{ f}$

$= 530$ fF

(e) At node $G2$ a pole is caused by C_{in2} and r_{O1}

$$f_{P2} = \frac{1}{2\pi \times 530f \times 100\text{ K}} = 3\text{ MHz}$$

(f) The total capacitance at the output node is

$C_{\text{out}} = C_{db2} + C_2$

where, using the Miller theorem, C_2 is

$$C_2 = C_{gd2}\left(1 + \frac{1}{g_{m2}r_{O2}}\right) = 5\text{ f}\left(1 + \frac{1}{100}\right) = 5.05\text{ fF}$$

$\Rightarrow C_{\text{out}} = 5\text{ f} + 5.05\text{ f} = 10.05\text{ fF}$

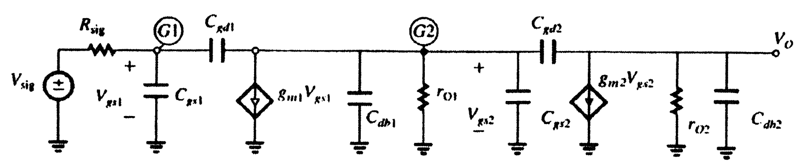
Thus a third pole is caused by C_{out} and r_{O2}

$$f_{P3} = \frac{1}{2\pi C_{\text{out}}.\ r_{O2}} = \frac{1}{2\pi \times 10.05f \times 100\text{ K}}$$

$= 158.4$ MHz

From the 3 poles: $f_{P1} = 30.3$ MHz, $f_{P2} = 3$ MHz, $f_{P3} = 158.4$ MHz f_{P2}, the pole formed at the interface of Q_1 and Q_2 is dominant.

(g) The pole formed at the interface of Q_1 and Q_2 is dominant pole. It is at the frequency of 3 MHz.



9.1 $A_F = \dfrac{A}{1+A\beta} = 100$

$A\beta = \dfrac{10^5}{100} - 1 = 999$

$\Rightarrow \beta = \dfrac{999}{10^5} = 9.99\times10^{-3}$

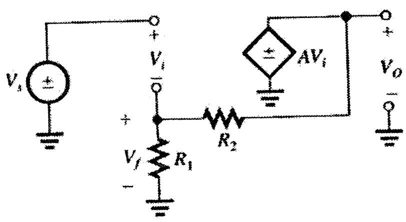
$A = 10^3, \quad A_F = \dfrac{10^3}{1+10^3(9.99\times10^{-3})}$

$= 90.99$

$\dfrac{\Delta A_f}{A_f} = \dfrac{90.99-100}{100} \Rightarrow -9\%$

9.2

(a) Replacing the op-amp with its equivalent circuit model:



$V_f = \beta V_O = \dfrac{R_1}{R_1+R_2}\cdot V_O$

$\Rightarrow \beta = \dfrac{R_1}{R_1+R_2}$

(b) $R_1 = 10\text{ k}\Omega$, $A_f = 10$ V/V, what is R_2 if:

(i) $A = 1000$ V/V

$A_f = \dfrac{A}{1+\beta A} \Rightarrow \beta = \dfrac{1}{A_f} - \dfrac{1}{A}$

$= \dfrac{1}{10} - \dfrac{1}{10^3} = 0.099$

$\beta = \dfrac{R_1}{R_1+R_2} \Rightarrow R_2 = R_1\left(\dfrac{1-\beta}{\beta}\right)$

$= 10\text{ K}\dfrac{(1-0.099)}{0.099} = 91.01\text{ k}\Omega$

(ii) $A = 100$ V/V

$\beta = \dfrac{1}{10} - \dfrac{1}{100} = 0.09;$

$R_2 = 10\text{ K}\dfrac{(1-0.09)}{0.09} = 101.11\text{ K}$

(iii) $A = 12$

$\beta = \dfrac{1}{10} - \dfrac{1}{12} = 0.0167;$

$R_2 = 10\text{ K}\dfrac{(1-0.0167)}{0.0167} = 588.8\text{ k}\Omega$

(c) if A decreases by 20%

(i) $A = 0.8\times1000 = 800$ V/V

$A_f = \dfrac{800}{1+(0.099)(800)} = 9.975$

$\dfrac{\Delta A_f}{A} = \dfrac{9.975-10}{10} = -0.25\%$

(ii) $A = 0.8\times100 = 80$ V/V

$A_f = \dfrac{80}{1+(0.09)(80)} = 9.756$

$\dfrac{\Delta A_f}{A} = \dfrac{9.756-10}{10} = -2.44\%$

(iii) $A = 0.8\times12 = 9.6$ V/V

$A_f = 9.6/(1+(0.0167)(9.6)) = 8.27$

$\dfrac{\Delta A_f}{A} = \dfrac{8.27-10}{10} = -17.26\%$

9.3

All output voltage is fed back $\therefore \beta = 1$

$A_f = \dfrac{100}{1+100\times1} = 0.99$

$1+A\beta = 1+100\times1 = 101 \equiv 40.1\text{dB}$

$V_O = 0.99V_s = 0.99\text{V}$

$V_i = V_s - V_o\beta = 1-0.99 = 10\text{mV}$

$A = 90 \Rightarrow A_f = \dfrac{90}{1+90\times1} \approx 0.989$

$\dfrac{\Delta A_f}{A_f} = \dfrac{0.989-0.99}{0.99} \equiv -0.1\%$

9.4

$A_F = \dfrac{A_0}{1+A_0\beta} = \dfrac{1}{1/A_0+\beta} = \dfrac{1}{\beta(1+1/A_0\beta)}$

so A_f + $1/\beta$ will be within $x\%$ when $1/(A_0\beta) = 0.01\times x$

(a) For 1% : $A_0\beta = 1/0.01 = 100$

Many possible solutions.

Let $A_0 = 10^5$ + $A_0\beta = 100 \Rightarrow \beta = 10^{-3}$

(b) For 5% : $A_0\beta = 1/0.05 = 20$

Let $A_0 = 10^5$ + $A_0\beta = 20 \Rightarrow \beta = 2\times10^{-4}$

(c) For 10% : $A_0\beta = 1/0.1 = 10$

Let $A_0 = 10^5$ + $A_0\beta = 10 \Rightarrow \beta = 10^{-4}$

(d) For 50% : $A_0\beta = 1/0.5 = 2$

Let $A_o = 10^5$: $A_o\beta = 2 \Rightarrow \beta = 2\times10^{-5}$

% error	A_o	$A_o\beta$	$1+A_o\beta$
1	10^5	100	101
5	10^5	20	21
10	10^5	10	11
50	10^5	2	3

9.5

$0 \le p \le 1$ linear

(a) For $A_O = 1$:

$$A_{f1} = \frac{A_O}{1+A_O\beta} = \frac{1}{1+0} = 1 \text{ V/V}$$

$$A_{f2} = \frac{1}{1+1\times0.5} = 0.667 \text{ V/V}$$

$$A_{f3} = \frac{1}{1+1\times1} = 0.5 \text{ V/V}$$

(b) For $A_o = 10$: $A_{f1} = \frac{10}{1+0} = 10 \text{ V/V}$

$$A_{f2} = \frac{10}{1+\frac{10}{2}} = 1.6 \text{ V/V}$$

$$A_{f3} = \frac{10}{1+10\times1} = 0.909 \text{ V/V}$$

(c) For $A_O = 100$: $A_{f1} = \frac{100}{1+0} = 100 \text{ V/V}$

$$A_{f2} = \frac{100}{1+\frac{100}{2}} = 1.96 \text{ V/V}$$

$$A_{f3} = \frac{100}{1+100} = 0.99 \text{ V/V}$$

(d) For $A_o = 10^4$: $A_{f1} = \frac{10^4}{1+0} = 10^4 \text{ V/V}$

$$A_{f2} = \frac{10^4}{1+10^4/2} = 1.99 \text{ V/V}$$

$$A_{f3} = \frac{10^4}{1+10^4} = 0.9999 \text{ V/V}$$

9.6

A_o : 2mV → 10V

$A_o = 10\text{V}/(2\times10^{-3})\text{V} = 5000 \doteq 74 \text{ dB}$

A_F : 200mV → 10V

$A_F = (10)/(200) = 500 \doteq 54\text{dB}$

$$A_F = \frac{A_o}{1+\beta A_o} = \frac{5000}{1+5000\beta} = 500$$

$\Rightarrow 1 + 5000\beta = 10$

$\Rightarrow \beta = 9/5000 = 0.0018 \doteq -54\text{dB}$

$(1+A_o\beta) = 10 \equiv 20\text{dB}$

$A_o\beta = 5000(9/5000) = 9 \equiv 19.08\text{dB}.$

9.7

$$\frac{dA_f}{A_f} = \frac{1}{1+A\beta}\cdot\frac{dA}{A}$$

$$\frac{dA_f/A_f}{dA/A} = \frac{1}{1+A\beta} = -20\text{dB}$$

$\Rightarrow 1 + A\beta = +20\text{dB} \cong 10$

$\therefore A\beta = 9$

Require $\frac{1}{1+A\beta} = \frac{1}{2} \Rightarrow A\beta = 1$

9.8

$$A_f = 25; \frac{\partial A_f}{A_f} = 1\%; \frac{\partial A}{A} = 10\%$$

$$\frac{\partial A_f}{A_f} = \frac{1}{1+A\beta}\cdot\frac{\partial A}{A} \Rightarrow 1 = \frac{10}{1+A\beta} \Rightarrow A\beta = 9$$

Since

$$A_f = \frac{A}{1+\beta A} \Rightarrow 25 = \frac{A}{1+9} \rightarrow A = 250 \text{ V/V}$$

thus $\beta = \frac{9}{250} = 0.036$

9.9

$$A_f = 25; \frac{\partial A_f}{A_f} = 1\%; \frac{\partial A}{A} = 10\%$$

$$\frac{\partial A_f}{A_f} = \frac{1}{1+A\beta}\cdot\frac{\partial A}{A} \Rightarrow 1 = \frac{10}{1+A\beta} \Rightarrow A\beta = 9$$

Since:

$$A_f = \frac{A}{1+A\beta} \Rightarrow 25 = \frac{A}{1+9} \rightarrow A = 250 \text{ V/V}$$

The lowest gain is $A - 10\% A = 250 - 25$

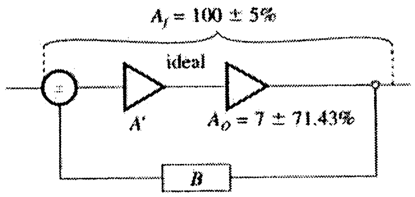
$= 225$ V/V and $\beta = \frac{9}{250} = 0.036$

9.10

For an output stage with gain varying between 2 and 12

$A_O = 7 \pm 5 V/V$ i.e $7\ V/V \pm 71.43\%$

$A_f = 100 \pm 5\%$



Referring to the schematic the total open-loop gain is: $A = A' \times 7$

Since the first stage is ideal the total open-loop gain variation is 71.43%

Thus: $\frac{\partial A_f}{A_f} = \frac{1}{1+\beta A} \cdot \frac{\partial A}{A} \rightarrow 5\% = \frac{71.43\%}{1+\beta A}$

$\rightarrow \beta A = 13.286$

Since

$A_f = 100 = \frac{A}{1+\beta A} = \frac{A}{14.286}$

$\Rightarrow A = 1428.6\ V/V$

Thus, $\beta = \frac{13.286}{1428.6} = 0.0093$

if $\frac{\partial A_f}{A_f} = 0.5 = \frac{71.43}{1+\beta A} \rightarrow \beta A = 141.86$

$\Rightarrow A_f = 100 = \frac{A}{142.86} \rightarrow A = 14286\ V/V$

and $\beta = \frac{141.86}{14286} = 0.00993$

Following the same procedure:

if $A_f = 10$ and $\partial A_f / A_f = 5\%$

$\beta A = 13.286$

$A = 142.86 \Rightarrow A' = \frac{142.86}{7} = 20.41\ V/V$

if $A_f = 10$ and $\partial A_f / A_f = 0.5\%$

$\beta A = 141.86$

$A = 1428.6 \Rightarrow A' = \frac{1428.6}{7} = 204.09\ V/V$

9.11

$$A(S) = Am \frac{S}{S+W_L}$$

$$A_f(S) = \frac{Am \frac{S}{S+W_L}}{1 + \frac{Am\,S}{S+W_L}\beta} = \frac{Am\,S}{S + W_L + Am\beta S}$$

$$= \frac{Am}{1+Am\beta} \cdot \frac{S}{S + \frac{W_L}{1+Am\beta}}$$

Thus

$$Am_f = \frac{Am}{1+Am\beta}$$

$$W_{Lf} = \frac{W_L}{1+Am\beta}$$

Both decreased by same amount

9.12

Worst case: A_{F1}

$$= \frac{A_O}{1+A_O\beta} = 9.8 \text{ (down 2\%)}$$

full battery: $A_{F2} = \frac{2A_O}{1+2A_O\beta} = 10$

from A_{F1}: $1 + A_O\beta = A_O/9.8$

$\therefore \beta = \frac{1}{9.8} - \frac{1}{A_O}$

Then $A_{F2} = \frac{2A_O}{1 + 2A_O\left[\frac{1}{9.8} - \frac{1}{A_O}\right]} = 10$

$\Rightarrow 1 + 2A_O\left[\frac{1}{9.8} - \frac{1}{A_O}\right] = \frac{2A_O}{10}$

$\Rightarrow 2A_O\left[\frac{1}{9.8} - \frac{1}{10}\right] = 2 - 1$

$\therefore 2A_O = 490$

[Check $\frac{2A_O}{1 + 2A_O\left[\frac{1}{9.8} - \frac{2}{2A_O}\right]\beta_{const}} = 10$

$\frac{A_O}{1 + A_O\left[\frac{1}{9.8} - \frac{1}{A_O}\right]} = 9.8$]

If β varies by $\pm 1\%$ the worst case for A_f is if β by 1%

$$A_{f1} = 9.8 = \frac{A_0}{1 + A_0\beta_{new}}$$

$$= \frac{A_0}{1 + A_0\beta 1.01} \quad \beta_{new} \triangleq 1.01\beta$$

$$\beta = \frac{1}{9.8} - \frac{1}{A_0} = \frac{1}{9.8} - \frac{2}{490} = \frac{48}{490}$$

$$9.8(1 + A_0\beta 1.01) = A_0$$

$$9.8\left(1 + A_0\frac{48}{490}1.01\right) = A_0$$

$$\Rightarrow 2A_0 = 645\frac{V}{V}$$

9.13

$$A_f = \frac{A_o}{1 + A_o\beta} = 10 = \frac{100}{1 + 100\beta}$$

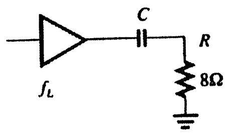
$$\therefore (1 + A_o\beta) = 100/10 = 10$$

$$f_L' = f_L/(1 + A_o\beta) = 100/10 = 10 \text{ Hz}$$

$$f_h' = f_H(1 + A_o\beta) = 10K \times 10 = 100 \text{ kHz}$$

9.14

For an $8\,\Omega$ loudspeaker and $f_L = 100$ Hz



$$f_L = \frac{1}{2\pi RC} \Rightarrow C = \frac{1}{2\pi \times 100 \times 8} = 199\ \mu\text{F}$$

If feed-back is used and: $A_f = 10$ V/V,

A = 1000 V/V

$$A_f = \frac{A}{1 + \beta A} \Rightarrow 1 + \beta A = \frac{1000}{10} = 100$$

$f_{Lf} = f_L/(1 + \beta A) = 100/100 = 1$ Hz

Since feed-back reduces the effective f_{Lf}, then a smaller capacitor C can be chosen for a larger value of f_L.

If f_{Lf} must now be 50 Hz:

$$50 = \frac{f_L}{100} \Rightarrow f_L = 5 \text{ KHz} = \frac{1}{2\pi \times 8 \times C}$$

$$\rightarrow C = 3.98\ \mu\text{F}$$

9.15

$$V_O = \frac{V_S \cdot A_1 A_2}{1 + A_1 A_2 \beta} + \frac{V_N \cdot A_1}{1 + A_1 A_2 \beta}$$

$= V_{Sf} + V_{Nf}$

Closed loop again

is: $$\frac{V_O}{V_S} = \frac{A_1 \cdot A_2}{1 + A_1 A_2 \beta} = 10\frac{\text{V}}{\text{V}} \quad (1)$$

if the output ripple V_N is ± 100 mV

$$\Rightarrow 100 \text{ mV} = \frac{1 \times 0.9}{1 + 0.9 A_2 \beta} \rightarrow A_2\beta = 8.88$$

Substituting in (1):

$$\frac{0.9 A_2}{1 + 0.9 \times 8.88} = 10 \rightarrow A_2 = 100\frac{\text{V}}{\text{V}}$$

$$\text{thus } \beta = \frac{8.88}{100} = 0.0888$$

Using the same procedure:

If $V_{Nf} = \pm 10$ mV, then $A_2\beta = 98.89$ and

$A_2 = 10\frac{\text{V}}{\text{V}}$

$\beta = 0.09899$

If $V_{Nf} = \pm 1$ mV, then $A_2\beta = 998.89$, and

$A_2 = 10\ \text{K}\frac{\text{V}}{\text{V}}$, $\beta = 0.099889$

9.16

Nominal $\frac{A}{1 + \beta A} = 100$, when A reduces

$$\text{to } \frac{1}{10} \Rightarrow \frac{A/10}{1 + \beta A/10} = 99$$

Compare these two:

$\beta A = 890 \quad \beta A + 1 = 891$

$\Rightarrow A = 100(1 + \beta A) = 89.1 \times 10^3$

$$\beta = \frac{890}{89.1 \times 10^3} = 0.01$$

when A increased 10 times

$$A_f = \frac{10A}{1 + 10\beta A} = \frac{89.1 \times 10^4}{1 + 8900} = 100.10$$

$$\text{when } A \rightarrow \infty \quad A_f = \frac{1}{\beta} = 100$$

9.17

A_1 has f_H high, A_2 has $A_M = 10$ V/V

with $f_L = 80$ Hz, $f_H = 8$ KHz.

$$A_F = \frac{A_1 A_2}{1 + A_1 A_2 \beta} = 100$$

Require $f_{Hf} = 40 \text{ KHz} = 8(1 + A_1 A_2 \beta)$

$\therefore 1 + A_1 A_2 \beta = 40/8 = 5$

$$\text{and } A_F = \frac{A_1 A_2}{5} = 100 \Rightarrow A_1 A_2 = 500$$

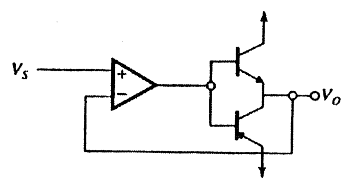
$\Rightarrow A_1 = 500/A_2 = 500/10 = 50$

$1 + A_1 A_2 \beta = 5 \Rightarrow \beta = 4/A_1 A_2 = 4/500$

$\therefore \beta = 0.008$

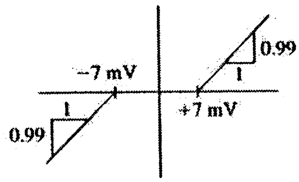
$f_{Lf} = f_L/(1 + A_1 A_2 \beta) = 80/5 = 16$ Hz

9.18



Dead band with be narrowed by the factor $1 + A\beta = 1 + A$ since $\beta = 1$ and since $A >> 1$, $1 + A \rightarrow A$

$$\therefore \text{new limits are } \pm\frac{0.7}{A} = \pm\frac{0.7}{100} = \pm 7 \text{ mV}$$



New slope ≡ gain $= A_f = \dfrac{A}{1+A}$

$$\Rightarrow \frac{100}{1+100} = 0.99$$

9.19

For $A = v_O/v_I = 10^2$ (select lowest A_O) to reduce · % change in gain by factor of 10

$$1 + A\beta = 10 \Rightarrow \beta = 9/10^2$$

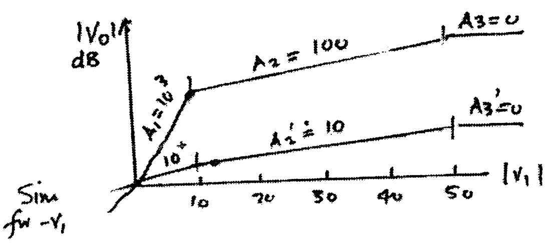
For $A_2 = 10^2$: $A_{2F} = 10^2/10 = 10$

For $A_1 = 10^3$: $A_{1F} = \dfrac{A}{1+A\beta}$

$$\therefore A_{1F} = \frac{10^3}{1+10^3(9/10^2)} = \frac{10^3}{91} = 10.98$$

For $A_3 = 0$: stays saturated

[For 10mV in and $A_1 = 10^3$, $v_O = 10$ V ?
For 10mV in and $A_2 = 10^2$. $v_O = 1$ V]



9.20

$$\beta = \frac{V_f}{V_O} = \frac{R_1}{R_1+R_2}$$ and open-loop gain is A

$$\Rightarrow A_f = \frac{A}{1+\beta A} = \frac{A}{1+\left(\dfrac{R_1 A}{R_1+R_2}\right)}$$

when: $A >> 1 \to A_f \simeq \dfrac{R_1+R_2}{R_1}$

$$A_f \simeq 1 + \frac{R_2}{R_1}$$

For: $A_f = 100$ V / V, $A = 10^4$, $R_1 = 1$ kΩ

$$100 = \frac{10^4}{1 + 10^4 \times \dfrac{1\text{ K}}{1\text{ K} + R_2}}$$

$$\to 1 + \frac{10^7}{10^3 + R_2} = 10^2$$

$$\to R_2 = 100.01 \text{ k}\Omega$$

If we use the approximate result for $A >> 1$

$$100 = 1 + \frac{R_2}{1\text{ K}} \to R_2 = 99 \text{ k}\Omega$$

If R_1 is removed:

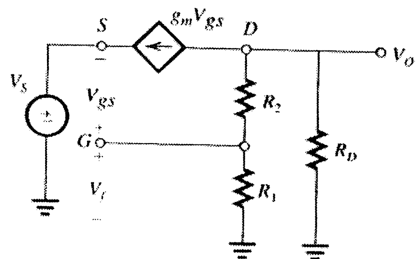
$$V_F = V_O \to \beta = 1 \to A_f = \frac{A}{1+A} \simeq 1$$

9.21

(a) If R_2 and R_1 are removed and the transistor gate is grounded then we have a CG amplifier

Thus: $A = g_m \cdot R_D$

Referring to Exercise 10.6, the equivalent small-signal circuit for Fig 10.7 c is:



$$\beta = \frac{V_f}{V_O} = \frac{R_1}{R_1+R_2}$$

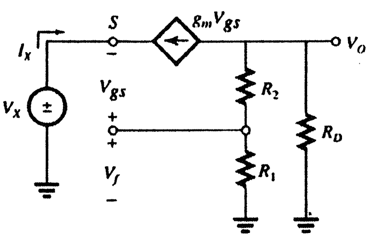
The amount of feed-back $1 + \beta A$ is:

$$1 + \frac{g_m R_D \cdot R_1}{R_1 + R_2}$$

(b) For a CG amplifier (with no feed-back)

$R_{in} = 1/g_m$ and $R_O = R_D$

(c) To obtain R_{inf} consider the following circuit



$$R_{inf} = \frac{V_X}{I_X}$$

$I_X = -g_m V_{gs}$ (Eq 1)

$V_O = I_X \cdot R_D$

$$V_f = \frac{V_O R_1}{R_1 + R_2} = (I_X R_D)\frac{R_1}{R_1 + R_2} \quad \text{(Eq 2)}$$

$V_X = -V_{gs} + V_f$ (Eq 3)

Sustituting Eq 1 and 2 into Eq 3:

$$V_X = \frac{I_X}{g_m} + \frac{I_X R_D R_1}{R_1 + R_2}$$

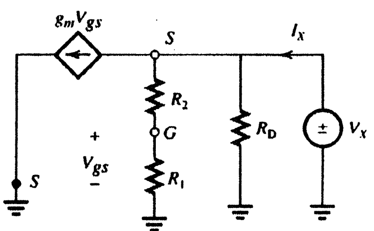
$$\Rightarrow \frac{V_X}{I_X} = \left(\frac{1}{g_m} + \frac{R_D R_1}{R_1 + R_2}\right)$$

Rearranging: $R_{inf} = \left(\frac{1}{g_m}\right)\left(1 + \frac{g_m R_D R_1}{R_1 + R_2}\right)$

Thus $R_{inf} = R_{in}(1 + A\beta)$

The input impedance is increased by a factor of $1 + A\beta$

To obtain R_{of} consider the following circuit:



$$R_{of} = \frac{V_X}{I_X}$$

$$I_X = g_m V_{gs} + \frac{V_X}{R_1 + R_2} + \frac{V_X}{R_D}$$

but $V_{gs} = \frac{R_1 \cdot V_X}{R_1 + R_2}$

$$\Rightarrow I_X = \frac{g_m R_1 V_X}{R_1 + R_2} + \frac{V_X}{R_1 + R_2} + \frac{V_X}{R_D}$$

$$= V_X \left\{\frac{g_m R_1 + 1}{R_1 + R_2} + \frac{1}{R_D}\right\}$$

$$\Rightarrow R_{of} = \frac{V_X}{I_X} = \frac{1}{\frac{g_m R_1 + 1}{R_1 + R_2} + \frac{1}{R_D}}$$

to re-arrange let's multiply by $\frac{R_D}{R_D}$

$$\Rightarrow R_{of} = \frac{R_D}{\frac{g_m R_1 R_D}{R_1 + R_2} + 1 + \frac{R_D}{R_1 + R_2}}$$

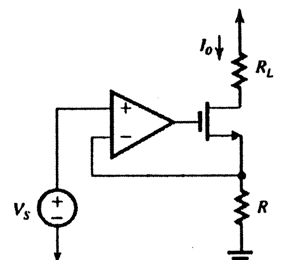
Since $R_1 + R_2 >> R_D \rightarrow \frac{R_D}{R_1 + R_2} \simeq 0$

$$R_{of} = \frac{R_D}{1 + \frac{g_m R_1 R_D}{R_1 + R_2}} = \frac{R_O}{1 + A\beta}$$

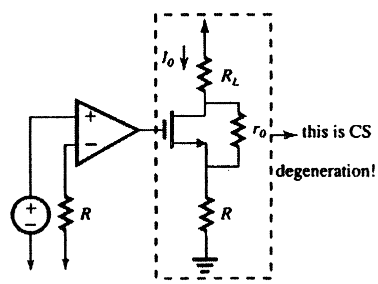
The output impedance is reduced by a factor of $1 + A\beta$

9.22

(a) A positive change in V_S results in a positive change at the gate of Q, which in turn will cause I_O to increase, causing a positive change is V_f



⇓ open circuit



$$Gm = \frac{g_m}{1 + g_m R + \frac{R}{r_o}}$$

b) $A = \mu \cdot G_m$

$$G_m = \frac{g_m}{1 + g_m R + \frac{R}{r_0}} \simeq \frac{g_m}{1 + g_m R}$$

c) $\beta = R$

d) $$A_f = \frac{A}{1 + \beta A} = \frac{\mu \cdot \frac{g_m}{1 + g_m R}}{1 + R\mu \frac{g_m}{(1 + g_m R)}}$$

e) when $\beta A = \frac{R\mu g_m}{1 + g_m R} >> 1$

$$A_f = \frac{1}{\beta} = \frac{1}{R}$$

9.23

$$\beta = \frac{V_f}{I_O}$$

$$V_f = \left(I_O \times \frac{R_M}{R_1 + R_2 + R_M}\right) \times R_1$$

$$\Rightarrow \beta = \frac{V_f}{I_O} = \frac{R_M R_1}{R_1 + R_2 + R_M}$$

To obtain A, remove R_1, R_2 and R_M and ground the negative input of the OP-AMP

$V_O = I_O \cdot R_L$

$V_O = \mu V_i \Rightarrow \mu V_i = I_O R_L$

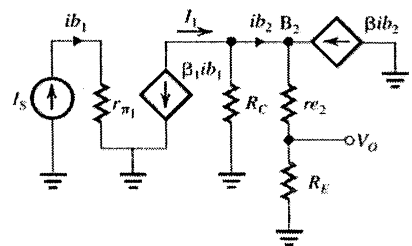
$$\rightarrow A = \frac{I_O}{V_i} = \frac{\mu}{R_L}$$

$$A_f = \frac{A}{1 + \beta A} = \frac{(\mu / R_L)}{1 + \left(\frac{R_M R_1}{R_1 + R_2}\right)(\mu / R_L)}$$

if βA is $>>1 \Rightarrow A_f \approx \frac{R_1 + R_2}{R_M \cdot R_1} \simeq \frac{1}{\beta}$

9.24

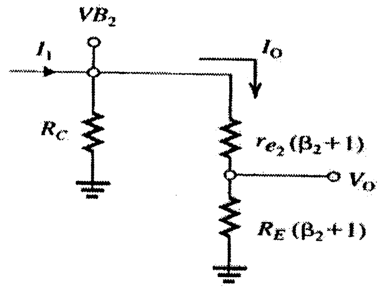
The equivalent small-signal circuit without the feed-back resistor R_F is:



$I_l = -\beta_1 I_S$

Reflecting r_{e2} and R_E towards the base B_2

But: $\beta_2 R_E >> R_C$ Therefore most of I_l will flow thru R_C and V_{B2} will be:



$V_{B2} = I_l R_C = -\beta_1 I_S \cdot R_C$

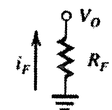
Thus: $$V_O = \frac{-\beta_1 I_S \cdot R_C \cdot R_E(\beta_2 + 1)}{(R_E + r_{e2})(\beta_2 + 1)}$$

we can also assume $R_E >> r_{e2}$ (e.g for $I_C = 1$ mA, $\beta = 100 \rightarrow r_e = 25\ \Omega$)

Then: $V_O \simeq -\beta_1 R_C I_S$

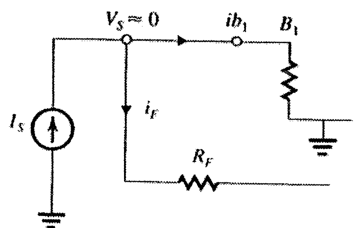
$$\Rightarrow A = \frac{V_O}{I_S} \equiv -\beta_1 \cdot R_C$$

To obtain β: if the signal voltage at the input is nearly zero:



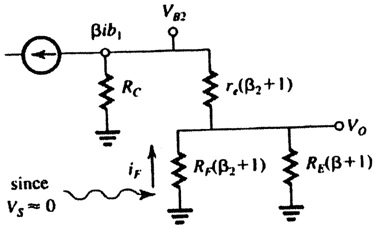
$$\Rightarrow \beta = \frac{I_f}{V_O} = \frac{-1}{R_F}$$

To obtain A_f: At the input side:



$I_S = ib_1 + i_F \qquad ib_1 = I_S - i_F$

At the output ride, after reflecting the emitter resistors towards the base:



Recall that: $R_C << \beta_2 R_E$

$R_E << R_F \rightarrow R_E\beta_2 << R_F\beta_2 \rightarrow R_C << R_F\beta_2$

Thus: $R_C \parallel \{(\beta + 1)(r_e + R_F \parallel R_E)\} \approx R_C$

Therefore:

$V_{B2} \simeq -\beta_1 ib_1 \times R_C = -\beta_1(I_S - i_F)R_C$

again, since we can neglect r_e

$V_o \simeq V_{B2} = -\overline{\beta}_1(I_S - i_F)R_C$

and $V_O = -R_F i_F \rightarrow i_F = \dfrac{-V_O}{R_F}$

$$V_O = -\beta_1 I_S R_C - \beta_1 \frac{R_C}{R_F} V_O$$

$$\Rightarrow V_O\left(1 + \beta_1\frac{R_C}{R_F}\right) = -\beta_1 R_C \cdot I_S$$

$$\Rightarrow A_f = \frac{V_O}{I_S} = \frac{-\beta_1 R_C}{1 + \beta_1 \frac{R_C}{R_F}}$$

which is the name result we would obtain from substituting A and β into: $A_f = A/(1 + A\beta)$

If $\beta_1 = 100$, $R_C = R_E = 10\ k\Omega$

$R_F = 100\ k\Omega$

$A = -100 \times 10\ K = -1 \times 10^6$ V/A ;

$$A_F = \frac{-10^6}{1 + 10^6 \times 10^{-5}} = -90.9\ \text{KV/A}$$

$\beta = -1/100\ K = -1 \times 10^{-5}$

9.25

To obtain A remove R_F and consider the small-signal response of the resulting CE:

$$V_O = -\beta I_i \cdot R_C \Rightarrow A = \frac{V_O}{I_i} = -\beta(R_C \parallel r_O)$$

if $r_O >> R_C \Rightarrow A \simeq -\beta R_C$

If the voltage at the input is near to cero volts

$$\Rightarrow I_f = -\frac{V_O}{R_F} \rightarrow \beta_F = \frac{I_C}{V_O} = \frac{-1}{R_F}$$

$$A_f = \frac{A}{1 + \beta_F A} = \frac{-\beta R_C}{1 + \frac{\beta R_C}{R_F}}$$

For $\beta = 100$, $R_C = 10$ K and $R_F = 100$ K

$$A_f = \frac{-100 \times 10\ \text{K}}{1 + (100 \times 10\ \text{K})/100\ \text{K}}$$

$= -90.9 \times 10^3$ V / A

9.26

$$A_F = \frac{A}{1 + A\beta} = \frac{10^3 \times 2}{1 + 2.10^3 \times 0.1} = 9.95\ V$$

$R_{if} = R_i(1 + A\beta) = 1(201) = 201\ K\Omega$

$R_{of} = R_o/(1 + A\beta) = 1/201 = 4.975\ K\Omega$

9.27

Here R_O is lowered by amount of feedback

i.e. $(1 + A\beta) = 80$

$\Rightarrow A\beta = 79$

$R_o = R_{of}(1 + A\beta) = 100 \times 80 = 8\ K\Omega$

9.28

The derivations are also valid for the case when A is a function of frequency

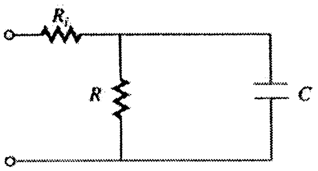
To obtain Z_{if} and Z_{of} we must replace A by its $A(S)$ form:

$$Z_{if} = R_i \cdot \left(1 + \frac{A_O}{\left(1 + \frac{S}{\omega_H}\right)} \cdot \beta\right)$$

$$Z_{of} = \frac{R_O}{1 + \frac{A_o}{1 + \frac{S}{\omega_H}}\beta}$$

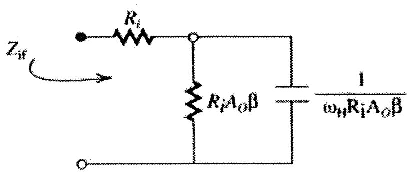
To obtain the equivalent circuits:

$$Z_{if} = R_i + \frac{R_i A_O \beta}{1 + \frac{S}{\omega_H}}$$ which corresponds to:



where $R \parallel C = \frac{R}{1 + RC_S}$; thus: $R = R_i A_O \beta$

$$RC = \frac{1}{\omega_H} \rightarrow C = \frac{1}{\omega_H \cdot R_i A_O \beta}$$

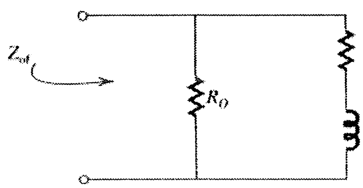


$$Z_{of} = \frac{R_O}{1 + \frac{A_O\beta}{\left(1 + \frac{S}{\omega_H}\right)}} = \frac{1}{\frac{1}{R_O} + \frac{(A_O\beta / R_O)}{1 + \frac{S}{\omega_H}}}$$

$$= \frac{1}{\frac{1}{R_O} + \frac{1}{\frac{1}{K} + \frac{S}{\omega_H K}}}$$

where $K = \frac{A_O\beta}{R_O}$

This is equivalent to a circuit of the form:

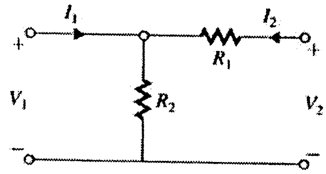


$$R = \frac{1}{K} = \frac{R_O}{A_O\beta}$$

$$L = \frac{1}{W_H \cdot K} = \frac{R_O}{W_H A_O \beta}$$

9.29

$$V_1 = h_{11} I_1 + h_{12} V_2$$
$$I_2 = h_{21} I_1 + h_{22} V_2$$



(a) $h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2 = 0} = R_1 \parallel R_2 = \frac{R_1 R_2}{R_1 + R_2}\ \Omega$

$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1 = 0} = \frac{R_2}{R_1 + R_2}$ V/V

$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2 = 0} = \frac{-R_2}{R_1 + R_2}$ A/A

$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1 = 0} = \frac{1}{R_1 + R_2}$ ℧

(b) $\beta = \left.\frac{V_1}{V_2}\right|_{I_1 = 0} = h_{12} = \frac{R_2}{R_1 + R_2}$

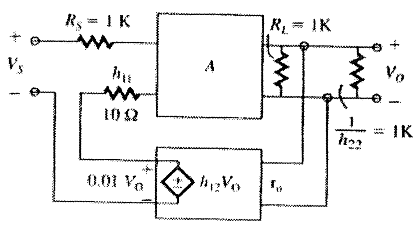
$\Rightarrow \frac{R_1}{R_2} = \frac{1}{\beta} - 1 = 99$

$\Rightarrow R_2 = R_1 / 99 = 10.1\ \Omega$

Thus $h_{11} = 10\ \Omega$; $h_{12} = 0.01$ V/V

$h_{21} = -0.01$ V/V; $h_{22} = 0.99 \times 10^{-3}$ ℧

(c)



9.30

$$R_{of} = \frac{R_O}{1 + A\beta} = \frac{1000}{1 + A\beta} = 100\ \Omega$$

$$\Rightarrow 1 + A\beta = 1000/100 = 10$$

$$\Rightarrow A_F = \frac{A}{1 + A\beta} = \frac{100}{10} = 10\ \text{V/V}$$

if $\beta = 1$: $R_{of} \Rightarrow \frac{R_O}{1 + A} = \frac{1000}{1 + 10^4} = 9.9\ \Omega$

9.31

(a) If the loop gain is large $\frac{V_O}{V_S} \approx \frac{1}{\beta}$

$$\beta = \frac{V_f}{V_O} = \frac{R_1}{R_1 + R_2} = \frac{1}{1+10}$$

$$= \frac{1}{11} \rightarrow \frac{V_O}{V_S} = \frac{1}{\beta} = 11 \text{ V/V}$$

(b) To solve for i_o are i_o:

At the base of Q_2: $iR_f + iB_1 = i_{e2} - I_2$

$$\left(\frac{i_{e1}R_S + 0.7}{R_1}\right) + \frac{i_{e1}}{(\beta+1)} = i_{e2} - I_2$$

Substituting and re-organizing:

$$(0.1099)\, i_{e1} + 1.7 \cdot 10^{-3} = i_{e2}$$

$$i_a + i_{b2} = I_1$$

$$\Rightarrow \frac{\beta_1}{\beta_1+1} i_{e1} + \frac{1}{\beta_2+1} i_{e2} = I_1$$

$$\Rightarrow i_{e2} = (\beta_2+1)\left(I_1 - \frac{\beta_1}{\beta_1+1} i_{e1}\right)$$

$$= 101\left(0.1\text{m} - \frac{100}{101} i_{e1}\right)$$

$$0.1099\, i_{e1} + 1.7 \times 10^{-3} = 10.1 \times 10^{-3} - 100 i_{e1}$$

$$\Rightarrow i_{e1} = 83.08\ \mu\text{A}$$

and: $i_{e2} = 1.708$ mA

At the base of Q_1:

$$V_{B1} = i_{e1} \cdot R_S + 0.7 = 0.708 \text{ V}$$

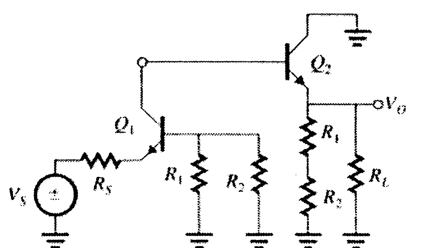
$$V_{R2} = V_{E2-B1} = R_2(i_{e2} - I_2)$$

$$= 10\text{ K}(1.708\text{ m} - 1\text{ m}) = 7.08 \text{ V}$$

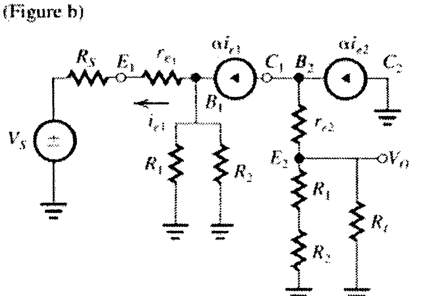
$$V_{E2} = V_{B1} + V_{E2-B1} = 0.708 + 7.08 = 7.788 \text{ V}$$

c) A-circuit

(Figure a)



(Figure b)



$$R_i = R_S + r_{e1} + \frac{R_1 \| R_2}{\beta+1}$$

$$r_{e1} = \frac{V_T}{i_{e1}} = \frac{25\text{ mV}}{83.08\ \mu\text{A}} = 301\ \Omega$$

$$R_i = 100 + 300 + \frac{1\text{ K} \| 10\text{ K}}{101}$$

$$R_i = 409\ \Omega$$

$$R_O \approx R_L \| (R_1 + R_2)$$

$$= 1\text{K} \| (1\text{ K} + 10\text{ K}) = 916\ \Omega$$

To obtain A: (see figure on previous page)

$$V_O = -\alpha i_{e1} \times (\beta+1)\{(R_1+R_2) \| R_L\} \ (1)$$

$$i_{e1} = -V_S / \{R_S + r_{e1} + (R_1 \| R_2)/(\beta+1)\} \quad (2)$$

Combining (1) & (2):

$$\Rightarrow V_O = \frac{\alpha V_S(\beta+1)\{(R_1+R_2) \| R_L\}}{R_S + r_{e1} + (R_1 \| R_2)/(\beta+1)}$$

$$A = \frac{V_O}{V_S} = \frac{\beta\{(R_1+R_2) \| R_L\}}{R_S + r_{e1} + (R_1 \| R_2)/(\beta+1)}$$

$$= \frac{\beta R_O}{R_i} = 100 \times \frac{916}{409}$$

$A \simeq 224$ V/V

d) As in part a) $\beta = \frac{R_1}{R_1+R_2} = \frac{1}{11} = 0.0909$

e) $A_F = \frac{A}{1+\beta A} = \frac{224}{1 + 224 \times 0.0909}$

$$= 10.48$$

$$R_{if} = R_i(1 + A\beta) = 409 \times 21.36 = 8.7\text{ k}\Omega$$

$$R_{of} = \frac{R_O}{1 + A\beta} = \frac{916}{21.36} \simeq 43\ \Omega$$

$$R_{in} = R_{if} - R_S = 8.6\text{ k}\Omega$$

$$R_{out} = 1/(1/43 - 1/1\text{ K}) = 44.9\ \Omega$$

$$\frac{\Delta A_f}{A_f} = \frac{10.48 - 11}{11} = -4.73\%$$

9.32

$Q_3 + Q_4$ form current multiplier
$\times\ 120/40 = \times 3$

$g_{m1} = 2\sqrt{\tfrac{1}{2}120(20/1)\,100} \doteq 693\ \mu A/V$

$g_{m5} = 2\sqrt{\tfrac{1}{2}60\ (20/1)\,1000} \doteq 1550\ \mu A/V$

$g_{m3} = 2\sqrt{\tfrac{1}{2}60\,(40/1)\,100} \doteq 693\ \mu A/V$

$g_{m4} = 2\sqrt{\tfrac{1}{2}60\,(120/1)\,300} \doteq 2078\ \mu A/V$

$g_{m2} = g_{m1} \qquad = 693\ \mu A/V$

$r_{o1} = 24/100 \Rightarrow 240\ K\Omega$

$r_{o2} = 24/100 \Rightarrow 240\ K\Omega$

$r_{o3} = 24/100 \Rightarrow 240\ K\Omega$

$r_{o4} = 24/300 \Rightarrow 80\ K\Omega$

$r_{o5} = 24/1000 \Rightarrow 24\ K$

(c) openloop gain $A\beta = g_{m1}(r_{o1}\|r_{o3})\times 1$

$\left[\ (3\times g_{m1})\left(\frac{r_{o4}}{3}\right) \equiv g_{m1} r_{o1}\right]$

$\beta = 1:\ \therefore A \approx g_{m1}(r_{o2}\|r_{o3})$

$\Rightarrow A \approx 693\times 120\times 10^{-3} = 84$

(d) $A_F = \dfrac{A}{1+A\beta} = \dfrac{84}{1+84} = 0.988\ V/V$

$R_o = r_{o5}\|r_{o5} = 12\ K$

$R_{of} = R_o/(1+A\beta) = 12/85 = 140\Omega$

(e) To obtain $V_o/V_s = 5$ we could change direct connection from Q_{5S} to Q_{2G} by voltage divider $R_1/(R_1+R_2)$ to change β from 1 to $1/5.3$ then

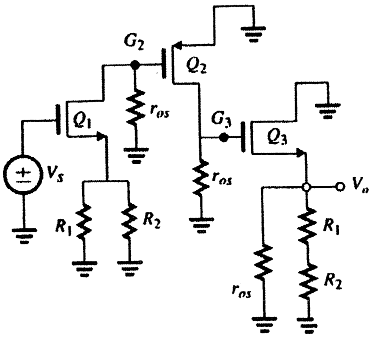
$A_F = \dfrac{84}{1+84\times 1/5.3} = \dfrac{84}{16.8} = 5$

Now $1 + A\beta = 16.8$

$R_{of}'' = R_o/(1+A\beta') = 12/16.8 = 714\Omega$

9.33

a) Transistors Q_1 and Q_2 are used in CS configuration. Therefore an increase in V_s causes the small-signal drain voltage of Q_1 to increase, followed by a voltage increase at the drain of Q_2. Transistor Q_3 is used in CD configuration. An increase in gate voltage at Q_3 results in an increase at the output V_o (source of Q_3) which through the voltage dividing feed-back causes V_f to increase. The feed-back is indeed negative.



b) If the loop gain $1 + A\beta$ is large then $A\beta >> 1$

$A_f = \dfrac{A}{1+A\beta} \doteq \dfrac{1}{\beta} = \dfrac{R_1+R_2}{R_1} = 10$

c) Find DC voltages : $V_{GS} - V_t = V_{OV} \rightarrow V_{GS} = V_{ov} + V_t$ for all transistors $|V_{Gs}| = 0.2 + 0.5 = 0.7$ V.

Then:

$V_{S1} = V_{DC} - V_{GS1} = 0.9 - 0.7 \rightarrow V_{S1} = 0.2$ V

$V_{GS} = 0.2$ V

$V_{G1} = V_{GS1} + V_{s1} = 0.7 + 0.2 = 0.9$V

$V_{G2} = V_{DD} - V_{SG2} = V_{DD} - 0.7$

For all current sources to operate in saturation

$|V_{DS}| \geq |V_{OV}| \qquad |V_{OV}| = 0.2$ V

For source I_1 : $|V_{DS}| = V_{DD} - V_{G2} = 0.7$ V

I_2: $V_{DS} = V_{G1} = 0.9$ V

I_3: $V_{DS} = V_{S1} = 0.2$ V.

d) obtain the A-circuit

Load of feed-back network at the input: $R_1 \| R_2$

Load of feed-back network at the output: $R_1 + R_2$

The A-circuit is:

Where $r_{os} = r_o$ is the output resistance of the current sources

Gain of each stage:

All g_m's $= 2I_D/V_{OV} = 2 \times 0.1\ \text{m}/0.2 = 1\ \text{mA/V}$

all r_o's $= V_A/I_D = 10/0.1\ \text{m} = 100\ \text{k}\Omega$

For Q_1: $A_{V1} = \dfrac{V_{G2}}{V_S} = g_{m\,eff}(r_{Oeff} \parallel r_{OS})$

$$\frac{V_{G2}}{V_S} = \frac{g_{m1}}{1 + g_{m1}R_S} \cdot (r_{O1}(1 + g_{m1}R_S) \parallel r_{OS});$$

$R_S = R_1 \parallel R_2$

$R_S = 2\ \text{K} \parallel 18\ \text{K} = 1.8\ \text{K}$ and $1 + g_m R_s$

$= 1 + 1.8 = 2.8$

$$\Rightarrow \frac{V_{G2}}{V_S} = \frac{1\ \text{m}}{2.8}[(100\ \text{K} \times 2.8) \parallel 100\ \text{K}]$$

$= 26.3$ V/V

For Q_2:

$$A_{V2} = \frac{V_{G3}}{V_{G2}} = g_{m2}(r_{O2} \parallel r_{OS})$$

$r_{o2} = r_{os}$

$g_{m2} = g_m$

$$\frac{V_{G3}}{V_{G2}} = g_m\frac{r_O}{2} = 1\ \text{m} \times \frac{100\ \text{K}}{2} = 50\ \text{V/V}$$

For Q_3:

$r_{OS} \parallel (R_1 + R_2) = 100\ \text{K} \parallel (18\ \text{K} + 2\ \text{K})$

$= 16.7\ \text{k}\Omega$

For a common-drain amplifier:

$$A_V = \frac{r_O \parallel R_L}{(r_O \parallel R_L) + \frac{1}{gm}}$$

where $R_L = r_{OS} \parallel (R_1 + R_2)$

$$\Rightarrow A_{V3} = \frac{V_O}{V_{G3}} = \frac{100\ \text{K} \parallel 16.7\ \text{K}}{(100\ \text{K} \parallel 16.7\ \text{K}) + 1/1\ \text{m}}$$

$= 0.93$

Find the overall voltage-gain:

$A = A_{V1} \cdot A_{V2} \cdot A_{V3} = 26.3 \times 50 \times 0.93$

$= 1223$ V/V

(e) Find β: $\beta = \dfrac{R_1}{R_1 + R_2} = \dfrac{2}{2 + 18} = 0.1$

(f) $A_f = \dfrac{V_O}{V_S} = \dfrac{A}{1 + A\beta} = \dfrac{1223}{1 + 0.1 \times 1223}$

$= 9.92$ V/V

which is $\approx \dfrac{1}{\beta} = 10$ as found in (b)

(g) For the common-drain stage:

$R_O \simeq \dfrac{1}{g_m} \parallel (r_{OS} \parallel R_1 + R_2) = 1\ \text{K} \parallel 16.7\ \text{K}$

$= 944\ \Omega$

$$R_{Of} = \frac{R_O}{1 + A\beta} = \frac{944}{1 + 0.1 \times 1223} = 7.66\ \Omega$$

Since $R_L = \infty \Rightarrow R_{out} = R_{of}$

9.34

(a) $i_{D6} = I_{ref}$,

$i_{D1} = i_{D2} = i_{D3} = i_{D4} = 50\ \mu\text{A} \Rightarrow i_{D7} = 100\ \mu\text{A}$

$i_{D8} = i_{D5} = 250\ \mu\text{A}$

For Q_6: $2.5\ \text{V} - I_{REF} \times 80\ \text{K} = -2.5\ \text{V} + V_{GS6} =$

$-1.5\text{V}, V_{ov} = 0.25\ \text{V}$

$V_{GS6} = V_{tn} + V_{ov} = 0.75 + 0.25 = 1\ \text{V}$

$$\text{Iref} = \frac{4}{800\ \text{K}} = 0.05\ \text{mA} = \frac{1}{2}\mu_n C_{ox}\left(\frac{\omega}{L}\right)_6 V_{OV}^2$$

$$= \frac{1}{2}100\mu\left(\frac{W}{L}\right)_6(0.25)^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_6 = \frac{16}{1}$$

For Q_7 and Q_8:

$$I_{D7} = 100\ \mu\text{A} \rightarrow \left(\frac{W}{L}\right)_7 = \frac{32}{1}$$

$$I_{D8} = 250\ \mu\text{A} \rightarrow \left(\frac{W}{L}\right)_8 = \frac{80}{1}$$

For Q_1 and Q_2:

$I_{D1,2} = 50\ \mu \rightarrow (W/L)_{1,2} = 16/1$

For Q_3 and Q_4: Since $\mu_p C_{ox} = \frac{1}{2}\mu_n C_{ox}$ and

$I_{D1} = I_{D2} = I_{D3} = I_{D4}$

$$\left(\frac{W}{L}\right)_{3,4} = 2 \times \left(\frac{W}{L}\right)_{1,2} = \frac{32}{1}$$

For Q_5: Transistor Q_5 must be sized such as

$V_{ODC} = 0\ \text{V}$

Since: $V_{D4} = V_{D3} = V_{G3,4}$

$\Rightarrow (2.5)\ \text{V} - V_{SG3} - V_{GS5} = 0\ \text{V}$

$V_{GS3} = V_{tp} = +0.75\ \text{V} + 0.25\ \text{V} = 1\ \text{V}$

$\Rightarrow V_{GS5} = 2.5 - V_{SG3} = 1.5\ \text{V}$

$\Rightarrow V_{ov5} - V_{DS5} - V_{th} = 1.5 - 0.75 = 0.75\ \text{V}$

$$\Rightarrow 250\mu = \frac{1}{2}100\mu\left(\frac{W}{L}\right)_5(0.75)^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_5 \approx 9$$

(b) The common-range is the range of common-mode input voltage in which Q_1, Q_2 and Q_7 remain in saturation.

Q_1 and Q_2 will enter the triode region when:

$V_{G1,2} = V_{D1,2} + V_{th} = 1.5 + 0.75 = 2.25\ \text{V}$

Q_7 will enter the triode region when:

$V_{G1,2} = V_{G7,2} - V_{th} + V_{GS1,2} = 1 - 0.75 + 1 = 1.25\ \text{V}$

Thus, the common-mode input range is:

1.25 to 2.25 V

(c) Find g_m: $g_m = \dfrac{2I_D}{V_{OV}} = \dfrac{100\ \mu}{0.25} = 0.4\dfrac{\text{mA}}{\text{V}}$

Q_5: $V_{OV} = V_{GS5} - V_{th} = V_{G5} - V_{S5} - V_{th} = 2.5\ \text{V}$

$- 1\ \text{V} - 0\ \text{V} - 0.75\ \text{V} - 0.75\ \text{V}$

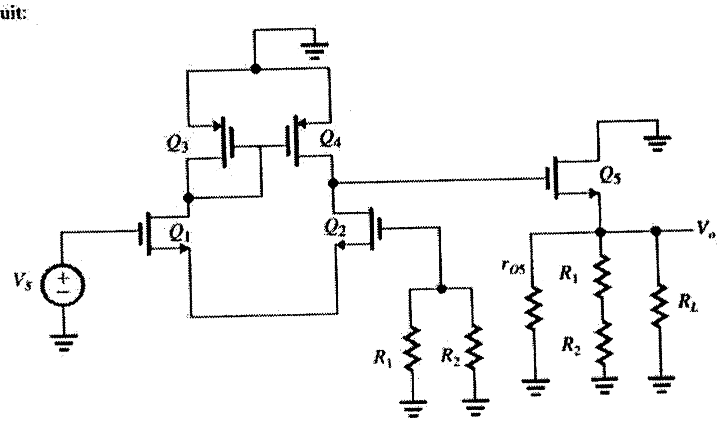
$$g_{m5} = \frac{500\ \mu}{0.75} = 0.67\frac{\text{mA}}{\text{V}}$$

(d) $r_O = V_A/I_O$

$$r_{O1} = r_{O2} = r_{O3} = r_{O4} = \frac{10}{50\ \mu} = 200\ \text{k}\Omega$$

$$r_{O5} = r_{O8} = \frac{10}{250\ \mu} = 40\ \text{k}\Omega$$

(e) A-circuit:



Gain: For the active-loaded differential path:

$$A_r = g_m(r_{o2} \| r_{o4}) = 0.4\ \text{m} \times \frac{200\ \text{K}}{2} = 40\frac{\text{V}}{\text{V}}$$

For the common drain

stage: $A_1 = \dfrac{r_{o5} \| R'_L}{(r_{os} \| R'_L) + \dfrac{1}{g_{ms}}}$

where $R'_L = r_{os} \| R_L\ (R_1 + R_2) = 22.2\ \text{k}\Omega$

$$\Rightarrow A_2 = \frac{40\ \text{K} \|\ 22.2\ \text{K}}{(40\ \text{K} \|\ 22.2\ \text{K}) + \dfrac{1}{0.67\ \text{m}}} = 0.91$$

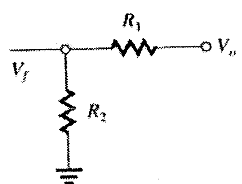
Total gain:

$$A = A_1 \times A_2 = 40 \times 0.91 = 36.4\frac{\text{V}}{\text{V}}$$

If

$$A_f = 10\frac{\text{V}}{\text{V}} = \frac{A}{1 + A\beta} \rightarrow \beta = \frac{1}{A_f} - \frac{1}{A} = \frac{1}{10} - \frac{1}{36.4}$$

$= 0.0725$



$$\beta = \frac{R_2}{R_1 + R_2}$$

and $R_1 + R_2 = 100\ \text{k}\Omega$

$\Rightarrow R_2 = 0.0725 \times 100\ \text{K} = 7.25\ \text{k}\Omega$

$R_1 = 100\ \text{K} - 7.25\ \text{K} = 92.25\ \text{k}\Omega$

(f) For the common drain stage

$$R_O = \frac{1}{g_{m5}} \| R'_L = 14.3\ \text{K} \|\ 1.5\ \text{K} = 1\ \text{k}\Omega$$

$$R_{of} = \frac{R_O}{1 + A\beta} = \frac{1\ \text{K}}{1 + 36.4 \times 0.0725} = 275\ \Omega$$

Excluding R_L:

$$R_{\text{OUT}} = 1 / \left(\frac{1}{275} - \frac{1}{100\ \text{K}}\right) = 276\ \Omega$$

9.35

(a) V_f is taken across R_1.

If V_s increases, so does the current at the output of A_1 and the voltage at the output of A_2. It follows that V_o increases and a portion of it is sampled by the Resistor divider $R_1 / (R_1 + R_2)$.

(b)Refer to circuit diagram.

(c) A-circuit

$$A = \frac{V_S}{V_O} = \left(\frac{82\ \text{K}}{82\ \text{K} + 9\ \text{K} + 10\ \text{K} \|\ 90}\right) \times \left(\frac{20 \times 5\ \text{K}}{3.2\ \text{K} + 5\ \text{K}}\right)$$

$$\times (20 \times (20 \| 20)) \times \left(\frac{100\ \text{K} \|\ 1\ \text{K}}{100\ \text{K} \|\ 1\ \text{K} + 1\ \text{K}}\right)$$

$A = (0.82) \times (12 \cdot 19) \times (200) \times (0.5)$

$A = 1000$ V/V

(d) $\beta = \dfrac{10\ \text{K}}{10\ \text{K} + 90\ \text{K}} = 0.1$

$1 + A\beta = 1 + 0.1 \times 1000 = 101$

(e) $A_f = \dfrac{1000}{1 + 100} = 9.9$ V/V

(f) $R_i = 82\ \text{K} + (10\ \text{K} \|\ 90\ \text{K}) + 9\ \text{K}$

$= 100\ \text{K}$

$R_{if} = R_i \cdot (1 + A\beta) = 100\ \text{K}(101)$

$= 10.1\ \text{M}\Omega$

$R_{in} = 10.1\ \text{M} - 9\ \text{K} \simeq 10.1\ \text{M}\Omega$

(g) $R_O = (1\text{ K} \parallel 100\text{ K} \parallel 1\text{ K}) = 500\ \Omega$

$$R_{of} = \frac{R_O}{1 + A\beta} = \frac{500}{101} \simeq 5\ \Omega$$

$$\text{W/O } R_L: R_{OUT} = \frac{1}{\frac{1}{5} - \frac{1}{1\text{ K}}} = 5.02\ \Omega$$

(h) If $f_H = 100\text{ Hz} \rightarrow f_{Hf} = 100 \times 101$
$= 10.1$ KHz

(i) if $A_1' = \frac{1}{2} A_1 \Rightarrow A = \frac{1000}{2} = 500$

$$A_f = \frac{500}{1 + 50} = 9.8$$

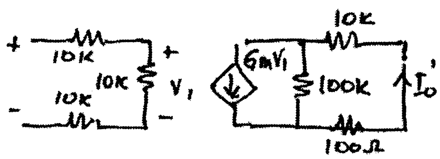
$$\frac{\Delta A_f}{A_f} = \frac{9.8 - 9.9}{9.9} = -1.01\%$$

9.36

$A = G_m = 100\text{ mA/V}$ $\beta = 0.1\text{ V/mA}$

$r_{in} = 10K$, $r_{out} = 100K$

A-circuit



$$V_i = V_1' \frac{10}{10 + 10 + 10} = V_1'/3$$

$$I_o' = \frac{G_m V_1\, 100}{100 + 10 + 0.1} = 30.28 V_1'\text{ mA}$$

$$A = \frac{I_o'}{V_i'} = 30.28\text{ mA/V}$$

$$A_F = \frac{A}{1 + A\beta} = \frac{30.28}{1 + 30.28(0.1)} = 7.52\ \frac{\text{mA}}{\text{V}}$$

$R_i = R_S + R_{id} + R_1 = 30\text{ K}\Omega$

$R_{if} = R_1(1 + A\beta) = 120.8\text{ K}\Omega$

$R_{in} = R_{if} - R_S = 110.8\text{ K}\Omega$

$R_O = R_L + R_{op} + R_2 = 110.1\text{ K}\Omega$

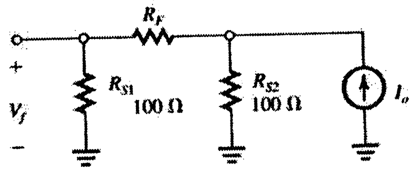
$R_{of} = R_O(1 + A\beta) = 443.4\text{ K}\Omega$

$R_{out} = R_{of} - R_L = 433.4\text{ K}\Omega$

9.37

(a) $A_f = \frac{A}{1\ 1\ A\beta}$ if A is large then $A_f \simeq \frac{1}{\beta}$
$= 0.1$ A/V

To obtain β:



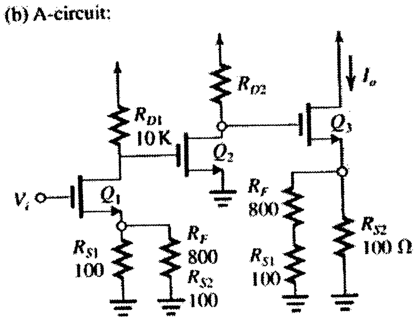
$$\beta = \frac{V_f}{I_o}$$

$$\beta = \frac{R_{S2} \cdot R_{S1}}{R_{S2} + R_F + R_{E1}} = \frac{100 \times 100}{100 + R_F + 100}$$

$$= \frac{10^4}{200\ 1\ R_F}$$

If $\frac{1}{\beta} = 0.1 \rightarrow R_F = 800\ \Omega$

(b) A-circuit:



To obtain A:

For the first stage:

$$\frac{V_{D1}}{V_i} = \frac{-g_m \cdot R_{D1}}{1 + g_m(R_S \parallel R_F\ 1\ R_{S2})}$$

Substituting: $g_m = 4$ mA / V

$R_{D1} = 10\text{ K}\ (R_{S1} \parallel R_F\ 1\ R_{S2}) = 90\ \Omega$

$$\frac{V_{D1}}{V_i} = -29.4\text{ V/V}$$

For the second stage:

$$\frac{V_{D2}}{V_{D1}} = -g_m R_{D2} = -4 \times 10 = -40\text{ V / V}$$

For the third stage: $\frac{I_O}{V_{D2}} = \frac{I_{D3}}{V_{G3}}$

$$= \frac{1}{1/g_m\ 1\ R_{S2} \parallel (R_{S1}\ 1\ R_F)}$$

Substituting

$g_m = 4$ mA / V $(R_{S2} \parallel R_{S1}\ 1\ R_F) = 90\ \Omega$

$$\frac{I_O}{V_{D2}} = 3\text{ mA / V}$$

Combining the gain of the three stages:

$A = \frac{I_O}{V_i} = -29.4 \times -40 \times 3 \times 10^{-3}$

$= 3.53$ A/V

(c) $1 + A\beta = 1\ 1\ 3.53 \times 10 = 36.3$

$A_f = \frac{V_O}{I_S} = \frac{A}{1\ 1\ A\beta} = \frac{3.53}{36.3} = 0.097$ A / V

The design value for A_f is 0.1 A/V

$\frac{\Delta A_f}{A_f} = -\frac{0.003}{0.1} = -3\%$

To change A_f such as it gets closer to 0.1 A/V R_f must be reduced, this will increase both the values of A and B

(d) $R_{D3} = r_{O3} + [R_{S2} \| (R_F + R_{S1})]$

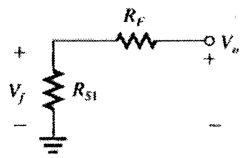
$= 20\text{ K} + 90 \simeq 20\text{ k}\Omega$

$R_{out} = R_{of} = (1 + A\beta)R_{o3} = 36.3 \times 20\text{ K}$

$= 726\text{ k}\Omega$

(e) If the output is taken at V_{o2}:

The feed-back network changes to:

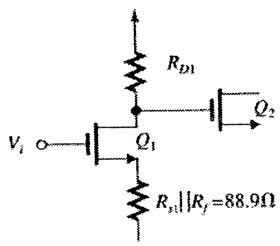


$\beta = \frac{100}{800 + 100} = 0.111$

Therefore the A-circuit changes at the source of Q_1 to:

$R_{S1} \| R_f = 100 \| 800$

$= 88.9\ \Omega$



For the first stage:

$\frac{V_{D1}}{V_i} = \frac{-g_m R_{D1}}{1 + g_m((R_{S1} \| R_f))}$

$= -29.5$ V / V

For the second stage: $\frac{V_{D2}}{V_{D1}}$

$= -40$ V / V (unchanged)

For the third stage:

$\frac{V_{O2}}{V_{D2}} = \frac{V_{S3}}{V_{G3}}$

$= \frac{R_{S2} \| (R_F + R_{S1})}{1/g_m + R_{S2} \| (R_F + R_{S1})}$

$= 0.265$ V / V

Combining the gains of all three stages:

$A = \frac{V_O}{V_S} = -29.5 \times -40 \times 0.265$

$= 312.7$ V / V

(f) $1 + A\beta = 1 + 312.7 \times 0.111 = 35.74$

$R_{out2} = (1 + A\beta)R_o$

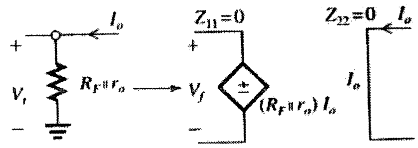
$= (1 + A\beta)(R_{S2} \| R_{S1} + R_F) \| r_O \| \frac{1}{g_m}$

$= 35.74 \times \{90 \| 20\text{ K} \| 250\}$

$= 2.35\text{ k}\Omega$

9.38

(a) The β circuit:



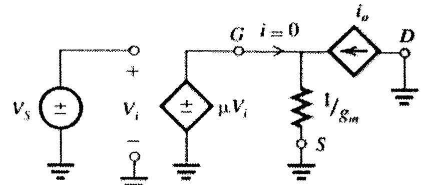
$\beta = \frac{V_f}{I_O} = \frac{R_f r_O}{R_f + r_O}$

$A_f = \frac{A}{1 + A\beta}$ for $A\beta >> 1$, $A_f \simeq \frac{1}{\beta}$ if

$A_f = 10\ \frac{\text{mA}}{\text{V}} \Rightarrow \beta = 100\ \Omega$

$R_F = 100.5\ \Omega$

(b) The A-circuit:



$\mu = 1000$ V / A

$g_m = 2$ mA / V

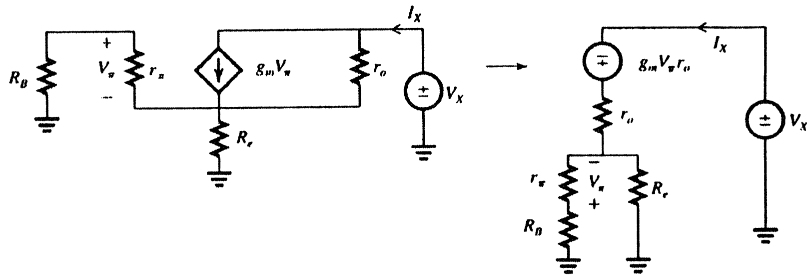
$r_O = 20$ K

$A = \frac{I_O}{V_S} = \mu g_m = 2$A / V

(c) $A\beta = \mu g_m(R_F \| r_O) = 2 \times 100 = 200$

$1 + A\beta = 201$

This figure is for 9.39



(d) $A_f = \frac{A}{1 + A\beta} = \frac{2}{201} = 9.95\ \text{mA/V}$

(e) $R_O = r_O \Rightarrow R_{of} = r_O(1 + A\beta)$

$= 20\ \text{K} \times 201 = 4.02\ \text{M}\Omega$

9.39

$$V_\pi = -\frac{r_\pi I_X \cdot R_e}{R_e + R_B + r_\pi}$$

$$V_X + g_m V_\pi r_o = I_X\{r_o + (R_e \| r_\pi + R_B)\}$$

$$V_X - \frac{r_\pi \cdot R_e}{R_e + R_B + r_\pi} g_m r_O I_X$$

$$\frac{V_X}{I_X} = r_O + (R_e \| r_\pi + R_B)$$

$$= I_x\{r_o + R_e \| (r_\pi + R_B)\}$$

$$\frac{V_X}{I_X} = r_O + \left(R_e \| (r_\pi + R_B) + g_m r_O \frac{r_\pi R_e}{R_e + R_B + r_\pi}\right)$$

$$\left\{1 + g_m r_O \cdot \frac{r_\pi \cdot R_e}{R_e + R_B + r_\pi} \times \frac{R_e + R_B + r_\pi}{R_e(r_\pi + R_B)}\right\}$$

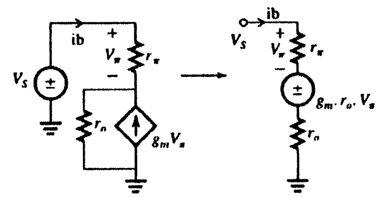
$$R_O = r_O + R_e \| (r_\pi + R_B)\left\{1 + g_m r_O \frac{r_\pi}{r_\pi + R_B}\right\}$$

If $R_B = 0$ $R_O = r_O + (R_e \| r_\pi)\{1 + g_m r_O\}$

R_O is maximum when $R_e >> r_\pi$, then:

$R_O = r_O + r_\pi\{1 + g_m r_O\}$

If $R_B = 0$ and $R_e = \infty$:



$$V_\pi = ib \cdot r_\pi$$

$$V_S - g_m r_O V_\pi = ib(r_\pi + r_O) \Rightarrow V_S = g_m r_O r_\pi ib + ib(r_\pi + r_O)$$

$$\Rightarrow ib = \frac{V_S}{g_m r_O r_\pi + r_\pi + r_O}$$

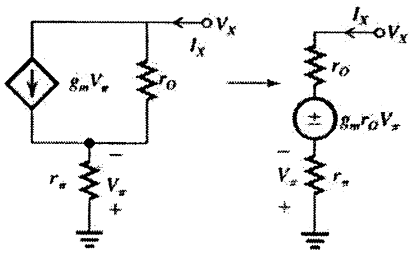
and:

$$g_m V_\pi = g_m r_\pi ib = \frac{g_m r_\pi V_S}{g_m r_O r_\pi + r_\pi + r_O}$$

and $ir_O = ib + g_m V_\pi$

$$= (1 + g_m r_\pi) \cdot \frac{V_S}{g_m r_O r_\pi + r_\pi + r_O}$$

9.40



$$V_\pi = -I_Xr_\pi$$

$$V_X - g_mr_Or_\pi I_X = I_X(r_O + r_\pi)$$

$$\frac{V_X}{I_X} = R_O = r_O + r_\pi + g_mr_Or_\pi$$

$$= r_\pi + r_O(1 + g_mr_\pi)$$

$$= r_\pi + r_O(1 + h_{fe})$$

$$\approx h_{fe}r_O$$

9.41

For: $g_{m1} = g_{m2} = 5\ \text{mA/V},\ r_{O2} = 20\ \text{K}$,
assume $R_D = 1\ \text{k}\Omega$

$$A = \frac{I_O}{V_S} = G_{m1}R_{Dgm2},\ G_{m1} = \frac{g_{m1}}{1 + g_mR_F}$$

$$= \frac{g_{m1}g_{m2}R_D}{1 + g_{m1}R_F}$$

$$= 16.67 \times 10^{-3}\frac{\text{A}}{\text{V}} = 16.67\ \text{mS}$$

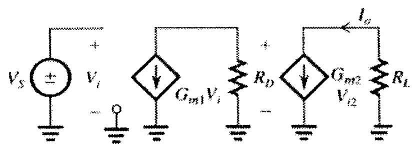
$$\beta = R_f = 100\ \Omega$$

$$1 + A\beta = 1 + 16.67 \times 10^{-3} \cdot 100 = 2.67$$

$$A_f = \frac{A}{1 + A\beta} = \frac{16.67\text{mS}}{267} = 6.25\ \text{mS}$$

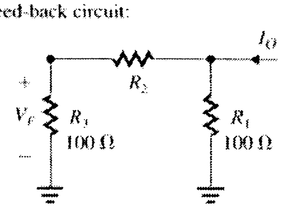
$$R_o = R_F//(r_{o2}\,R_L) = 99.5\ \Omega$$

$$R_{of} = R_o(1 + A\beta) = 99.5\ \Omega \cdot 2.67 = 266\ \Omega$$



9.42

Feed-back circuit:



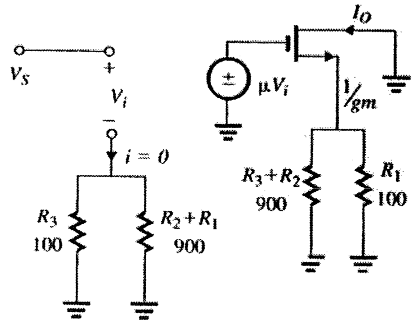
$$\beta = \frac{V_f}{I_O} = \frac{R_1 \cdot R_3}{R_1 + R_2 + R_3}$$

$$\text{if } A\beta >> 1 \Rightarrow A_f \simeq \frac{1}{\beta} = \frac{R_1 + R_2 + R_3}{R_1R_3}$$

$$\text{If } A_F = 100\ \text{m}\frac{\text{A}}{\text{V}}\ 0.1 = \frac{200 + R_2}{100 \times 100}$$

$$\rightarrow R_2 = 800\ \Omega\ \ \beta = 10$$

A-circuit:



$$I_O = \frac{\mu V_S}{1/g_m + R_1 \parallel (R_1 + R_2)}$$

$$A = \frac{I_O}{V_S} = \frac{\mu}{1/g_m + R_1 \parallel (R_1 + R_2)}$$

For $gm = 1$ mA $\quad A = \frac{\mu}{1\ \text{K} + 90} = \frac{\mu}{1090}$

Amount of feed-back: $1 + A\beta = 1000$
i.e. 60 dB

and $\beta = 10 \Rightarrow A = \frac{999}{10} = 99.9$

$$\mu = 1090 \times 99.9 \simeq 109 \times 10^3\ \text{V/V}$$

R_o is degenerated by $R_1//(R_2 + R_3)$

$$R_O = r_0 + g_mr_0[R_1//R_2 + R_3] + [R_1//R_2 + R_3]$$

$$= 50\ \text{k}\Omega + 90\ \Omega + 4.5\ \text{k}\Omega = 54.59\ \text{k}\Omega$$

$$R_{out} = R_0(1 + A\beta) = 54.59\ \text{M}\Omega$$

9.43

$$I_{D1} = I_{D2} = I_{D3} = I_{D4} = 0.1\ \text{mA}$$

$$I_{D5} = 0.8\ \text{mA}$$

$$g_{m1} = g_{m2} = g_{m3} = g_{m4} = \frac{2 \times 0.1\ \text{m}}{0.2}$$

$$= 1\ \text{mA/V}$$

$$gm_5 = \frac{2 \times 0.8\ \text{m}}{0.2} = 8\ \text{mA/V}$$

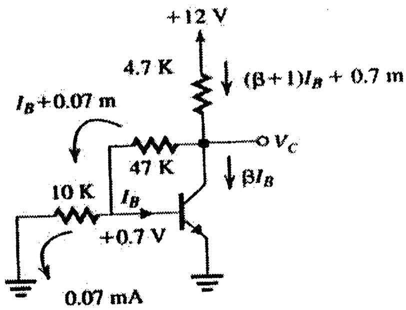
$$r_{O2} = \frac{|V_A|}{I_D} = \frac{20}{0.1\ \text{m}} = 200\ \text{k}\Omega$$

$$r_{O5} = \frac{20}{0.8\ \text{m}} = 25\ \text{k}\Omega$$

$$|V_{G3}| = 0.2 + 0.4 = 0.6\ \text{V}$$

9.44

(a)



$V_C = 0.7 + (I_B + 0.07)47 = 3.99 + 47\,I_B$

and $\frac{12 - V_C}{4.7} = (\beta + 1)I_B + 0.07$

Solving both equations results in: $I_B \simeq 0.015$ mA

$I_C \simeq 1.5$ mA and $V_C = 4.7$ V

$V_\pi = I_i(R_S \| R_f \| r_\pi)$

$V_O = -g_m V_\pi (R_f \| R_c)$

$\Rightarrow A = \frac{V_o}{I_i} = -g_m(R_f \| R_c)(R_s \| R_f \| r_\pi)$

$g_m = \frac{I_C}{V_T} = \frac{1.5\text{ m}}{25\text{ m}} = 60\,\frac{\text{mA}}{\text{V}}$ and

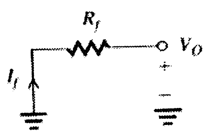
$r_\pi = \frac{\beta}{g_m} = \frac{100}{60\text{ m}} = 1.67\text{ k}\Omega$

Substituting: $A = -358.7\text{ k}\Omega$

$R_i = R_S \| R_f \| r_\pi = 1.4\text{ k}\Omega$

$R_O = R_C \| R_f = 4.27\text{ k}\Omega$

(d) To determine β:



$\beta = \frac{I_f}{V_O} = \frac{-1}{R_f} = \frac{-1}{47\text{ k}\Omega}$

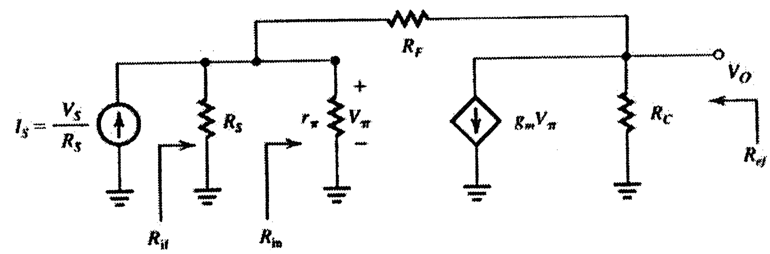
$A\beta = \frac{358.7}{47} = 7.63 \quad 1 + A\beta = 8.63$

(e) $A_f = \frac{V_O}{I_S} = \frac{A}{1 + A\beta} = \frac{-358.7}{8.63}$

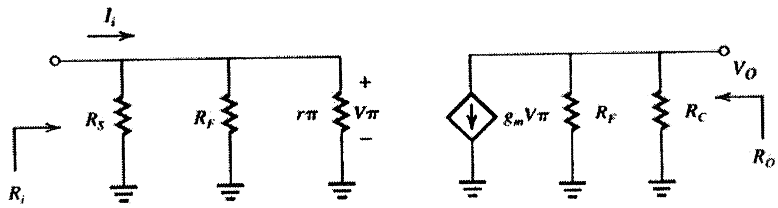
$= -41.6\text{ k}\Omega$

$R_{if} = \frac{R_i}{1 + A\beta} = \frac{1.4\text{ K}}{8.63} = 162.2\ \Omega$

(b)



(c) A-circuit:



To find A:

$R_{of} = \frac{R_O}{1 + A\beta} = \frac{4.27\text{ K}}{8.63} = 495\ \Omega$

$R_{in} = \frac{1}{\frac{1}{R_{if}} - \frac{1}{R_S}} = \frac{1}{\frac{1}{162.2} - \frac{1}{10\text{ K}}} = 164.87\ \Omega$

$R_{out} = \frac{1}{\frac{1}{R_{of}} - \frac{1}{R_L}} = \frac{1}{\frac{1}{495} - \frac{1}{\infty}} = 495\ \Omega$

$V_S = I_S \cdot R_S$

$\frac{V_O}{V_S} = \frac{V_O}{I_S} \cdot \frac{1}{R_S} = \frac{-41.6\text{ K}}{10\text{ K}} = -4.16$ V/V

if $A\beta >> 1$

$\Rightarrow A_f \simeq -47\text{ k}\Omega$

$\rightarrow \frac{V_O}{V_S} = \frac{-47}{10} = -4.7$ V/V

9.45

(a) if $A\beta >> 1 \Rightarrow A_f \simeq \frac{1}{\beta}$ with $\beta = \frac{-1}{R_F}$

$A_f = \frac{V_O}{I_S} = -R_F$

Since $V_S = I_S \cdot R_S \Rightarrow \frac{V_O}{V_S} = \frac{A_F}{R_S} = \frac{-R_F}{R_S}$

For $\frac{V_O}{V_S} \cong -10\,\frac{\text{V}}{\text{V}} = \frac{-R_F}{1\text{ k}\Omega} \Rightarrow R_F = 10\text{ k}\Omega$

(b) $\mu = 10^3$ V/V $R_{id} = 100\text{ k}\Omega$

$r_O = 1\text{ k}\Omega$

$R_L = \infty,\ R_f = 10\text{ K},\ R_S = 1\text{ K}$

$R_i = R_{id} \| R_F \| R_S = 100 \| 10 \| 1 = 901\ \Omega$

$A = \frac{V_o}{I_i} = -\mu R_i \frac{(R_F \| R_L)}{r_O + (R_F \| R_L)}$

$= \frac{-10^3 \times 901(10\text{ K})}{1\text{ K} + 10\text{ K}} = -819\text{ k}\Omega$

$A\beta = \frac{819}{10} = 81.9$

$1 + A\beta = 82.9$

$A_f = \frac{A}{1 + A\beta} = \frac{-819 \times 10^3}{82.9} = -9.88\text{ k}\Omega$

$\frac{V_O}{V_S} = \frac{-9.88}{1} = -9.88\text{ V/V}$

To find R_{in}: Since R_{id} is large

(Eq 10.58) $R_{in} \simeq \frac{R_F}{\mu'}$

$\mu' = \frac{\mu(R_F \| R_L)}{r_O + (R_F \| R_L)} = 909$

$R_{in} = \frac{10\text{ K}}{909} = 11\ \Omega$

Since: $R_t >> \frac{r_o}{1 + \mu\frac{R_i}{R_F}}$

$R_{out} \simeq \frac{R_F}{R_i} \cdot \frac{r_O}{\mu} = \frac{10}{0.901} \times \frac{1}{1} = 11.1\ \Omega$

$r_0 = 1000\Omega \quad \mu = 1000$

(e) $f_H = 1\text{ kHz} \Rightarrow f_{Hf} = 1\text{ K}(1 + A\beta)$

$= 1\text{ K} \times 82.9 = 82.9\text{ kHz}$

9.46

(a) $V_{OV} = V_{GS} - V_t \Rightarrow V_{GS_{1,2}} V_{OV} + V_t$

$= 0.2 + 0.5 = 0.7\text{ V}$

$V_{G1} = V_{GS1} = 0.7\text{ V}$

$V_O = V_{GS1} = 0.7\text{ V}$

$V_{G2} = V_O + V_{GS2} = 0.7 + 0.7 = 1.4\text{ V}$

(b) $I_{D1} = I_{D2} = 0.5\text{ mA}$

$$g_m = \frac{2I_D}{V_{OV}} \rightarrow g_{m1,2} = \frac{2 \times 0.5\text{ m}}{0.2} = 5\ \frac{\text{mA}}{\text{V}}$$

$$r_O = \frac{V_A}{I_D} \rightarrow r_{O1,2} = \frac{10}{0.5\text{ m}} = 20\text{ k}\Omega$$

(c) A-circuit:

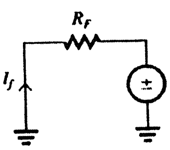
$V_1 = I_S R_F \quad V_2 = -g_{m1} r_{O1}(I_S R_F)$

$$V_O = \frac{V_2 \cdot (R_F \| r_{O2})}{1/g_{m2} + (R_F \| r_{O2})}$$

$$= \frac{-g_{m1} \cdot r_{O1}(I_S R_F)(R_F \| r_{O2})}{1/g_{m2} + (R_F \| r_{O2})}$$

$$A = \frac{V_O}{I_S} = \frac{-g_{m1} \cdot r_{O1} \cdot R_F\,(R_F \| r_{O2})}{1/g_{m2} + (R_F \| r_{O2})}$$

(d)



$$\beta = \frac{I_f}{V_O} = \frac{-1}{R_F}$$

$$A\beta = \frac{1 \cdot r_{O1} \cdot (R_F \| r_{O2})}{1/g_{m2} + (R_F \| r_{O2})}$$

$$1 + A\beta = \frac{1/g_{m2} + (1 + g_{m1} r_{O1})(R_f \| r_{O2})}{1/g_{m2} + (R_f \| r_{O2})}$$

(e) $A_f = \dfrac{A}{1 + A\beta}$

$$= \frac{-g_{m1} \cdot r_{O1} \cdot R_F\,(R_F \| r_{O2})}{1/g_{m2} + (1 + g_{m1} \cdot r_{O1})(R_F \| r_{O2})}$$

(f) $R_i = R_F$

$R_O = R_F \| r_{O2} \| 1/g_{m2}$

$$R_{if} = \frac{R_i}{1 + A\beta} \Rightarrow \frac{1}{R_{if}} = \frac{1}{R_i} + \frac{A\beta}{R_i} = \frac{1}{R_i} + \frac{A\beta}{R_F}$$

$$\frac{1}{R_i} + \frac{g_{m1} \cdot r_{O1} \cdot (R_F \| r_{O2})}{(1/g_{m2} + (R_F \| r_{O2})) \cdot R_F}$$

If we define: $\mu = \dfrac{g_{m1} \cdot r_{O1}(R_F \| r_{O2})}{(1/g_{m2} + (R_F \| r_{O2}))}$

$$\Rightarrow \frac{1}{R_{if}} = \frac{1}{R_i} + \frac{\mu}{R_f} \Rightarrow R_{if} = R_i \| \frac{R_f}{\mu}$$

$$= R_f \| \frac{R_F}{\mu} = \frac{R_F}{1 + \mu}$$

Since $R_{if} = R_S \| R_{in}$ and $R_S = \infty$

$$\Rightarrow R_{in} = \frac{R_F}{1 + \mu} = \frac{R_F}{1 + \dfrac{g_{m1} r_{O1}(R_F \| r_{O2})}{\dfrac{1}{g_{m2}} + (R_F \| r_{O2})}}$$

$R_{of} = \dfrac{R_O}{1 + A\beta}$ Since: $R_{of} = R_L \| R_{out}$ and

$R_L = \infty \quad \Rightarrow R_{of} = R_{out}$

$$R_{out} = \frac{(R_f \| r_{O2} \| 1/g_{m2})(1/g_{m2} + (R_f \| r_{O2}))}{1/g_{m2} + (1 + g_{m1} r_{O1})(R_f \| r_{O2})}$$

but: $1/g_{m2} << R_f \| r_{o2} \Rightarrow R_{out} = \dfrac{(1/g_{m2})}{(1 + g_{m1} \cdot r_O)}$

(g) $g_{m1} = g_{m2} = 5\text{ mA/V}$

$r_{O1} = r_{O2} = 20\text{ k}\Omega$

$R_f = 10\text{ K}$

$R_F \| r_{O2} = 10 \| 20 = 6.67\text{ k}\Omega$

$\dfrac{1}{g_{m2}} + R_F \| r_{O2} = 200 + 6.67\text{ K} = 6.87\text{ k}\Omega$

$$A = \frac{-5 \times 20 \times 10\text{ K} \times 6.67}{6.87} = -970.8\text{ k}\Omega$$

$$\beta = \frac{-1}{10\text{ K}} = -100\ \mu\text{A/V}$$

$A\beta = 97.08$

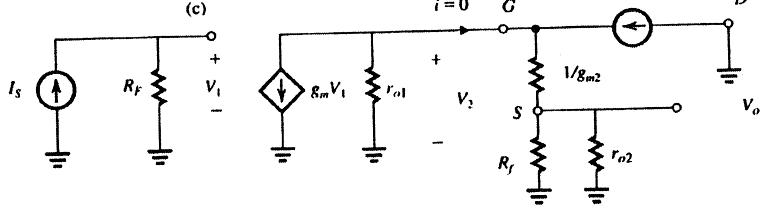
$A_f = -970.8\text{ K}/(98.08) = -9.9\text{ k}\Omega$

$R_i = 10\text{ K};\ R_O = 6.67\text{ K} \| 200 = 194.2\ \Omega$

$$R_{in} = \frac{10\text{ K}}{1 + \dfrac{5 \times 20 \times 6.67}{6.87}} = 102\ \Omega$$

$$R_{out} = \frac{200}{1 + 5 \times 20} \cong 1.98\ \Omega$$

(c)

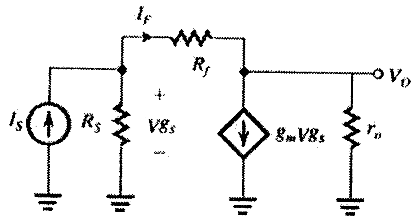


9.47

$$A_f = \frac{V_O}{I_S}$$

Across R_f: $I_F = \frac{V_{gs} - V_O}{R_F} = \frac{V_{gs}}{R_F} - \frac{V_O}{R_F}$ (1)

At the output: $I_F = g_m V_{gs} + \frac{V_o}{r_o}$ (2)



Combining (1) and (2):

$$\frac{V_{gs}}{R_F} - \frac{V_O}{R_F} = g_m V_{gs} + \frac{V_O}{r_O}$$

$$\Rightarrow V_{gs}\left(\frac{1}{R_F} - g_m\right) = V_O\left(\frac{1}{r_O} + \frac{1}{R_F}\right)$$

$$= \frac{V_O}{(r_O \parallel R_F)} \quad (3)$$

At the input: $I_F = I_S - \frac{V_{gs}}{R_S} \Rightarrow$ Combining with (1)

$$\frac{V_{gs}}{R_F} - \frac{V_O}{R_F} = I_S - \frac{V_{gs}}{R_S} \Rightarrow V_{gs}\left(\frac{1}{R_S} + \frac{1}{R_F}\right)$$

$$= I_S + \frac{V_{gs}}{R_S} \Rightarrow \frac{V_{gs}}{(R_F \parallel R_S)} = I_S + \frac{V_O}{R_f}$$

$$\Rightarrow V_{gs} = I_S(R_F \parallel R_S) + V_O\frac{(R_F \parallel R_S)}{R_F} \quad (4)$$

To simplify let's call: $K_1 = \frac{1}{R_F} - g_m$

$K_2 = R_F \parallel R_S$ and $K_3 = r_O \parallel R_F$

The Eq (3) and (4) become:

$$V_{gs}K_1 = \frac{V_O}{K_3} \qquad V_{gs} = I_S K_2 + V_O\frac{K_2}{R_f}$$

Combining: $I_S K_2 \cdot K_1 + V_O\frac{K_2 \cdot K_1}{R_f} = \frac{V_O}{K_3}$

$$I_S K_1 \cdot K_2 = V_O\left(\frac{1}{K_3} - \frac{K_1 \cdot K_2}{R_f}\right)$$

$$\rightarrow \frac{V_O}{I_S} = \frac{K_1 \cdot K_2}{\frac{1}{K_3} - \frac{K_1 \cdot K_2}{R_f}}$$

$$\frac{V_O}{I_S} = \frac{K_1 \cdot K_2 \cdot K_3}{1 - \frac{K_1 \cdot K_2 \cdot K_3}{R_f}}$$ where we recognize an

Eq of the form: $\frac{A}{1 + A\beta}$ with $\beta = \frac{-1}{R_f}$

Substituting for K_1, K_2, K_3 and re-arrenging the signs:

$$\frac{V_O}{I_S} = \frac{-\left(g_m - \frac{1}{R_F}\right)(R_f \parallel R_S)(r_O \parallel R_f)}{1 + \left(g_m - \frac{1}{R_F}\right)(R_f \parallel R_S)\frac{(r_O \parallel R_f)}{R_f}}$$

For the feed-back analysis method to be accurate

$$g_m \gg \frac{1}{R_F}$$

9.48

A-circuit:

$V_i = g_{m1}V_{gs} \cdot R_{D1}$ $V_{gs} = -I_S(R_f \parallel 1/g_{m1})$

$$\Rightarrow V_i = g_{m1} \cdot R_{D1}(R_f \parallel 1/g_{m1})$$

$$V_O = -g_{m2}V_i(R_{D2} \parallel R_f)$$

$$\Rightarrow \frac{V_O}{I_S} = -g_{m2}(R_{D2} \parallel R_f) \times g_{m1}R_{D1}(R_f \parallel 1/g_{m1})$$

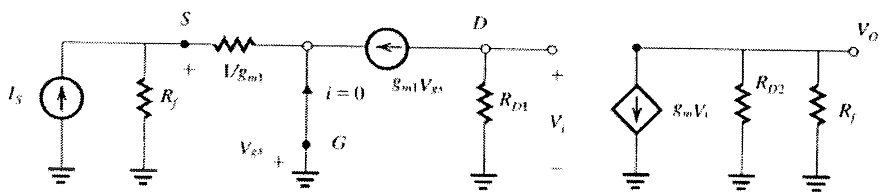
$$= g_{m1} \cdot g_{m2} \cdot R_{D1} \cdot (R_{D2} \parallel R_f)(R_f \parallel 1/g_{m1})$$

$$\beta = \frac{-1}{R_F} \Rightarrow A_F = \frac{A}{1 + A\beta}$$

$$= \frac{-g_{m1} \cdot g_{m2} \cdot R_{D1}(R_{D2} \parallel R_f)(R_f \parallel 1/g_{m1})}{1 + \frac{g_{m1}g_{m2}R_{D1}}{R_f}(R_{D2} \parallel R_f)(R_f \parallel 1/g_{m1})}$$

$R_i = \left(R_f \parallel \frac{1}{g_m}\right)$ and $R_{if} = \frac{R_i}{1 + A\beta}$

$R_O = (R_{O2} \parallel R_f)$ and $R_{of} = R_O/(1 + A\beta)$



This figure is for 9.48

9.49

(a) Due to the feed-back we can assume that I_f is very small.

$\Rightarrow V_O \simeq V_{B1} = 0.7\ \text{V}$

$V_{B2} = V_O + 0.7 = 1.4\ \text{V}$

$$I_{RE} = \frac{0.7 + 5}{10\ \text{K}} = 0.57\ \text{mA} \simeq I_{E2}$$

$$I_{RC} = \frac{5 - 1.4}{10\ \text{K}} = 0.36\ \text{mA}$$

$$I_{C1} = I_{RC} - \frac{I_{C1}}{\beta} = 0.36\ \text{mA} - \frac{0.56\ \text{mA}}{100}$$

$= 0.35\ \text{mA}$

(b) A-circuit: neglecting r_{e1} and r_{e2}

$$g_{m1} = \frac{0.35\ \text{m}}{25\ \text{m}} = 14\ \frac{\text{mA}}{\text{V}}$$

$$r_{\pi 1} = \frac{100}{14\ \text{m}} = 7.14\ \text{k}\Omega$$

$$r_{e2} = \frac{25\ \text{m}}{0.58\ \text{m} \times 0.99} = 43.5\ \Omega$$

$V_{\pi 1} = I_S (r_{\pi 1} \parallel R_f) = I_S (7.14\ \text{K} \parallel 10\ \text{K})$

$= I_S \times 4.16\ \text{K}$

$$V_O = \frac{-g_{m1} \cdot V_{\pi 1} \cdot R_C \cdot R_F \parallel R_E(\beta + 1)}{R_C + (\beta + 1)(r_{e2} + R_F \parallel R_E)}$$

$$= -\frac{14\ \text{m} \times 10\ \text{K} \times 5\text{K} \times (101)}{10\ \text{K} + 101(43.5 + 5\ \text{K})} V_{\pi 1}$$

$= -136\ V_{\pi 1}$

$V_O = (-136 \times 4.16\ \text{K}) I_S$

$$\Rightarrow A = \frac{V_O}{I_S} \cong -566\ \text{k}\Omega$$

$R_i = R_f \parallel r_\pi = 4.16\ \text{k}\Omega$

$$\left(\frac{R_C + r_{e2}}{\beta + 1}\right) \parallel R_E \parallel R_f$$

$$= \left[\frac{10\ \text{K} + 43.5}{101}\right] \parallel 5\ \text{K} = 97.5\ \Omega$$

(C) $\beta = -\frac{1}{R_F} = \frac{-1}{10\ \text{K}} = -100\ \mu\frac{\text{A}}{\text{V}}$

$A\beta = 100\ \mu \times 566\ \text{k}\Omega = 56.6$

$1 + A\beta = 57.6$

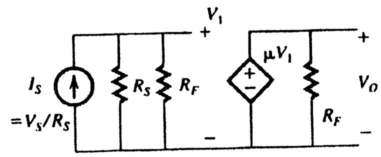
(d) $A_f = \frac{A}{1 + A\beta} = 9.8\ \text{k}\Omega$

$$R_{if} = \frac{R_i}{1 + A\beta} = \frac{4.16\ \text{K}}{58.4} = 73.14\ \Omega$$

$$R_{of} = \frac{R_O}{1 + A\beta} = \frac{140.5}{58.4} = 2.4\ \Omega$$

9.50

A-circuit



$\beta = -1/R_F$

$V_O = -\mu V_1$

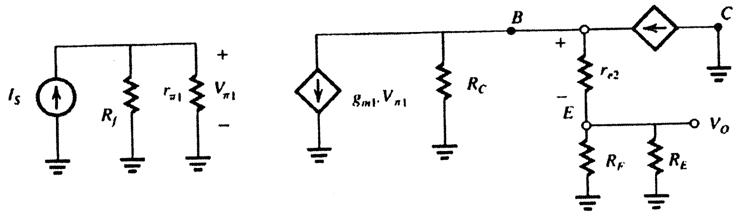
$V_1 = I_S (R_S \parallel R_F)$

$\therefore V_O = -\mu\, I_S (R_S \parallel R_F)$

$$A = \frac{V_O}{I_S} = -\mu \frac{R_S R_F}{(R_S \parallel R_F)}$$

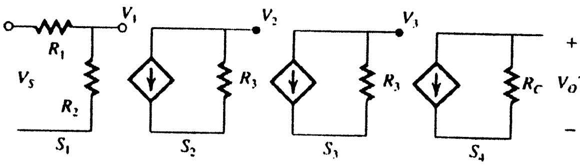
$$A_F = \frac{A}{1 + A\beta} = \frac{-\mu(R_S \parallel R_F)}{1 + \mu(R_S \parallel R_F) R_F}$$

$$= \frac{-\mu(R_S \parallel R_F)}{1 + \mu\left(\frac{R_S \parallel R_F}{R_F}\right)}$$



This figure is for 9.49

This figure is for 9.50(b)



$$\frac{-\mu R_S R_F}{R_S + R_F + \mu R_S} = \frac{-R_F}{1 + \frac{R_F + R_S}{\mu R_S}} \approx -R_F \text{ if } \mu \text{ is}$$

large

$$\frac{V_O}{V_S} = \frac{V_O I_S}{I_S V_S} = -R_F \frac{1}{R_S} = \frac{-R_F}{R_S}$$

(b) For circuit of Fig p8.46 (b)

$$I_B = \frac{V_{CC}\frac{10}{10+15} - V_{BE}}{[(15 \parallel 10) + (4.7 \times 101)] \times 10^3}$$

$$= \frac{15(10/25) - 0.7}{(6 + 474.4) \times 10^3} = \frac{5.3\text{ V}}{480.7\text{ K}} \approx 0.011\text{ mA}$$

$I_C = 100\, I_B = 1.1$ mA

$r_e = V_T / I = 22.6\ \Omega$

$r_\pi = (\beta + 1) r_e = 2.286$ K

$g_m = \beta / r_\pi = 100/2.286\text{ K} \rightarrow 43.7$ mA / V

$R_{in} = (15 \parallel 10 \parallel 2.286)\text{ K} = 1.5$ K

$R_C \parallel R_B \parallel r_\pi = (7.5 \parallel 6 \parallel 2.286) = 1.35$ K

For S_1: $R_I = R_S$, $R_2 = R_{in} \parallel R_F$

$\therefore V_1 = 1.5/11.5\ V_S = 0.13\ V_S$

For S_2: $R_3 = R_C \parallel R_B \parallel r_\pi$

$$\therefore \frac{V_2}{V_1} = -g_m R_3 = -43.7 \times 1.35$$

$= -59$ V / V

For S_3: Same as S_2 $\therefore \frac{V_3}{V_2} = -59$ V / V

For S_4:

$$\frac{V_O}{V_3} = -g_m R_C = -43.7 \times 7.5 = -327.75\text{ V/V}$$

$$\frac{V_O}{V_S} = -0.13 \times 59 \times 59 \times 327.75$$

$$\rightarrow \frac{V_O}{V_S} = -1.488 \times 10^5$$

Because we have ignored r_o, etc let us estimate $V_O / V_S = -1 \times 10^5$ which is quite large.

Then $A_F = \frac{A}{1 + A\beta} \approx 100$ needed

$$= \frac{10^5}{1 + 10^5\beta} \approx \frac{1}{\beta} = 100$$

Select R_F so that

$R_F / R_S = 100 \rightarrow R_F = 100 \times 10\text{ K} = 1\text{ M}\Omega$

We can ignore leading effect of R_F in A-circuit. R_L will cause loading of R_C

$$V_L = \left(\frac{R_L}{(R_C + R_L)}\right) V_o$$

$V_L = (1/8.5) = 0.11\ V_O$

Now $A_O \approx 1.65 \times 10^4$

$$A_F' = \frac{10^4}{1 + 10^4/100} = 99.4$$

9.51

(a) To lower R_{in} and raise R_{out} SHUNT - SERIES

(b) To raise R_{in} and R_{out} SERIES - SERIES

(c) To lower R_{in} and R_{out} SHUNT - SHUNT

9.52

$A_f = -100$ A / A and $1 + A\beta$ is 40 dB

$\Rightarrow 1 + A\beta = 100$

$A\beta = 99$

and since $A_F = A/(1 + A\beta) \Rightarrow$

$A = -100 \times 100 = -10.000$ A / A

and $\beta = -0.0099$

$R_i = R_S \parallel R_{id} \parallel (R_I + R_2)$ and

$R_S = R_{id} = \infty \Rightarrow R_i = R_1 + R_2$

$$R_{in} \cong \frac{R_2}{\mu} \Rightarrow 1\text{ K} = \frac{R_2}{\mu}$$

If we assume that $\frac{1}{g_m} << (R_1 \parallel R_2 \parallel r_{O2})$ then we can use eq (10.73)

$$A\beta = \frac{\mu R_l}{R_2} \rightarrow A\beta = \frac{(R_1 + R_2)}{(R_2 / \mu)}$$

$$\rightarrow 99 = \frac{R_1 + R_2}{1\text{ K}} \qquad (1)$$

$$\beta = \frac{-R_1}{R_1 + R_2}$$

$\Rightarrow$ For $\beta = -0.0099$

$R_2 = (100.01)R_1 \qquad (2)$

Combining (1) and (2): $R_1 + R_2 = 99\text{ K}$

$R_2 = (100.01)R_1$

$\Rightarrow R_1 = 980.1\ \Omega$

$R_2 = 98.02\text{ k}\Omega$

$$99 = \frac{\mu .99\text{ K}}{98.02\text{ K}} \rightarrow \mu = 98.02\text{ V/V}$$

Given $g_m = 5\text{ m}\frac{\text{A}}{\text{V}}$ and $r_O = 20\text{ K}$ we observe that the assumption $\frac{1}{g_m} <<$ $(R_1 \parallel R_2 \parallel r_{o2})$ is not valid

$\left(\frac{1}{5\text{ m}} = 200\ \Omega\right)$

$\approx ((980.1 \parallel 98.02\text{ K} \parallel 20\text{ K}) = 925.5)$

, cannot be used.

Instead we use:

$$-10.000 = \frac{-\mu \cdot 99\text{ K}}{200 + 925.5}\left(\frac{20\text{ K}}{20\text{ K} + 970.4}\right)$$

$\Rightarrow \mu = 119.2$

To calculate R_{out} we cannot use .

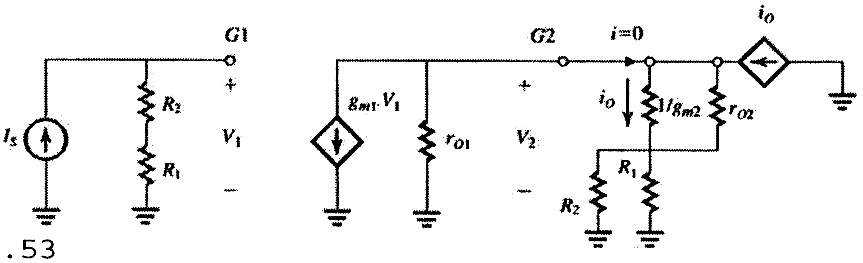
$R_o = r_{O2} + g_{m2}r_{O2}(R_1 \parallel R_2) + (R_1 \parallel R_2) = 119\text{ k}\Omega$

$R_{of} = R_O(1 + A\beta) = 11.950$

Since: $R_{out} = R_{of} - R_L$ and $R_L = 0$

$\Rightarrow R_{out} = R_{of} = 9.7\text{ M}\Omega$

This figure is for 9.53(c)



9.53

(a) $V_{GS} = V_{OV} + V_t$

$= 0.2 + 0.5 = 0.7\text{ V}$

$\Rightarrow V_{GS1} = V_{G1} = 0.7\text{ V}$

Since $I_{G1} = 0 \rightarrow V_{S2} = 0.7$

$$I_{D2} = \frac{V_{S2}}{R_I} = \frac{0.7}{3.5\text{ K}} = 0.2\text{ mA}$$

$$\text{(b)}\, g_m = \frac{2I_D}{V_{OV}} = \frac{0.4\text{ m}}{0.2} = 2\text{ mA/V}$$

$$r_o = \frac{V_A}{I_D} = \frac{10}{0.2\text{ m}} = 50\text{ k}\Omega$$

(c) The A-circuit:

$V_1 = I_S(R_1 + R_2)$

$V_2 = -g_{m1}r_{O1}V_1 = -g_{m1}r_{O1}(R_1 + R_2)I_S$

$$i_O = \frac{V_2}{1/g_{m2} + R_2 \| R_1 \| r_{O2}}$$

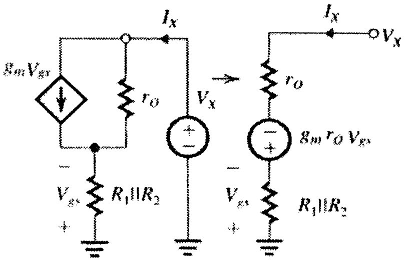
$$= \frac{-g_{m1}r_{O1}(R_1 + R_2)I_S}{1/g_{m2} + R_2 \| R_1 \| r_{O2}}$$

$$\Rightarrow A = \frac{i_O}{I_S} = \frac{-g_m \cdot r_O \cdot (R_1 + R_2)}{1/g_m + (R_2 \| R_1 \| r_O)}$$

$$= \frac{-2 \times 50(3.5 + 14)\text{ K}}{500 + (3.5 \| 14 \| 50)\text{ K}} = -555.3\ \frac{\text{A}}{\text{A}}$$

$R_i = R_2 + R_1 = 3.5 + 14 = 17.5\text{ k}\Omega$

To get R_O:



$V_{gs} = -I_X(R_1 \| R_2)$

$V_X + gm\, r_O V_{gs} = I_X(r_O + R_1 \| R_2)$

$$R_O = \frac{V_X}{I_X} = r_O + R_1 \| R_2 + gm\, r_O\,(R_1 \| R_2)$$

$R_O = 50\text{ K} + (3.5 \| 14)\text{ K} + 2 \times 50(3.5 \| 14)\text{ K}$

$R_O = 332.8\text{ k}\Omega$

$$\text{(d)}\ \beta = \frac{-R_1}{R_1 + R_2} = \frac{-3.5}{17.5} = -0.2$$

(e) $A\beta = 555.3 \times 0.2 = 111.06$

$$A_f = \frac{A}{1 + A} = \frac{-555.3}{112.06} = -5\text{ A/A}$$

(f) $R_{in} = R_{if} \| R_S$ since $R_S = \infty \Rightarrow R_{in} = R_{if}$

$$R_{in} = \frac{R_i}{1 + A\beta} = \frac{17.5\text{ K}}{112.06} = 156.2\ \Omega$$

$R_{out} = R_{of} - R_L$ but $R_L = 0 \Rightarrow R_{out} = R_{of}$

$R_{out} = R_o(1 + A\beta)$

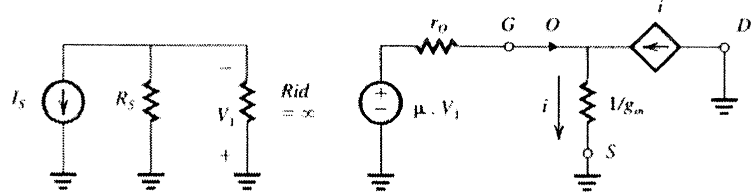
$= 332.8\text{ K} \times 112.06 = 37.3\text{ M}\Omega.$

9.54

(a) if μ is large, the loop gain is large and the current at the negative input is $\sim 0 \Rightarrow V_{in} \sim -R_S \cdot I_S$

$$\text{Since } I_S \simeq I_O \Rightarrow \frac{I_O}{I_S} \simeq 1\text{ A/A}$$

(b) A-circuit:



$V_1 = I_S \cdot R_S$

$\Rightarrow I_O = i = \mu \cdot R_S g_m \cdot I_S$

$i = \dfrac{(\mu \cdot V_1)}{1/g_m}$

$\Rightarrow A = \dfrac{I_O}{I_S} = \mu \cdot R_S \cdot g_m$

$R_i = R_S\ R_O = r_{02}$

(c) $\beta = 1$

(d) $A\beta = \mu \cdot R_S \cdot g_m$

$A_f = \dfrac{A}{1 + A\beta} = \dfrac{\mu \cdot R_S \cdot g_m}{1 + \mu \cdot R_S \cdot g_m}$

If $r >> 1 \Rightarrow A_f \simeq 1$

(e) $R_{if} = \dfrac{R_i}{1 + A\beta} = \dfrac{R_S}{1 + \mu R_S \cdot g_m} \Rightarrow \dfrac{1}{R_{if}}$

$= \dfrac{1}{R_S} + \mu \cdot g_m$

$R_{if} = R_S \parallel 1/\mu \cdot g_m$

since $R_{if} = R_S \parallel R_{in}$

$\Rightarrow R_{in} = \dfrac{1}{\mu \cdot g_m}$ if $r >> 1 \rightarrow R_{in} = 0$

$R_{of} = R_O(1 + A\beta) = r_{02}(1 + \mu R_S g_m)$

$R_{of} = R_{out} + R_L$ and

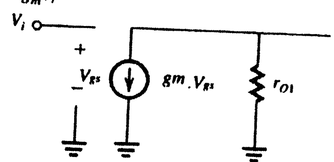
$R_L = 0 \Rightarrow R_{out} = \dfrac{1}{g_m} + \mu R_S$

if $\mu >> 1 \Rightarrow R_{out} = \infty$

For Q_1:

This is equivalent to circuit with:

$I_s = g_m V_i$



$R_s = r_{01}$

$I_O = V_2 g_{m1} \dfrac{\mu g_{m2} r_{01}}{1 + \mu g_{m2} r_{01}}$

$V_1 g_{m1}$ if $\mu g_{m2} r_{01} > 1$

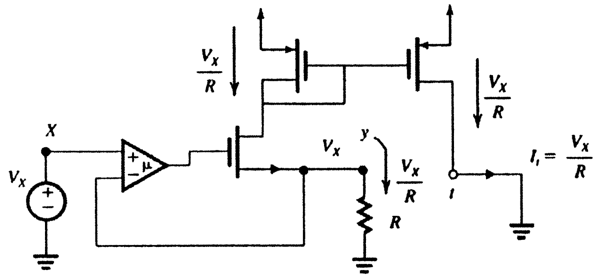
$R_{out} = r_{02}(1 + \beta A)$

$= r_{02}(1 + \mu r_{01} g_{m2})$

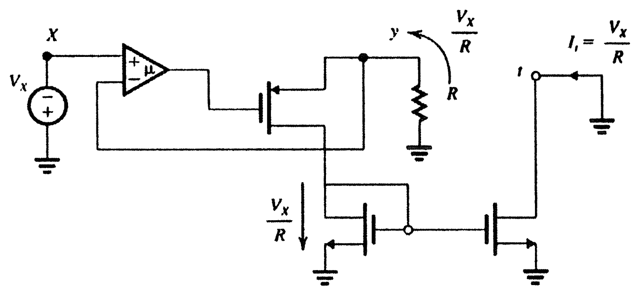
$\approx \mu(r_{02} g_{m2} r_0)$

9.55

(a) When V_X is positive:



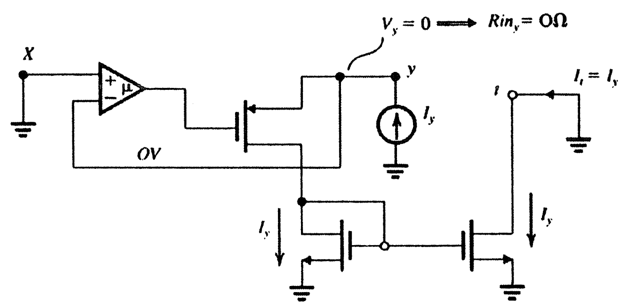
When V_X is negative:

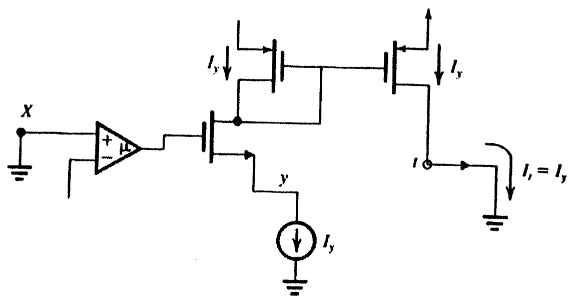


(b) When I_y is positive:

$$R_{in} = \frac{1}{\mu(g_{mp} + g_{mn})} \sim 0 \text{ (if } \mu >> 1)$$

When I_y is negative:





(c) When R_{zout} is $r_{O2} \parallel r_{O4}$

9.56

Neglect I_{B2}; $I_{B1} \approx \frac{200}{100} = 2\mu A$

$V_{BE} = 0.7V \quad \therefore V_{B1} = +0.7V$

But no d.c. component in V_s

$\therefore I_{RS}$ (into V_s) $= 0.7/10K = 0.07\mu A$

Thus $I_F = I_{RS} + I_{B1} = 0.07 + 0.002$

$= 0.072\ mA$

$V_{E2} = 0.7 + 10 \times 0.072 = 0.7 + 0.72 = 1.42V$

$I_{C2} = 1.42/140 + 0.072 = 10.2mA$

$I_{B2} = I_{E2}/(\beta+1) = 0.1\ \mu A \approx \frac{1}{2} 200\mu A$

Iterate:

$I_{B1} = \frac{200\mu A - 100\mu A}{100} = 0.001\mu A$

$V_{E2} = 0.7 + 10 \times 0.073 = 1.41V$

$I_{E2} = 1.41/0.140 + 0.071 = 10.1mA$

$I_{B2} = 10.1/101 = 100\mu A \quad \therefore I_{C2} = 10mA$

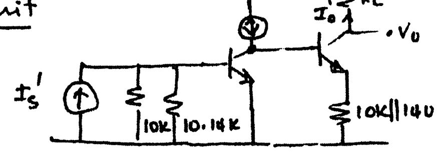
$V_{B2} = V_{E2} + V_{BE} = 1.41 + 0.7 = 2.11V$

$V_O = 10 - 10 \times 500\Omega = +5V$

$r_{\pi1} = \frac{V_T}{I_E} = \frac{25mV}{0.1mA} = 250\Omega$, $r_{e2} = 25\Omega$

$\beta = \frac{R_E}{R_F + R_E} = \frac{140}{10K + 140} \approx 0.0138$

A-circuit



$V_{B1} = 10.14K \parallel 10K \parallel \beta(250) I_s'$

$= 4.2 \times 10^3\ I_s'$

$\Rightarrow \frac{I_o'}{I_s'} = \frac{4.2 \times 10^3 (\beta+1)(r_{e2} + 10K\parallel 140)}{250} \times \frac{1}{(r_{e2} + 10K\parallel 140)}$

$= 1.69 \times 10^3\ A/A$

$A_F \equiv \frac{I_o}{I_s} = \frac{A}{1+A\beta} = \frac{1.69 \times 10^3}{1 + 1.69 \times 10^3 \times (0.0138)}$

$= 69.6$

$\Rightarrow \frac{V_o}{V_s} = \frac{I_o R_L}{I_s R_s} = \frac{500}{10^4} \cdot 69.6 = 3.5 V/V$

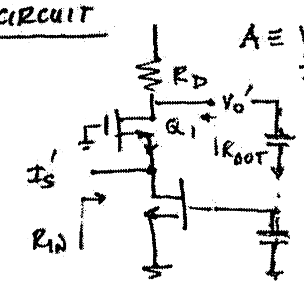
$R_i = 10K \parallel 10.14K \parallel 25K = 4.2\ K\Omega$

$R_{if} = R_i/(1+A\beta) = \frac{4.2}{1+23.3} = 172.8\Omega$

$\Rightarrow = R_{IN} \parallel R_s \Rightarrow R_{IN} = 175.8\Omega$

9.57

A-CIRCUIT

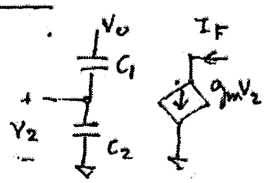


$A \equiv \frac{V_o'}{I_s'} = [R_D \| X_C] = R_D$ kΩ

Neglecting loading of C's

$A_F = \frac{A}{1+A\beta}$

B-CIRCUIT



$\beta = \frac{C_1}{C_1+C_2} g_{mf}$

$= \frac{1}{10} \times 1 \frac{mA}{V}$

Here. $g_{m1} = 5mA/V$ $g_{mf} = 1mA/V$
$R_D = 10K$

Thus $A = 10 K\Omega$

$A_F = \frac{10K}{1 + 10K(0.1)} = 5 K\Omega$

$R_{IN} = (R_D \| r_o) \longrightarrow R_D$

$R_{if} = R_D/(1+A\beta)$ shunt $= \frac{R_D}{2} = 5K\Omega$

$R_{out} = R_D/(1+A\beta) = \frac{R_D}{2} = 5K\Omega$

9.58

(a) $V_{B1} \simeq 12\frac{15}{100+15} = 1.57$ V

$V_{E1} \simeq 1.57 - 0.7 = 0.87$ V

$I_{E1} = 0.87/0.87 = 1$ mA $\rightarrow g_{m1}$

$= \frac{\alpha I_{E1}}{V_T} = \frac{0.99 \times 1}{25} \simeq 40 \frac{mA}{V}$

$V_{C1} \simeq 12 - 10 \times 1 = 2$ V

$V_{E2} \simeq 2 - 0.7 = 1.3$ V

$I_{E2} \simeq 1.3/3.4 \simeq 0.4$ mA $\rightarrow g_{m2}$

$= \frac{0.99 \times 0.4}{25} \simeq 16 \frac{mA}{V}$

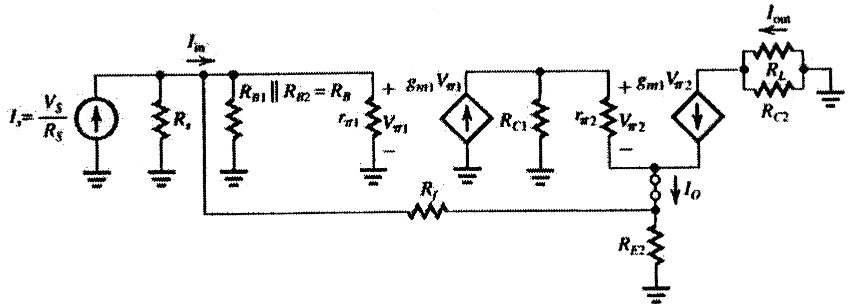
$V_{C2} \simeq 12 - 0.4 \times 8 = 8.8$ V

$r_{\pi 1} = \frac{\beta}{g_{m1}} = \frac{100}{40 \text{ m}} = 2.5$ kΩ

$r_{\pi 2} = \frac{100}{16 \text{ m}} = 6.25$ kΩ

$r_{e2} = \frac{V_T}{I_{E2}} = \frac{25}{0.4} = 62.5$ Ω

(b)



To obtain A:

$R_B = R_{B1} \| R_{B2}$

$V_{\pi 1} = I_i[R_S \| (R_{E2} + R_f) \| R_B \| r_{\pi 1}]$

$= 1.535\ k \times I_i$

$V_{b2} = -g_{m1} V_{\pi 1}$

$\{R_{C1} \| [r_{\pi 2} + (\beta + 1)(R_{E2} \| R_f)\}$

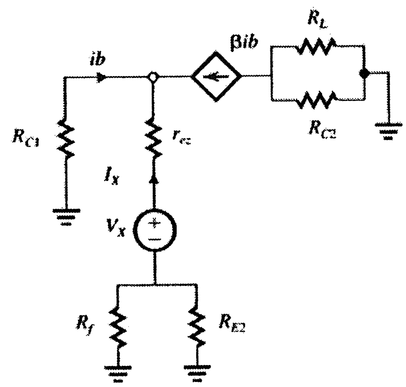
$= I_O \simeq \frac{V_{b2}}{r_{e2} + (R_{E2} \| R_f)}$

Combining these equations we obtain:

$A = \frac{I_O}{I_S} = -201.45$ A/A

$R_i = R_S \| (R_{E2} + R_f) \| R_B \| r_{\pi 1} = 1.535\ \text{k}\Omega$

R_O is obtained by looking into nodes Y and Y', with I_i set to zero



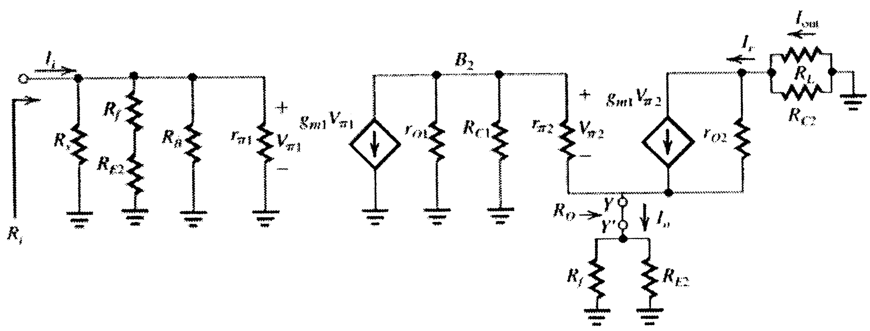
$R_O = \frac{V_x}{I_x}$

$I_x = -(\beta + 1)ib$

$-R_{C1} \times ib = (\beta + 1)ib[r_{e2} + R_f \| R_{E2}] + V_x$

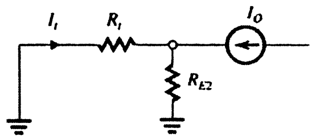
Replacing $-ib$ by $I_x/(\beta + 1)$

See Figure below.



$$R_O = \frac{V_x}{I_x} = \frac{R_{C1}}{\beta+1} + r_{e2} + (R_f \| R_{E2}) = 2.69\ \text{k}\Omega$$

(d)



$$\beta = \frac{I_f}{I_O} = -\frac{R_{E2}}{R_{E2}+R_f} = -\frac{3.4}{13.4} = -0.254$$

(e) $A\beta = 51.1$

$1 + A\beta = 52.1$

$$R_{if} = \frac{R_i}{1+A\beta} = 29.5\ \Omega$$

$$A_f = \frac{I_O}{I_i} = \frac{A}{1+A\beta} = -3.87\ \text{A/A}$$

$$R_{of} = R_O(1+A\beta) = 140.1\ \text{k}\Omega$$

(f) $\frac{I_{out}}{I_{in}} \simeq \frac{I_{out}}{I_S} = \frac{R_{C2}}{R_L+R_{C2}} \cdot \frac{I_C}{I_S} \simeq \frac{R_{C2}}{R_L+R_{C2}} \cdot \frac{I_O}{I_S}$

$$\rightarrow \frac{I_{out}}{I_{in}} = -3.44\ \text{A/A}$$

$$R_{in} = \frac{1}{\frac{1}{R_{if}} \cdot \frac{1}{R_S}} \simeq 29.5\ \Omega$$

Notice that $R_{in} = R_{if}$

$$A_f = \frac{I_O}{I_i} = \frac{I_O}{I_S}$$

To obtain R_O: recall from problem 10.49 that:

$$R_{out} = r_{o2}[1 + g_{m2}(r_{\pi 2} \| R_{of})]$$

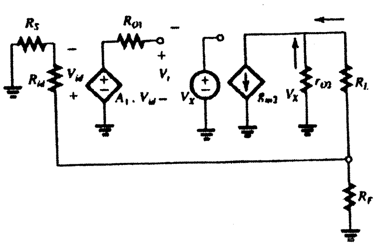
$$r_{O2} = \frac{75}{0.4} = 187.5\ \text{k}\Omega \quad r_{\pi 2} = 6.25\ \text{k}\Omega$$

$g_{m2} = 16$ mA/V

$R_{of} = 140.1$ kΩ

$\Rightarrow R_{out} = 18.1$ MΩ

9.59



$$\frac{-V_t}{V_x} = \frac{g_{m2} \cdot r_{O2}}{r_{O2} + R_L + R_F \| (R_{id}+R_S)} \times \frac{R_F}{R_F + R_{id} + R_S} \times R_{id} \times A_1$$

Re-grouping to put this equation in the form of

$$A\beta = (A_1 g_{m2}) \cdot \left(\frac{R_{id}}{R_{id}+R_S+R_F}\right) \cdot \frac{r_{O2}}{(r_{O2} + R_L + R_F \| (R_{id}+R_S))} \cdot R_F$$

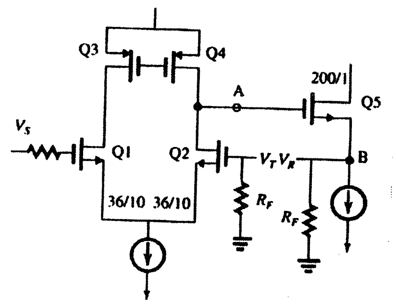
Since R_F is usually << R_{id}

$$A\beta \simeq \left(A_1 \cdot g_{m2}\left(\frac{R_{id}}{R_{id}+R_S+R_F}\right)\left(\frac{r_{O2}}{r_{O2}+R_L+R_F}\right) \cdot R_F\right)$$

where the term R_F is the only difference recall that

$\beta = R_F$ thus R_F is small

9.60



$V_T = V_{G2}$ and $V_S \rightarrow 0$

$$V_A = -g_{m2}(r_{O2} \| r_{O4})$$

$$V_B = V_A \frac{(R_F \| r_{O2})}{(R_F \| r_{O5}) + 1/g_{m5}}$$

$$A\beta = \frac{-V_T}{V_R} = 1 g_{m2} \frac{(r_{O2} \| r_{O4})(R_F \| r_{O5})}{(R_F \| r_{O5}) + 1/g_{m5}}$$

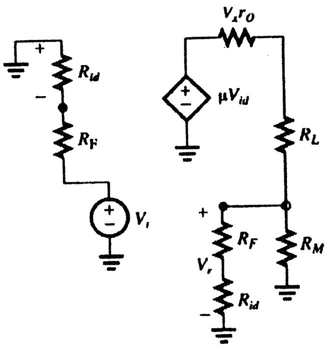
9.61

$$\frac{V_x}{V_t} = \mu \frac{R_{id}}{R_F + R_{id}}$$

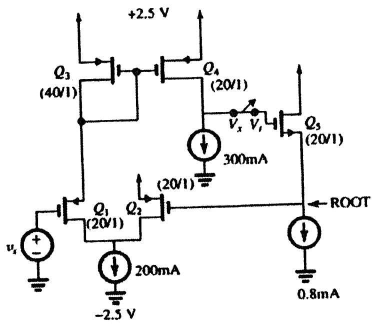
$$\frac{V_r}{V_x} = \frac{(R_F + R_{id}) \| R_M}{(R_F + R_{id}) \| R_M + R_L + r_O}$$

$$A\beta = \mu \frac{R_{id}}{R_F + R_{id}} \frac{(R_F + R_{id}) \| R_M}{(R_F + R_{id}) \| R_M + R_L + r_O}$$

$$\approx \mu \frac{(R_F + R_{id}) \| R_M}{(R_F + R_{id}) \| R_M + R_i + r_O} \approx \mu \frac{R_M}{R_M + R_L + r_O}$$



9.62



Breaking at the gate of QS:

$$\frac{V_r}{V_t} = \left(\frac{g_{m5} r_{05}}{1 + g_{m5} r_{05}}\right)\left(\frac{1}{2} g_{m2} r_{04}\right)$$

$$= \frac{2 \times 10^{-3} \cdot 30 \times 10^3}{1 + 2 \times 10^{-3} \cdot 30 \times 10^3}$$

$$\left(\frac{1}{2} \cdot 0.693 \times 10^{-3} \cdot 80 \times 10^3\right)$$

$$= 27.26$$

$$1 + A\beta = 28.26$$

$$R_{out} = \frac{1}{g_{m5}} \frac{1}{1 + A\beta} = 17.7\ \Omega$$

$$k'n = 2k'_p = 120\ \mu A / V^2$$

$$V_t = 0.7V$$

$$V_A = 24V / \mu m$$

$$r_{O5} = \frac{V_A}{I_{DS}} = 30\ k\Omega$$

$$r_{O4} = \frac{V_A}{I_{DS}} = \frac{24}{0.3 \times 10^{-3}} = 80\ k\Omega$$

$$g_{m2} = \sqrt{2k'n\left(\frac{W}{L}\right) I_{DS2}}$$

$$= \sqrt{2.120 \times 10^{-6} \cdot 20 \cdot 100 \times 10^{-6}}$$

$$= 0.693 \times 10^{-3} = 0.693\ mS$$

$$g_{m5} = \sqrt{2k'_n\left(\frac{W}{L}\right) I_{DS5}}$$

$$= 2\ mS$$

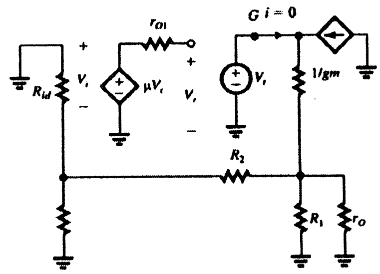
9.63

$$\frac{-V_r}{V_t} = \frac{(R_2 + R_S \parallel R_{id}) \parallel (R_1 \parallel r_O)}{\frac{1}{g_m} + (R_2 + R_S \parallel R_{id}) \parallel (R_1 \parallel r_O)}$$

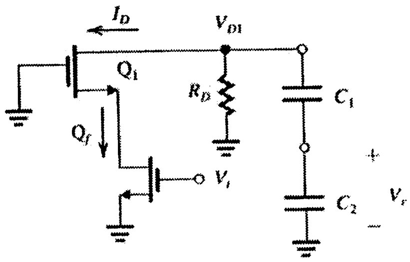
$$\cdot \frac{R_S \parallel R_{id}}{R_S \parallel R_{id} + R_2} \cdot \mu$$

Since $R_{id} = R_S = \infty \Rightarrow$ The expression reduces to

$$A\beta = \frac{-V_r}{V_t} = \mu \frac{R_1 \parallel r_O}{\frac{1}{g_m} + (R_1 \parallel r_O)} = 997$$



9.64



Assume that C_1 and C_2 are small and do not load the output.

Neglect r_{of} and r_{o1}

Since $I_D = I_{D1} = I_{Df}$

$$V_{D1} = -gm_f V_i \cdot R_D$$

$$V_r = V_{D1} \cdot \frac{1/SC_2}{\frac{1}{SC_2} + \frac{1}{SC_1}}$$

$$\Rightarrow \frac{-V_r}{V_i} = \frac{C_1}{C_2 + C_1} \cdot gmf \cdot R_D$$

$$A\beta = \frac{0.9}{0.1 + 0.9} \cdot 1 \times 10 = 9$$

9.65

$$A(S) = \frac{10^5}{1 + 5/100}$$

$$\text{Ang}(A) = -\tan^{-1}\frac{\omega}{100} - 2\tan^{-1}\frac{\omega}{10^4}$$

at ω_{180} : Ang$(A) = -180°$ for $\omega_{180} >> 100$

$$\Rightarrow 18° = 90° + 2\tan^{-1}\left[\frac{\omega_{180}}{10^4}\right]$$

hence $\tan^{-1}\frac{\omega_{180}}{10^4} = \frac{90°}{2}$

i.e. $\frac{\omega_{180}}{10^4} = \tan(45°) = 1$

$\therefore \omega_{180} = 10^4$ rad / s

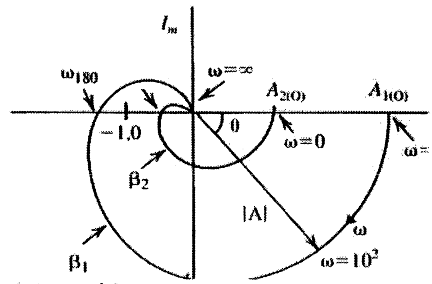
$$|A\beta| = \frac{10^5\beta}{\sqrt{1 + (10^4/10^2)^2}} \cdot \frac{1}{(\sqrt{1+1})^2} = 1$$

$\Rightarrow \beta = 0.002$

$$A_f(0) = \frac{10^5}{1 + 10^5(0.002)} = 500 \text{ V/V}$$

9.66

ω	Ang(A)	$\lvert A\rvert B_1$	$\lvert A\rvert B_2$
0	0	10^5	10^2
10^2	45	7.07×10^4	70.7
10^3	95.7	9.85×10^3	9.85
10^4	180	500	0.5
∞	0	0	0



(not to scale)

9.67

$$A(S) = \frac{10^3}{1 + 5/10^4}$$

$$\beta(S) = \frac{K}{(1 + 5/10^4)^2}$$

$$\text{Ang}(A\beta) = -\tan^{-1}\frac{\omega}{10^4} - 2\tan^{-1}\frac{\omega}{10^4}$$

$$= -3\tan^{-1}\frac{\omega}{10^4}$$

For 180° $\omega_{180} = \sqrt{3} \times 10^4$ rad/s

For $|A\beta(\omega_{180})| < 1$

$$\frac{10^3}{\sqrt{1 + (\sqrt{3})^2}} \cdot \frac{K}{1 + (\sqrt{3})^2} < 1$$

$\Rightarrow K < 0.008$

9.68

$$A(S) = \frac{1000}{(1 + S/10^4)(1 + S/10^5)^2}$$

and β is independent of frequency

$$\text{Ang}(A) = -\tan^{-1}\frac{\omega}{10^4} - 2\tan^{-1}\frac{\omega}{10^5}$$

try $\omega = 10^4$: θ 5 45° + 2 × 5.7 = 56.4°

try $\omega = 10^5$: θ 5 84.2° + 2 × 45 = 174.2°

Iteraction yields $\omega = 1.1 \times 10^5$ rad/s

For oscillations: $|AB(\omega_{180})| \geq 1$

$$\frac{\beta\ 10^3}{(\sqrt{1 + 11^2})(\sqrt{1 + 1.1^2})} \geq 1$$

$\Rightarrow \beta \geq 0.0244$

9.69

$$A(jf) = \frac{(10 \times 10^6)/10^4}{1 + jf/10^4}$$

$$\therefore\ A(jf) = \frac{10^3}{1 + jf/10^4}$$

$\beta = 0.1$ independent of frequency

$$A_f(jf) = \frac{10^3}{1+10^3(0.1)} \cdot \frac{1}{1 + \frac{jf}{10^4(1+10^3(0.1))}}$$

$$= \frac{9.9}{1 + jf/(101 \times 10^4)}$$

$A_f(0) = 9.9$ V/V

$f_{Pf} = 10^4(101) = 1.01$ MHz

for $\frac{f}{f_{Pf}} \gg 1$: $A_f \approx 9.9\,\frac{10^4(101)}{f}$

for $A_f = 1$: $f = f_t = 10$ MHz

Pole is shifted by $(1 + A(0)\beta) = 101$

9.70

$$A(j_f) = \frac{10^3}{(1 + j_f/10^4)(1 + j_f/10^5)}$$

(a) closed-loop poles given by

$1 + A(f)B = 0$

using $P = j_f$

$P^2 + P(10^4 + 10^5) + (1 + 10^3\beta)10^4 \cdot 10^5 = 0$

i.e. $P^2 + (1.1 \times 10^5)P + 10^9(1 + 10^3\beta) = 0$

compare terms with

$(P + f_{Pf})^2 = P^2 + 2f_{Pf} \cdot P + f_{Pf}^2$

$2f_{Pf} = (1.1 \times 10^5)$

$(1 + 10^3\beta) \times 10^9 = f_{Pf}^2$

$\Rightarrow f_{Pf} = 5.5 \times 10^5$

and $(1 + 10^3\beta) = 3.025 \Rightarrow$

β 5 2.025×10^{-3}

(b) At 55 kHz

$$A(f) = \frac{10^3}{\left(1 + j\frac{55 \times 10^3}{10^4}\right)\left(1 + j\frac{55 \times 10^3}{10^5}\right)}$$

$$= \frac{10^3}{(1 + j5.5)(1 + j0.55)}$$

$$= \frac{10^3}{1 + j6.05 - 3.025} = \frac{10^3}{-2.025 + j6.05}$$

$= -24.75(2.025 + j6.05) = -49.75 - j149.74$

$|A(55\text{ kHz})| = 157.7$

$$A_f(55\text{ kHz}) = \frac{-49.75 - j149.74}{1 - (49.75 + j149.74)2.025 \times 10^{-3}}$$

$$= -\frac{49.75 + j149.74}{0.9}(0.9 + j0.3) = 0.16 - j166.3$$

$|A_f(55\text{ KHz})| = 166.3$

(c) from $S^2 + (\omega_O/Q)S + \omega_O^2$ cf above

$$Q = \frac{f_{Pf}}{2f_{Pf}} = \frac{1}{2}$$

(d) $P^2 + 1.1 \times 10^5P + (1 + 10^3\beta) = 0$

$\Rightarrow P^2 + 1.1 \times 10^5\,P + 21.25 \times 10^9 = 0$

$$P = \frac{-1.1 \times 10^5 \pm \sqrt{(1.1 \times 10^5)^2 - 4(21.25 \times 10^9)}}{2}$$

$$= \frac{-1.1 \times 10^5 \pm j\,2.7 \times 10^5}{2}$$

$= -5.5 \times 10^4 \pm j\,1.35 \times 10^5$ Hz

$$Q = \frac{|P|}{2(5.5 \times 10^4)}$$

$$= \frac{\sqrt{(5.5 \times 10^4)^2 + (1.35 \times 10^5)^2}}{1.1 \times 10^5}$$

$= 1.33$

9.71

$$A(jf) = \frac{10^3}{(1 + jf/10)(1 + jf/f_P)}$$

$$A_f(0) = \frac{10^3}{1 + 10^3\beta} = 100$$

$\Rightarrow \beta = 9 \times 10^{-3}$ V/V

Maximally flat when $Q = 0.707 = 1/\sqrt{2}$

from

$p^2 + p(f_1 + f_2) + (1 + A_0\beta)(f_1 \times f_2) = 0$

$$Q = \frac{\sqrt{(1 + A_0\beta)f_1f_2}}{f_1 + f_2}$$

$$\Rightarrow \frac{\sqrt{(1 + 10^3\beta)\,10^3 f_{Pf}}}{10^3 + f_{Pf}} = \frac{1}{\sqrt{2}}$$

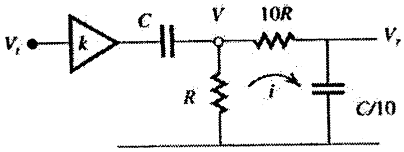
$\Rightarrow f_{Pf}^2 + (2 \times 10^3)f_{Pf} + 10^6$

$= 2(1 + 10^3\beta)^2 10^6$

$$\Rightarrow f_{Pf} = \frac{18 \times 10^3 \pm \sqrt{(18 \times 10^3)^2 - 4(10^6)}}{2}$$

$= 17.94$ KHz.

9.72



$i = \frac{SCV_r}{10} \Rightarrow V = V_r + SCRV_r$

$= V_r(1 + SCR)$

By KCL : $\Sigma_i = 0$ a + V

$\Rightarrow \frac{(KV_i - V)}{1/SC} = \frac{V}{R} + i = \frac{V}{R} + \frac{SCV_r}{10}$

$KSCV_i - SCV_r(1 + SCR)$

$= V_r\left(\frac{1 + SCR}{R} + \frac{SC}{10}\right)$

Collecting terms:

$K\,SCV_iR = V_r[(SCR)^2 + 2.1\,SCR + 1]$

Thus $L(S) \triangleq -V_r/V_i$

$= \frac{-K/CR\ S}{S^2 + \frac{2.1\ S}{CR} + \frac{1}{CR^2}}$

$\equiv \frac{-K/CR\ S}{S^2 + \left(\frac{\omega_O}{Q}\right)S + \omega_O^2}$

from which $\omega_o = 1/CR$

$Q = \frac{1}{2.1 - K}$

Poles coincide when $Q = 1/2$

$\Rightarrow K = 2.1 - 2 = 0.1$

maximally flat when $Q = 1/\sqrt{2}$

$\Rightarrow K = 2.1 - 1.414 = 0.686$

Oscillates when $Q \to \infty$

$\Rightarrow K = 2.1$

9.73

$A(f) = \frac{-K}{1 + j_f/10^5}$

for $\beta = 1$: $A\beta = \frac{K^3}{\left(1 + \frac{j_f}{10^5}\right)^3}$

For oscillations to occur : $|A\beta| \geq 1$ at

$\phi(A\beta) = 180°$

3tan^{-1}

$\left(\frac{f_{180°}}{10^5}\right) = 180° \Rightarrow f_{180°} = \sqrt{3} \times 10^5$ Hz

$f_{180°} = 173.2$ KHz

Amplifier is unstable if $|A\beta| \geq 1$ at $f_{180°}$

$\left[\frac{K}{\sqrt{1 + (\sqrt{3})^2}}\right]^3 \geq 1 \to K \geq 2$

9.74

$A(f) = \frac{10^5}{1 + j_f/10}$

for $\beta = 1$: $A(f)B = \frac{10^5}{1 + j_f/10}$

for $f >> 10$: $|A\beta| \approx 10^5 \cdot \frac{10}{f_1}$

$\Rightarrow f_1 = 1$ MHz

at f_1: phase margin $= 180° - \tan^{-1}\frac{10^6}{10}$

$= 90°$

9.75

$A(f) = \frac{10^5}{1 + j_f/10\,(1 + j_f/10^4)}$

$A\beta(0) = 10^5\beta$

$A_F(0) = 100 = \frac{10^5}{1 + 10^5\beta} \Rightarrow \beta \approx 0.01$

$|A\beta| = 1 \Rightarrow |1 + j_f/10| \cdot |1 + j_f/10^4|$

$= 10^5\beta = 10^3$

$(1 + f^2/10^2)(1 + f^2/10^8) = 10^6$

$f^4 + f^2(10^8 + 10^2) - (10^8)(10^2)10^6 = 0$

$f \simeq \frac{-10^8 + \sqrt{10^{16} + 4 \times 10^6}}{2}$

$\Rightarrow 61.8 \times 10^6 \Rightarrow f = 7.86$ KHz

Phase margin

$= 180 - \left(\tan^{-1}\frac{7.86 \times 10^3}{10} + \tan^{-1}\frac{7.86}{10}\right)$

$\approx 180° - 90° - 38.16° = 51.8°$

For $PM \geq 45°$: $\tan^{-1}\frac{f_1}{10^4} \leq 45°$

$\Rightarrow f_1 \leq 10^4$

thus

$|A\beta| = 1 = \frac{10^5\beta}{\sqrt{1 + (10^3)^2} - \sqrt{2}}$

$\Rightarrow \beta = \frac{\sqrt{2}}{100} = 0.0141$

9.76

$|1 + e^{-j\theta}| = |1 + \cos\theta - j\sin\theta|$

$= [(1 + \cos\theta)^2 + (\sin\theta)^2]$

$= [1 + 2\cos\theta + \cos^2\theta + 1 - \cos^2\theta]^{1/2}$

$= \sqrt{2}(1 + \cos\theta)^{1/2}$

for 5%: $1 + \cos\theta = \frac{1}{1.05^2(2)} = 0.4535$

$\theta = 123.13°$ and $PM = 180 - \theta = 56.87°$

for 10%: $1 + \cos\theta = \dfrac{1}{1.1^2(2)} = -0.586$

$\theta = 125.93°$ and $PM = 54.07°$

for 0.1 dB $\equiv 10^{0.1/20} = 1.0116$

$\cos\theta = \dfrac{1}{2(1.0116)^2} - 1 = -0.5114$

$\theta = 120.76°$ and $PM = 59.24°$

for 1 dB $\equiv 10^{1/20} = 1.122$

$\cos\theta = \dfrac{1}{2(1.122)^2} - 1 = -0.6028$

$\theta = 127.07°$ and $PM = 52.93°$

9.77

$$A(j_f) = \frac{10^5}{\left(1 + \frac{j_f}{10^5}\right)\left(1 + \frac{j_f}{3.16 \times 10^5}\right)\left(1 + \frac{j_f}{10^6}\right)}$$

Assume β independent of frequency

For 45° PM: $\theta = 180 - 45$

$$\tan^{-1}\frac{f_1}{10^5} + \tan^{-1}\frac{f_1}{3.16 \times 10^5} + \tan^{-1}\frac{f_1}{10^6} = 135°$$

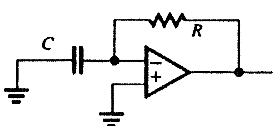
solve $\Rightarrow f_1 = 3.16 \times 10^5$ Hz

$$|A\beta(f_1)| = 1 = \frac{10^5\beta}{\sqrt{1 + (3.16)^2} \cdot \sqrt{2} \cdot \sqrt{1 + (0.316)^2}}$$

$\Rightarrow \beta = 49 \times 10^{-6}$

$$Af(o) = \frac{10^5}{1 + 10^5(4.9 \times 10^{-6})} = 16.9 \times 10^3$$

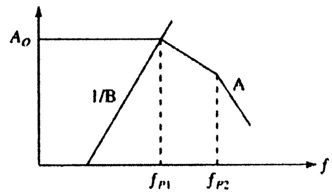
9.78



$$\beta = \frac{1/SC}{R + 1/SC} = \frac{1}{1 + SCR}$$

$$\beta(f) = \frac{1}{1 + j2\pi fCR}$$

$$A(j_f) = \frac{10^3}{\left(1 + \frac{j_f}{10^6}\right)\left(1 + \frac{j_f}{10^7}\right)}$$



From sketch, we need

$$A_O \frac{1}{2\pi f_{\phi 1} CR} = 1 = A\beta$$

$$\Rightarrow RC = \frac{A_0}{2\pi f_{\phi 1}} = \frac{10^3}{2\pi \times 10^6} = 159.2\ \mu s$$

At 1 MHz $Ang(\beta) = -90°$

$Ang(A) = -\tan^{-1}1 - \tan^{-1}0.1$

$= -45 - 5.7 = -50.7°$

$\therefore PM = 180 - (90 + 50.7) = 39.3°$

Gain margin exists at ω_{180}

then $\tan^{-1}\dfrac{f_1}{10^6} + \tan^{-1}\dfrac{f}{10^7} = 90°$

$\therefore f_{180} = \sqrt{10^6 \cdot 10^7}$ = geometric mean

= 3.16 MHz

$A\beta(f_{180}) = 20\log|A| - 20\log|1/\beta|$

$|A|$ has fallen 10 db, $|\beta|$ has risen 10 dB

thus $GM = 1\,(10) - (-10) = 20$ dB

9.79

For 90° PM:

$$\tan^{-1}\frac{f_1}{10^5} + \tan^{-1}\frac{f_1}{10^6} + \tan^{-1}\frac{f_1}{10^7} = 90°$$

From graph $f_1 = 3 \times 10^5$ Hz

thus $71.6 + 16.7 + 1.72 = 89.9°$ (close)

$$|A(f_1)| = \frac{10^5}{\sqrt{1 + 3^2} \cdot \sqrt{1 + 0.3^2} \cdot \sqrt{1 + 0.03^2}}$$

$|A\beta| = 1 \Rightarrow B = 33.0 \times 10^{-6}$

$$\therefore Af(0) = \frac{10^5}{1 + 10^5\beta} = 2.32 \times 10^4$$

For $PM = 45°$ $f_1 \approx 10^6$ Hz from graph

thus $84.3 + 45 + 5.7 = 135°$ (ok)

$$|A(f_2)| = \frac{10^5}{\sqrt{1 + 10^2} \cdot \sqrt{2} \cdot \sqrt{1 + 0.1^2}} = 7 \times 10^3$$

$|A\beta| = 1 \Rightarrow \beta = 1.43 \times 10^{-4}$

$$\therefore Af(o) = \frac{10^5}{1 + 10^5\beta} = 6.54 \times 10^3$$

9.80

$f_1 = 2\text{MHz}$

$A_o = 80\text{ dB} \equiv 10^4$

$\Rightarrow f_p = f_1/A = (2\times10^6)/10^4 = 200\text{Hz}$

9.81

$f_{P1} = 2\text{MHz}, \quad f_{P2} = 10\text{MHz}$

$A_o = 80\text{ dB} \equiv 10^4$

$$f_D = \frac{f_P}{A_o} = \frac{10\times10^6}{10^4} = 10^3\text{ Hz}$$

$$f_D' = 1/(C_x + C_c)2\pi R_x \rightarrow C \times \frac{2\times10^6}{10^3} = 2000C$$

9.82

$R_1 = R_2 = R$

$$C_2 = \frac{C_1}{10} = C$$

$$C_f >> C;\ g_m = \frac{100}{R}$$

$$\therefore \omega_1 = \frac{1}{C_1R_1} = \frac{1}{10\cdot C\cdot R}$$

$$\omega_2 = \frac{1}{C_2R_2} = \frac{1}{C\cdot R}$$

$$\omega'_{P1} = \frac{1}{g_mR_2\cdot C_fR_1} = \frac{1}{100\cdot C_f\cdot R}$$

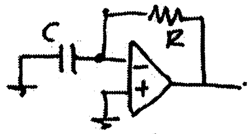
$$\omega'_{P2} = \frac{g_mC_f}{C_1C_2 + C_f(C_1+C_2)}$$

$$= \frac{g_mC_f}{C_1(C_2+C_f) + C_fC_2}$$

$$\text{for } C_f >> C \Rightarrow \omega'_{PS} = \frac{g_m}{C_1+C_2}$$

$$= \frac{100}{11\times CR} = \frac{9.1}{CR}$$

9.83



$A_o = 10^4$

poles at $10^5, 10^6, 10^7$ Hz

For $\beta = 1$, f_p must be kept $\times 10^4$ lower than lowest amplifier pole at 10^5 Hz

$$\Rightarrow f_p = \frac{10^5}{10^4} = 10\text{ Hz}$$

$$f_P = \frac{1}{2\pi CR} \quad \text{and } R = 1\text{M}\Omega$$

$$\Rightarrow C = \frac{1}{2\pi\, 10^6\,(10)} = 15.9\text{nF}$$

9.84

$A_O = 80\text{ dB} \equiv 10^4$

$$f_{P1} = 10^5 = \frac{1}{2\pi C_1R_1} \Rightarrow R_1 = \frac{1}{2\pi f_{P1}C_1}$$

$$\Rightarrow R_1 = \frac{1}{2\pi\cdot10^5\cdot(150\times10^{-12})} = 10.62\text{ k}\Omega$$

$$f_{P2} = 10^6 = \frac{1}{2\pi C_2R_2}$$

$$\Rightarrow R_2 = \frac{1}{2\pi\,10^6\,(5\times10^{-12})} = 31.85\text{ k}\Omega$$

Assuming $f_{P2} >> f_{P3}$

$$f_{P1} = \frac{f_{P3}}{10^4} = \frac{2\times10^6}{10^4} = 200\text{ Hz}$$

$$\text{and } f_{P1} = \frac{1}{2\pi g_mR_1R_2C_f}$$

$$\Rightarrow C_f = \frac{1}{2\pi g_mR_1R_2f_{P1}}$$

$\therefore C_f$

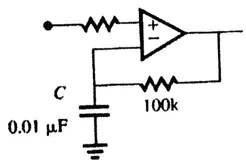
$$= \frac{1}{2\pi(40\times10^{-3})(10.62\times10^3)(31.85\times10^3)200}$$

$= 58.8\ P_f$

$$f_{P2} = \frac{1}{2\pi}\frac{g_mC_f}{C_1C_2 + C_f(C_1+C_2)}$$

$$= \frac{1}{2\pi}\frac{40\times10^{-3}(58.8\times10^{-12})}{(150\times5)10^{-24} + 58.8(155)10^{-24}} = 37.95\text{ MHz}$$

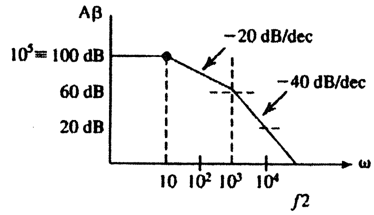
9.85



$$\beta(S) = \frac{1}{1 + SCR}$$

$$= \frac{1}{1 + 10^{-3}S}$$

(a) $A\beta(S) = \frac{10^5}{1 + S/10} \cdot \frac{1}{1 + S/10^3}$



(b) From plot $|A\beta| = 20$ dB at $10^4 = \omega$

Hence $|A\beta| = 1$ at 31.6 Krad/s $\left(\frac{1}{2}\text{dec}\right)$

At $\omega = 10^4$ the phase is $-180°$ decreasing at a rate of 45°/dec, at 31.6 K$\frac{\text{rad}}{\text{S}}$ $\frac{1}{2}$ (dec above $\omega = 10^4$) the phase margin is $-22.5°$.
The circuit will oscillate.

(c) $A_f(S) = \dfrac{\dfrac{10^5}{1 + S/10}}{1 + \dfrac{10^5}{1 + S/10} \cdot \dfrac{1}{1 + S/10^3}}$

$$= \frac{10^5(1 + S/10^3)}{(1 + S/10)(1 + S/10^3) + 10^5}$$

$$\therefore A_f(S) = \frac{1 + S/10^3}{1 + S/10^6 + S^2/10^9} = \frac{10^6S + 10^9}{S^2 + 10^3S + 10^9}$$

Zero at $S = -10^{-3}$ rad/s

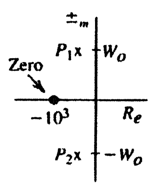
poles at $\dfrac{-10^3 \pm \sqrt{10^6 - 4 \times 10^9}}{2}$

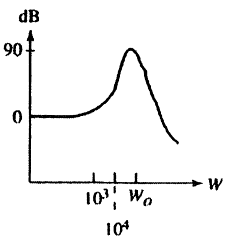
$$= \frac{-10^3 \pm j\,63.2 \times 10^3}{2}$$

$= -500 \pm j\,31.6 \times 10^3$ rad/s

$\omega_0 = 31.6$ Krad/s

$Q = 31.6$





10.1

$V_{ICM(max)} \le V_{DD} - |V_{tp}| - |V_{OV1}| - |V_{OV5}|$
$\le +2.5 - 0.7 - 0.3 - 0.3$
$\le +1.2V$

$V_{ICM(min)} \ge -V_{SS} + V_{OV3} + V_{tn} - |V_{tp}|$
$\ge -2.5 + 0.3(+0.7 - 0.7)$
$\ge -2.2V$

$-V_{SS} + V_{OV6} \le V_O \le V_{DD} - |V_{OV7}|$
$-2.5 + 0.3 \le V_O \le +2.5 - 0.3$
$-2.2V \le V_O \le +2.2V$

10.2

$V_A' = 25V/\mu m$, $|V_p'| = 20V/\mu m$, $L = 0.8\mu m$
Hence $V_A = 20V$ and $|V_p| = 16V$
For all devices $V_{OV} = 0.25V$

$A = A_1 A_2 = G_{m1}(r_{o2} \| r_{o4})\, G_{m2}(r_{o6} \| r_{o7})$

$[r_{op} \| r_{on}] = \left[\frac{V_A}{I} \times \frac{V_p}{I}\right] \times \frac{I}{V_A + V_p} = \left[\frac{V_A \| V_p}{I}\right]$

For A_2: $R_o = \frac{8.89}{I} \rightarrow \frac{8.89V}{0.4\,mA} = 22.2K\Omega$

To avoid systematic output dc offset

$$\frac{(W/L)_6}{(W/L)_4} = \frac{2(W/L)_7}{(W/L)_5}$$

Since Q_5, Q_6, Q_7 carry I and Q_4 only $I/2$
satisfy requirement by making Q_4 have $(W/L)/2$
Since $g_m = \sqrt{2(\mu C_{ox})(W/L)I} = 2K(V_{OV})$
$g_{m1} = 2I_1/V_{OV} = 0.4mA/0.25V = 1.6mA/V$
$g_{m6} = 2I_6/V_{OV} = 3.2mA/V$
$\therefore A = (1.6)(44.4)(3.2)(22.2) = 5047\ V/V$
For unity gain amplifier

$$A_F = \frac{A}{1 + A\beta} = \frac{5047}{1 + 5047\beta} = 1$$

Thus $(1 + A\beta) = 5047$
Then $R_{of} = R_o/(1 + A\beta)$
$= 22.2K/5047 \approx 4.4\Omega$

10.3

$A = A_1 A_2 = G_{m1}(r_{o2} \| r_{o4})\, G_{m2}(r_{o6} \| r_{o7})$

$$= \frac{2I_1}{V_{OV}} \cdot \frac{1}{2}\frac{V_A}{I_1} \cdot \frac{2I_2}{V_{OV}} \cdot \frac{1}{2}\frac{V_A}{I_2}$$

$$= \left[\frac{V_A}{V_{OV}}\right]^2 = 2500$$

where $V_A = 10V/\mu m \times 1\mu m = 10V$
Hence $V_{OV} = V_A/50 = 10/50 = 0.2V$

10.4

$CMRR = g_{m1}(r_{o2} \| r_{o4}) \times 2 g_{m3} R_{SS}$ with

$R_{SS} = r_{o5}$

$$80dB = \frac{2I_1}{V_{OV}}\left(\frac{1}{2} \times \frac{V_A}{I_1}\right) \times 2 \times \frac{2I_3}{V_{OV}}\left(\frac{V_A}{I_5}\right)$$

$$= \left(\frac{V_A}{V_{OV}}\right)^2 \times 2 \text{ (Note } I_5 = 2I_3)$$

$$10000 = 2\left(\frac{V_A}{V_{OV}}\right)^2.$$

$$V_A = V_{ov}\sqrt{\frac{10,000}{2}}$$

$$V_A = .15 \times \frac{100}{\sqrt{2}}$$

$$V_A = V_A' \times L \Rightarrow 20\,\frac{V}{\mu m} \times L$$

$$L = \frac{0.15 \times 100}{20 \times \sqrt{2}} \Rightarrow L = \frac{15}{20\sqrt{2}} = 0.53\ \mu m$$

10.5

$G_{m1} = 0.3$ mA/V $G_{m2} = 0.6$ mA/V $C_2 = 1$p

$r_{02} = r_{04} = 222$ kΩ $r_{06} = r_{07} = 111$ kΩ

(a) $f_{p2} = \frac{G_{m2}}{2\pi C_2} = \frac{0.6 \times 10^{-3}}{2\pi 10^{-12}} = 95.5$ MHz

(b) $R = \frac{1}{G_{m2}} = \frac{1}{0.6} = 1.66$ kΩ

(c) For $pm = 80^\circ$: $\tan^{-1}\frac{f_t}{f_{P2}} = 10^\circ$

$f_t = f_{p2} \tan 10^\circ$
$= 95.5 \times 0.176 = 16.84$ MHz

$C_C = \frac{G_{m1}}{2\pi f_t} = \frac{0.3 \times 10^{-3}}{2\pi 16.84 \times 10^6} \Rightarrow 2.83$ pF

$A = A_1 A_2 = G_{m1}(r_{o2} \| r_{o4}) G_{m2}(r_{o6} \| r_{o7})$
$= 0.3 \times 111 \times 0.6 \times 55.5$
$= 1109 = 60.8$ dB

Donamant pole $f_{p1} = f_t / |A|$

Thus fp1 is approx 3 decades below ft i.e. at 16.84 KHz providing uniform 20 dB/dec slope drawn to ft.

(d) $f_t = \frac{G_{m1}}{2\pi C_C}$ ∴ to double f_t, halve C_C

$C_{c(new)} = 1.4$pF

$\tan^{-1}\frac{f_t}{f_P} = \tan^{-1}\frac{33.7}{95.5} = 19.4^\circ$

The zero must be moved to reduce the 19.1 − 10 = 9.4°

$\tan^{-1}\frac{f_t}{f_z} = 9.4^\circ \rightarrow \frac{f_t}{f_z} = 0.16$

$\Rightarrow f_z = 0.16\, f_t = 0.16 \times 33.7$
$= 5.6$ MHz

$f_z = \frac{1}{2\pi C_C\left[R - \frac{1}{G_m}\right]} \rightarrow \left[R - \frac{1}{G_m}\right] = \frac{1}{2\pi f_z C_C}$

Hence $[R - 1/G_m] = \frac{10^{12} \cdot 10^{-6}}{2\pi\, 5.6 \times 1.4}$

$R = 1.67 + 20.3 = 21.97$ kΩ

10.6

Two-stage amp with $C_2 = 1$pF

$f_t = 100$MHZ, PM = 75°

For PM = 75°: $\tan^{-1}\frac{f_t}{f_{P2}} = 15^\circ$

∴ $f_{P2} = f_t \tan 15^\circ = 3.73 f_t = 373$MHZ

$f_{P2} = \frac{G_{m6}}{2\pi C_2} = \frac{1}{2\pi R_2 10^{-12}} = 373$MHZ

$\Rightarrow R_2 = \frac{10^{12}}{2\pi(373 \times 10^6)} = 426\Omega$

$\Rightarrow G_{m6} = \frac{1}{R_2} = 2.35 \times 10^{-3}$ mA/V

To move zero to infinity $R = \frac{1}{G_{m6}} = 426\Omega$

$SR = \frac{I}{C_c} = \frac{200\mu A}{C_c}$

$SR = 2\pi f_t V_{ov1} = 2\pi 10^8 \times 0.2 = 1.26 \times 10^8$ V/S

$\Rightarrow C_c = \frac{200 \times 10^{-6}}{1.26 \times 10^8} \Rightarrow 1.6$pF

10.7

$G_{m1} = 1$ mA/V, $G_{m2} = 2$ mA/V,

R = 500 Ω

$f_t \approx \frac{G_{m1}}{2\pi C_c} \Rightarrow C_c = \frac{G_{m2}}{2\pi f_t}$

$= \frac{1 \times 10^{-3}}{2\pi 100 \times 10^6} \Rightarrow 1.59$pF

For $\frac{1}{G_{m2}} - R = \frac{10^3}{2} - 500 = 0$

Zero has been moved to ∞

For $PM = 60$: $f_t = f_{p2} \tan(90 - 60)^\circ$

$\Rightarrow f_{P2} = f_t / \tan 30^\circ = 173$ MHz

$C_2 \approx \frac{G_{m2}}{2\pi f_{P2}} = \frac{2 \times 10^{-3}}{2\pi(173 \times 10^6)} = 1.84$ pF

10.8

$SR = 60$ V/μs

$f_t = 50$ MHz

(a) $SR = 2\pi f_t V_{ov1}$

$\Rightarrow V_{ov1} = SR / 2\pi f_t$

$= \frac{60 \times 10^6}{2\pi(50 \times 10^6)} \approx 0.2$ V

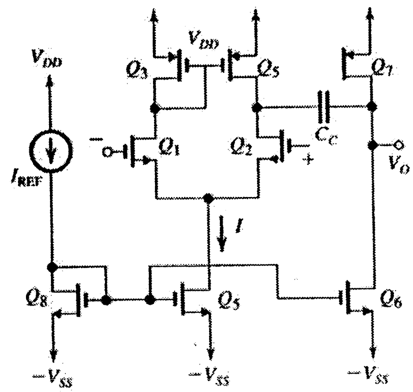
(b) $SR = \frac{I}{C_C} \Rightarrow C_C = \frac{100\ \mu A}{60 \times 10^6} = 1.67$ pF

(c) $I = \frac{1}{2}\mu\, C_{ox}\, (W/L)[V_{ov}]^2$

$\Rightarrow \left(\frac{W}{L}\right) = \frac{2I}{50[0.2]^2} = \frac{100}{1}$

10.9

Invert circuit leaving $V_{DD} \times V_{SS}$ and neverse all arrows on FETS.



$$V_{ICM(max)} = V_{DD} - |V_{OV9}| + V_{tn}$$
$$= +1.65 - 0.2 + 0.5$$
$$= +1.75V$$

$$V_{ICM(min)} = -V_{SS} + V_{OV11} + V_{OV1} + V_T$$
$$= -1.65 + 0.2 + 0.2 + 0.5$$
$$= -0.75V$$

$$-V_{SS} + 2V_{OV} + V_t \leq V_O \leq +V_{DD} - 2V_{OV}$$
$$-1.65 + 0.4 + 0.5 \leq V_O \leq +1.65 - 0.4$$
$$-0.75V \leq V_O \leq +1.25V$$

10.10

a)

$$PSRR = g_{m1}(r_{o2} \| r_{o4})g_{m6}r_{o6}$$

$$= \frac{2 \times I/2}{V_{ov}}\left(\frac{1}{2}\frac{V_A}{(I/2)}\right)\frac{2I}{V_{ov}}\frac{V_A}{I} = 2\left(\frac{V_A}{V_{ov}}\right)^2$$

b) $|V_{ov}| = 0.2\text{V}$, $PSRR = 80\text{dB}$,

$|V'_A| = 20V/\mu\text{m}$

$$PSRR = 2\left|\frac{V_A}{V_{ov}}\right|^2 \Rightarrow 80\text{ dB} = 10000$$

$$= 2\left|\frac{20 \times L}{0.2}\right|^2 \Rightarrow \frac{100}{\sqrt{2}} = 100L \Rightarrow L = 0.7\mu\text{m}$$

10.11

V_{BIAS1}: V_S can rise to $V_{DD} + V_T - V_{OV}$
V_{D3} can rise to $V_{S3} - V_{OV}$

$$\therefore V_{BIAS1} = V_{DD} - V_{OV10} - V_{OV4} + V_T$$
$$= 1.65 - 0.2 - 0.2 + 0.5$$
$$= 1.75V$$

$$V_{BIAS2} = V_{DD} - V_{OV10}$$
$$= +1.65 - 0.2 = +1.45V$$

$$V_{BIAS3} = -V_{SS} + V_{OV11}$$
$$= -1.65 + 0.2 = -1.45V$$

10.12

$I = 125\mu A$, $I_B = 150\mu A$, $V_t = 0.2v$

For Q_9, Q_{10}: $I_B = 150\mu A$

$$I = \frac{1}{2}(\mu C_{ox})(W/L)(V_{ov})^2$$
$$150 = \frac{1}{2}\ 90\ (W/L)(0.2)^2$$
$$\Rightarrow (W/L)_{9,10} = (83.33/1)$$

For Q_1, Q_2: $I = 125\mu A/2$

$$\frac{1}{2}.125 = \frac{1}{2}\ 250\ (W/L)(0.2)^2$$
$$\Rightarrow (W/L)_{1,2} = (12.5/1)$$

For Q_{11}: $I = 125\mu A$

$$125 = \frac{1}{2}\ 250\ (W/L)(0.2)^2$$
$$\Rightarrow (W/L)_{11} = (25/1)$$

For $Q_3\ Q_4$: $I = 125\mu A/2$

$$\frac{1}{2}125 = \frac{1}{2}\ 60\ (W/L)(0.2)^2$$
$$\Rightarrow (W/L)_{3,4} = (52/1)$$

For $Q5, Q_6, Q_7, Q_8$: $I = 125\mu A/2$

$$\frac{1}{2}125 = \frac{1}{2}\ 250(W/L)(0.2)^2$$
$$\Rightarrow (W/L)_{5,6,7,8} = (12.5/1)$$

10.13

$G_m = \frac{I}{V_{ov}}$ $r_o = \frac{V_A}{I}$

$R_o = R_{o4} \| R_{o6}$

$R_{o4} = g_{m4} r_{o4} (r_{o2} \| r_{o10})$

$R_{o6} = g_{m6} (r_{o6} \| r_{o8})$

$A = G_{m1} R_o$

$Q_{10}: I = I_B \Rightarrow g_{m10} = 0.75\ mA/V$

$r_{o10} = 66.6\ K\Omega$

$Q_x: I = 125\mu A \Rightarrow g_x = 0.31\ mA/V$

$r_{ox} = 160\ K\Omega$

$\therefore R_{o4} = 0.31 \times 160 \times 47K = 2359K$

$R_{o6} = 0.31 \times 160 \times 160K = 8000K$

$R_o = 2.35 \| 8.0 = 1.8\ M\Omega$

$A = g_{m1} R_o = 0.31 \times 1800 = 558$

$\frac{V_o}{V_i} = 1 + \frac{(1/sC)}{(1/S9C)} = 1 + \frac{9sC}{sC} = 10$

$\therefore \beta = 1/10 = 0.1$

$A_F = \frac{A}{1+A\beta} = \frac{558}{1+558\times 0.1} = 9.8$

$R_{oF} = \frac{R_o}{1+A\beta} = \frac{1.8}{56.8} = 31.69\ K\Omega$

10.14

$SR = \frac{I}{C_L} \Rightarrow I = SR \times C_L$

$= 10 \times 10^6 \times 10 \times 10^{-12}$

$= 100\ \mu A$

(1) $I_B = 1.2I = 120\ \mu A$

(2) $f_p = \frac{1}{2\pi C_L R_o} \cdot ft = \frac{Gm}{2\pi C_L}$

$G_m = \frac{2I/2}{V_{ov}} = \frac{100\ \mu A}{0.2\ V} = 0.5\ mA/V$

$f_t = \frac{0.5 \times 10^{-3}}{2\pi 10 \times 10^{-12}} = 7.96\ MHz$

(4) $R_{of} \simeq \frac{1}{G_m} = \frac{1000}{0.5} = 2\ k\Omega$

$A_V = f_t/f_p = G_m R_o$

But $R_{of} = \frac{R_o}{1+G_m R_o} \Rightarrow R_o = 2\ m\Omega$

$\therefore A_V = 0.5 \times 10^{-3} \times 2 \times 10^6 = 1000$

$f_p = f_t/1000 = 7.96\ KHz$

(5) $\theta = -\tan^{-1}\frac{f_t}{f_p} - 2\left(\tan^{-1}\frac{f_t}{f_p^2}\right)$

$Pm@\ f_p^2 = 90^\circ - \tan^{-1}\frac{f_t}{f_p^2}$

$= 90^\circ - \tan^{-1}\left[\frac{7.96\ MHz}{25\ MHz}\right]$

$= 90^\circ - 17.7^\circ = 72.3^\circ$

(4) For $pm = 75^\circ$: $\tan^{-1}\frac{f_t}{f_p^2} = 15^\circ$

Thus $f_t^* = f_p^2 \tan 15^\circ$

$= 25\ MHz \times 0.27 = 6.7\ MHz$

(5) $\frac{ft^*}{ft} = \frac{C_L}{C_L^*} \Rightarrow \frac{6.7}{7.96} = \frac{10\ pF}{C_L^*}$

$\Rightarrow C_L^* = C_L \frac{7.96}{6.7} = C_L \times 1.19$

$\therefore$ Increase C_L by 19%

(6) $SR^* = \frac{I}{C_L^*} \Rightarrow \frac{SR}{1.19} = 8.4\ V/\mu s$

10.15

$A = 80\ dB$ $f_t = 10\ MHz$ $C_L = 10\ pF$

$I_B = I$ All same $|V_{ov}|$, $L = 0\mu m$

$|V_A| = 20V$

$g_m = \frac{2I}{V_{ov}}$ and $f_t = \frac{g_m}{2\pi C_L}$

$A = g_{m1}[g_{m4} r_{04}(r_{02} \| r10)] \| [g_{m6}\ r_{06}\ r_{08}]$

Consider $Q_1: I_1 = \frac{1}{2} kn[W/L]_1 [V_{ov}]^2$

$= \frac{1}{2} 200 [W/L]_1 [V_{ov}]^2$

$Q_1\ Q_2\ Q_5\ Q_6\ Q_7\ Q_8$ are same

$g_{m1} = \frac{2I_1}{V_{ov}}$ and $r_{o1} = \frac{V_A}{I_1}$

Consider $Q_3, Q_4: I_3 = \frac{1}{2} kp[W/L]_3 [V_{ov}]^2$

$I_1 = \frac{1}{2}\frac{200}{2.5}[W/L]_3 [V_{ov}]^2$

$\Rightarrow [W/L]_3 = 2.5[W/L]_1$

$g_{m3,4} = g_{m1}$ and $r_{03,4} = r_{01}$

Consider $Q_9\,Q_{10}: I_{10} = \frac{1}{2}kp[W/L]_{10}[V_{ov}]^2$

$$2I_1 = \frac{1}{2}\frac{200}{2.5}[W/L]_{10}[V_{ov}]^2$$

$\Rightarrow [W/L]_{10} = 5[W/L]_1$

$g_{m10} = 2g_{m1}$ and $r_{09,10} = r_{01/2}$

Consider $Q_{11}: I_{11} = \frac{1}{2}kn[W/L]_{11}[V_{ov}]^2$

$$2I_1 = \frac{1}{2}200[W/L]_{11}[V_{ov}]^2$$

$\Rightarrow [W/L]_{11} = 2[W/L]_1$

$g_{m11} = 2g_{m1}$ and $r_{011} = r_{01/2}$

Thus

$$A = g_{m1}\left[g_{m1}r_{01}\left(r_{01} \| \frac{r_{01}}{2}\right)\right] \| \; [g_{m1}\, r_{01}\, r_{01}]$$

$$= g_{m1}[g_{m1}r_{01}]\left(r_{01} \| \frac{r_{01}}{2} \| \; r_{01}\right)$$

$$10^4 = \frac{1}{4}g_{m1}\, g_{m1}\, r_{01}\, r_{01}$$

$\Rightarrow g_{m1}\, r_{01} = 200$

Non $g_{m1}\, r_{01} = \frac{2I}{V_{ov}} \cdot \frac{V_A}{I}$

$\Rightarrow V_{ov} = 2VA/200 = 2(20)/200 = 0.2\text{V}$

Hence $g_{m1} = \frac{1}{2\pi f_t C_L} = 0.628 \text{ mA/V}$

$\rightarrow r_{01} = 200/g_{m1} = 318 \text{ k}\Omega$

$g_m = \frac{2I}{V_{ov}} \Rightarrow I = \frac{g_m V_{ov}}{2}$

$\rightarrow I_1 = \frac{g_{m1}V_{ov}}{2} = \frac{0.678 \text{ mA/V} \times 6.7\text{V}}{2}$

$= 62.8\ \mu\text{A}$

$SR = 2\pi f_t V_{ov} = 2\pi 10 \times 10^6 \times 0.2 = 12.5\text{V}/\mu\text{s}$

$Q_1\,Q_2\,Q_5\,Q_6\,Q_7\,Q_8$:

$I = \frac{1}{2}kn[W/L][V_{ov}]^2$

$62.8 = \frac{1}{2}200[W/L][V_{ov}]^2$

$\Rightarrow [W/L]_1 = 15.7$

For Q_3,Q_4: $I = \frac{1}{2}\frac{200}{2.5}[W/L][V_{ov}]^2$

$$62.8 = \frac{1}{2}\frac{200}{2.5}[W/L][0.2]^2$$

$\Rightarrow [W/L]_3 = 2.5[W/L]_1 = 39.25$

For Q_9,Q_{10}:

$[W/L]_9 = 5[W/L]_1 = 78.5$

For Q_{11}: $[W/L]_{11} = 2[W/L]_1 = 31.4$

For $L = 1\ \mu\text{m}$: $W_x = [W/L]_x\ \mu\text{m}$

$\therefore$ width for $Q_1,Q_2,Q_5,Q_6,Q_7,Q_8 = 15.7\ \mu\text{m}$

for Q_3,Q_4 $= 39.25\ \mu\text{m}$

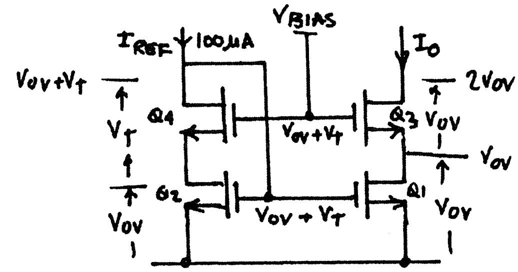
for Q_9,Q_{10} $= 78.5\ \mu\text{m}$

for Q_{11} $= 31.4\ \mu\text{m}$

10.16

Simply invert circuit relative to V_{DD}, V_{ss} and reverse all arrows on FETS

10.17



All same $k(W/L)$ $\therefore I_o \approx I_{REF}$

All same $r_o = V_A/I = 10V/100\mu A = 100k\Omega$

$I = \frac{1}{2}k[W/L][V_{ov}]^2$

$V_{D3} = V_o$: $V_o(\min) = 2V_{ov}$

R_o (looking into Q3D and assuming I_o current source is ideal. $r_o' = \infty$)

$= r_{o3}(1 + g_{m3}r_{o1}) \approx g_m r_o^2$

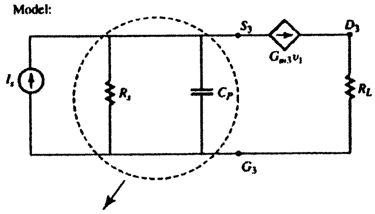
$g_m = \frac{2I}{V_{ov}} = \frac{2 \times 100}{0.2} \Rightarrow 1\text{mA/V}$

Then $R_o \approx g_m r_o^2 = 1 \times 10^5 \times 10^5$

$\approx 10^4\ \text{M}\Omega$

10.18

Model:



$$[Z] \Rightarrow R_S \parallel \frac{1}{C_pS} = \left[\frac{\frac{1}{R_SC_pS}}{R_S + \frac{1}{C_pS}}\right]\frac{C_pS}{C_pS} = \frac{R_s}{R_SC_pS + 1}$$

Summing currents at node S_3:

$$-I_S + g_{m3}V_1 + V_1\frac{(1 + R_SC_PS)}{R_S} = 0$$

$$\frac{V_1}{I_S} = \frac{1}{g_{m3} + \frac{(1 + R_SC_PS)}{R_S}} = \frac{1}{\left(g_{m3} + \frac{1}{R_S}\right) + C_PS}$$

$$W_{3\,dB} = \frac{\left(g_{m3} + \frac{1}{R_S}\right)}{C_P} \text{ so}$$

$$f_{3\,dB} = \frac{\left(g_{m3} + \frac{1}{R_S}\right)}{2\pi C_P} \approx \frac{g_{m3}}{2\pi C_P}$$

$$p\mu = 180 - \phi_{total} = 90° - \tan^{-1}\left(\frac{f_t}{f_{p2}}\right)$$

For $Pm = 75°$: $\frac{f_t}{f_{p2}} = \tan 15° = 0.27$

$$\frac{f_t}{f_{p2}} = \frac{C_P}{C_L} = 0.27$$

$$\therefore C_P = 0.27C_L$$

10.19

$$I_3 = I_1\sqrt{\frac{I_{S3}\,I_{S4}}{I_{S1}\,I_{S2}}}$$

$$= 154\sqrt{\frac{10^{-14}\cdot 10^{-14}}{3\times 10^{-14}\cdot 6\times 10^{-14}}}$$

$$= 36.3\mu A$$

10.20

$$I_{E_{TOT}} = 0.73 \text{ mA}$$

$$I_{EA} = 0.25(0.73) = 0.1825 \text{ mA}$$

$$I_{EB} = 0.75(0.73) = 0.5475 \text{ mA}$$

$$V_{EB_A} = V_T\ln\frac{0.1835\times 10^{-3}}{0.25\times 10^{-14}} = 0.625\text{V}$$

$$g_{mA} = \frac{I_C}{V_T} = \frac{I_E}{V_T} = 7.3 \text{ mA/V}$$

$$r_{eA} = \frac{\alpha}{g_{mA}} = 134.3\ \Omega$$

$$r_{\pi A} = (\beta + 1)r_{eA} = 6.85\text{ k}\Omega$$

$$r_{oA} = \frac{V_A}{I_{CA}} = 274\text{ k}\Omega$$

$$V_{EB_B} = V_{EB_A} = 0.625\text{V}$$

$$g_{mB} = \frac{0.5475}{25} = 21.9\text{ mA/V}$$

$$r_{e_B} = \frac{\alpha}{g_{mB}} = 44.7\ \Omega$$

$$r_{\pi B} = (\beta + 1)r_{e_B} = 2.28\text{ k}\Omega$$

$$r_{oB} = 91.3\text{ k}\Omega$$

10.21

Let $V_{BE} = 0$

For breakdown $V_{ID} = V_{Bi} - V_{B2}$

$> V_{BE1} + V_{BE2} + 7 + 50$

or $V_{ID} \geq 58.4$ V

10.22

$$V_{SG1} + V_{GS2} = V_{SG4} + V_{GS3}$$

since V_t's are equal

$$\sqrt{\frac{I_1}{K_1}} + \sqrt{\frac{I_1}{K_2}} = \sqrt{\frac{I_3}{K_4}} + \sqrt{\frac{I_3}{K_3}}$$

$$\sqrt{I_1}\left[\frac{1}{\sqrt{K_1}} + \frac{1}{\sqrt{K_2}}\right] = \sqrt{I_3}\left[\frac{1}{\sqrt{K_4}} + \frac{1}{\sqrt{K_3}}\right]$$

or

$$\sqrt{\frac{I_1}{I_3}} = \frac{\sqrt{\frac{1}{K_3}} + \frac{1}{\sqrt{K_4}}}{\sqrt{\frac{1}{K_1}} + \sqrt{\frac{1}{K_2}}}$$

$K_1 = K_2$, $K_3 = K_4 = 16\,K_1$

$$\sqrt{I_1} = \sqrt{I_3}\sqrt{\frac{K_2}{K_4}}$$

$$\text{or } I_1 = I_3\frac{K_2}{K_4}$$

$$= \frac{I_3}{16} = \underline{100\mu A}$$

10.23

As $V_{BE} = 0.7$

$$I_{ref} = \frac{5 - 14 - (-5)}{R_s}$$

$= 220.5\ \mu A$

At this current level

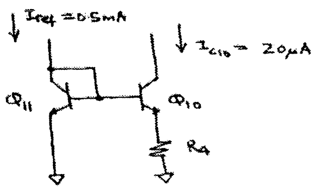
$$V_{BE} = V_T \ln \frac{220.6 \times 10^{-3}}{10^{44}} = 595\ mV$$

$$\Rightarrow I_{ref} = \frac{10 - 2(0.595)}{39\ k} = 226\ \mu A$$

For $I_{ref} = 0.75$ mA $V_{BE} = 0.625$ V

$$R_5 = \frac{10 - 2(0.625)}{0.73 \times 10^{-3}} = 12\ k\Omega$$

10.24



$$I_{C10} R_4 = V_{BE11} - V_{BE10}$$

$$= V_T \ln \frac{I_{ref}}{I_{C10}}$$

$$\therefore R_4 = \frac{25 \times 10^{-3}}{20 \times 10^{-6}} \ln \frac{0.5 \times 10^{-3}}{20 \times 10^{-6}} = 4.02\ k\Omega$$

$$V_{BE11} = V_T \ln \frac{0.5 \times 10^{-3}}{10^{-14}} = 616\ mV$$

$$V_{BE10} = V_T \ln \frac{20 \times 10^{-6}}{10^{-14}} = 535\ mV$$

10.25

Assume $\beta_p >> 1$

$$I_{C10} = \frac{2I}{1 + 2/\beta_p} + \frac{2I}{\beta_p}$$

$$\simeq 2I\left(1 - \frac{2}{\beta_p} + \frac{1}{\beta_p}\right)$$

$$= 2I(1 - 1/\beta_p)$$

$$\Rightarrow I \simeq \frac{I_{C10}}{2}\left(1 + \frac{1}{S_p}\right)$$

Thus $\frac{1}{\beta_p} = 0.1 \Rightarrow \beta_p = 10$

Without the above assumption and using the exact relationship $\beta_p = 7.79$.

10.26

In this case

$$\frac{4I}{1 + 2/\beta_p} + \frac{2I}{\beta_p} = I_{C10}$$

For $\beta_p >> 1$

$I_{C10} \simeq 4I$ or $I = 4.75\ \mu A$

To correct we need $I_{C10} = 38\ \mu A$

$$\Rightarrow R_4 = \frac{V_T}{I_{C10}} \ln \frac{0.73\ mA}{I_{C10}} = 1.94\ k\Omega$$

10.27

At $I = 9.5\ \mu A$

$V_{BE5} = V_{BE6} = 517$ mV

and $V_{B6} = V_{BE6} + IR_2$

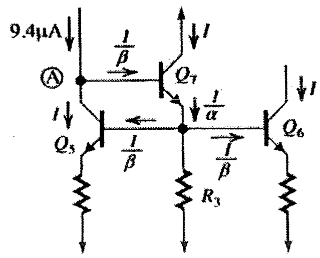
$= 526.5$ mV

If R_2 is shorted $V_{BE6} = V_{BG} = 526.5$ mV

and $I_{C6} = I_S\ e^{V_{BE6}/VT}$

$= 14\ \mu A$

10.28



ΣI @A $I = I + I/\beta = 9.4\ \mu A$

$$\Rightarrow I = \frac{9.4}{I + r\beta} = 9.353\ \mu A$$

$$I_{R3} = \frac{I}{\alpha} = \frac{2I}{\beta} = 9.307\ \mu A$$

$$V_{B5} = I_{R3} R_3 = V_{BE5} + \frac{IR}{\alpha}$$

$$V_{BE5} = V_T \ln \frac{9.353\ \mu}{10^{-14}} = 516.4\ mV$$

Thus $V_{B5} = 525.8$ mV

and $R_3 = \frac{V_{B5}}{I_{R3}} = 56.5\ k\Omega$

10.29

Assume equal collector current

$I_{C1} = I_{C2} = 9.5\ \mu A$

$$I_{os} = I_{B1} - I_{B2} = \frac{I_{C1}}{\beta_1} - \frac{I_{C2}}{\beta_2}$$

$= 15.7$ mA

$$I_B = \frac{1}{2}(I_{B1} - I_{B2})$$

$= 55.4$ mA

10.30

$I_B = 40\,nA$, $I_{os} = 4\,nA$

Thus, base currents are

$I_{B1} = (I_B \pm \frac{I_{os}}{2})$

$= \frac{9.5}{\beta_N}\ \mu A$

$\hat{\beta}_N = \frac{9.5\,\mu A}{38\,nA} = \underline{\underline{250}}$

$\check{\beta}_N = \frac{9.5\,\mu A}{42\,nA} = \underline{\underline{226}}$

$\Rightarrow \Delta\beta_N = 24$

$\bar{\beta}_N = \frac{\hat{\beta}_N + \check{\beta}_N}{2} = 238$

10.31

$I_{C1} + I_{C2} = 19\,\mu A$

Mirror forces $I_{C2} = 0.9\, I_{C1}$

Thus $I_{C1} = \frac{19}{1.9}\ \mu A = 10\,\mu A$

and $I_{C2} = 9\,\mu A$

$V_{OS} = \Delta V_{BE}$

$= V_{BE1} - V_{BE2}$

$= V_T \ln \frac{10}{9} = \underline{\underline{2.63\ mV}}$

10.32

At $I_{c17} = 550\ \mu A$, $V_{BE17} = 618\ mV$

$I_{B17} = \frac{550}{200} = 2.75\ \mu A$

$\Rightarrow I_{C16} = 9.5\ \mu A = I_{B17} + \frac{I_{B17}R_8 + V_{BE17}}{R_9}$

or $R_9 = 99.7\ k\Omega$

10.33

Neglecting base currents

$I_{C18} = I_{C19} = \frac{180}{2} = 90\ \mu A$

$V_{BE18} = V_T \ln \frac{90 \times 10^{-6}}{10^{-14}} = 573\ mV$

Thus $R_{10} = \frac{V_{BE18}}{I_{C18}} = 6.37\ k\Omega$

$I_{C14} = 3 \times 10^{-14}\, e^{573/25} = 270\ \mu A = I_{C20}$

10.34

$I_{V_{CC}} = I_{C12} + I_{C13A} + I_{C13B} + I_{C14} + I_{C9} + I_{C8} + I_{C7} + I_{C16}$

$= (730 + 180 + 550 + 154 + 19 + 19 + 10.5 + 162)\ \mu A$

$= 1.68\ mA$

$P_{Diss} = P_Q = I_{V_{CC}}(V_{CC} + V_{EE})$

$= 1.68(15 + 15)\ mW$

$= 50.4\ mW$

10.35

Series connection of devices assures the same bias currents.

$R_{id} = (\beta + 1)(6r_e)$

$r_e = \frac{V_T}{9.5\mu A} = 2.63\ k\Omega$

$R_{id} = 3.17\ M\Omega$

$r_e = \frac{v_{id}}{6r_e}, i_o = 2i_e$

$\Rightarrow G_{m1} = \frac{i_o}{v_{id}} = \frac{2}{6r_e} = \frac{1}{3r_e}$

$= 127\ \mu A/V$

$R_{o4} = r_o(1 + gm(R_E \| r_\pi))$

$g_m = 1/r_e$

$R_E = 2r_e = 5.36\ k\Omega$

$r_\pi = (\beta_p + 1)r_e = 134\ k\Omega$

Thus$R_{o4} = 15.4\ M\Omega$

$R_{o6} = 18.2\ M\Omega$ (from text)

$R_{o1} = R_{o4} \| R_{o6} = 8.34\ M\Omega$

$G_{m1}R_{o1} = 127 \times 8.34 = 1059\ V/V$

See gain decreases due to negative feedback

10.36

$R_o = r_{o6}(1 + g_{m6}(R_2 \| r_{\pi 6}))$

need to double the second factor

Since $r_{\pi 6} >> R_2$

$R_{o6} \simeq r_{o6}(1 + g_{m6}R_2)$

Thus

$1 + g_{m6}R'_2 = 2(1 + g_{m6}R_2)$

$g_{m6} = \frac{1}{2.63\ k\Omega}, R_2 = 1\ k\Omega$

$R'_2 = 4.63\ k\Omega$

10.37

$I_{c5} = I_{c6} = I_{c7}$

$\Rightarrow r_{e5} = r_{e6} = r_{e7} = 2.63\ \text{k}\Omega$

(a) $V_{b6} = (r_{e6} + R_2)i_e = 4.63\ \text{k}\Omega \times i_e$

(b) $R_B = (50\text{k} \| r_{\pi 5} \| r_{\pi 6})$

$= 45.1\ \text{k}\Omega$

$\Rightarrow i_{c7} = \dfrac{V_{b6}}{R_B} = 0.103 i_e$

(c) $i_{b7} = \dfrac{i_{e7}}{\beta + 1} = \dfrac{0.103}{201} i_e = 510\ \mu\text{A} \times i_e$

(d) $V_{b7} = V_{b6} + r_{e7}\, i_{e7}$

$= (4.63\ \text{k}\Omega + 2.63\ \text{k}\Omega \times 0.103)i_e$

$= 4.9\ \text{k}\Omega \times i_e$

(e) $R_{in} = \dfrac{V_{b7}}{i_e} = 4.9\ \text{k}\Omega$

10.38

$$\frac{\Delta I}{I} = \frac{\Delta R}{R + \Delta R + r_e}$$

$\Delta I \approx cami\ V_{DS} = \dfrac{V_{DS}}{2r_e}$

Thus

$$\frac{V_{DS}}{2r_{el}} = \frac{\Delta R}{R + \Delta R + r_e};\ r_e I = V_T$$

$$\frac{V_{DS}}{2r_e I} = \frac{\Delta R}{R}\left[\frac{1}{1 + \frac{r_e}{R} + \frac{\Delta R}{R}}\right] (*)$$

$$\frac{V_{DS}}{2V_T}\left(1 + \frac{r_e}{R}\right) = \frac{\Delta R}{R}\left(1 - \frac{V_{DS}}{2V_T}\right)$$

$$\frac{\Delta R}{R} = \frac{V_{DS}}{2V_T}\,\frac{1 + (r_e/R)}{1 - \frac{V_{DS}}{2V_T}}$$

(b) $V_{DS} = 5\ \text{mV},\ r_e = 2.63\text{k}\Omega,\ R = 1\ \text{k}\Omega$

$$\frac{\Delta R}{R} = \frac{5}{2(25)}\,\frac{1 + 2.63}{1 - \frac{5}{2(25)}} = 0.40$$

(c) R_2 completely shorted

$\Rightarrow \dfrac{\Delta R}{R} = -1$

From (*) $\dfrac{V_{DS}}{2V_T} = -1\dfrac{1}{r_e/R}$

$\Rightarrow V_{DS} = -19\ mV$ (or $19mV$)

10.39

Current in the collector of Q_3 remains unchauged at 9.5 μA

Thus $I_{E3} = I_{E4} = \dfrac{51}{50}\, 9.5\ \mu\text{A} = 9.69\ \mu\text{A}$

$I_{C4} = \dfrac{20}{21} \cdot I_{E4} = 9.23\mu\text{A}$

$\Delta I = 9.5 - 9.23 = 0.27\mu\text{A}$

$$V_{OS} = \frac{\Delta I}{G_{m1}} = 2r_e \Delta I$$

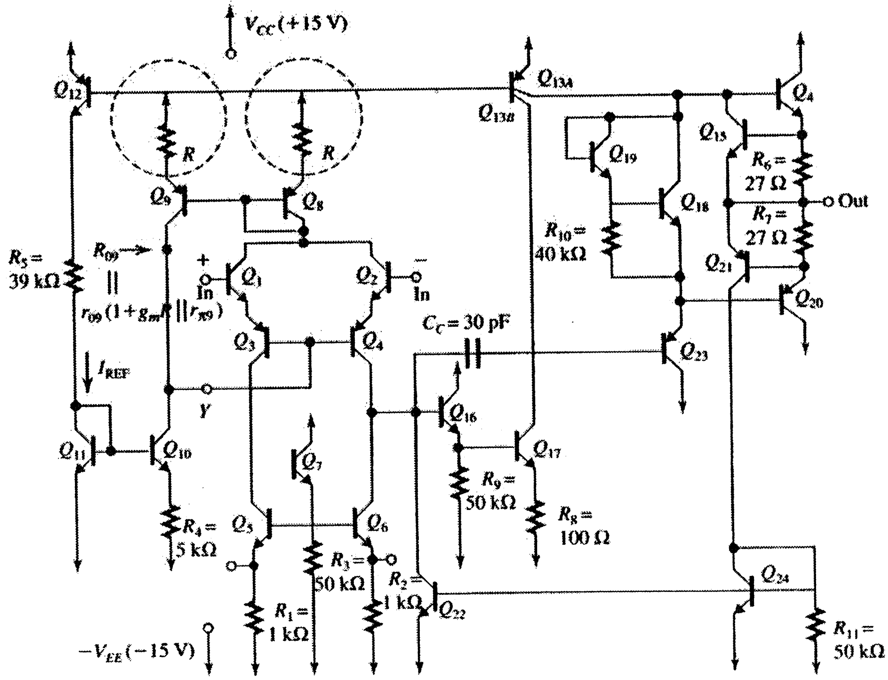
with

$r_e = 2.63\ \text{k}\Omega$:

$V_{OS} = 2 \times 2.63 \times 10^3 \times 0.27 \times 10^{-6} = 1.4\ \text{mV}$

10.40

Ignoring body effect



$R = 5\ \text{k}\Omega$ to make $R_{o9} = R_{o10}$ since I is the same for Q_9 & Q_{10}

$$R_O(\text{left of Y}) = R_{o9} \| R_{o10} = \frac{1}{2} r_O[1 + g_m(5\text{ K} \| r_\pi)]$$

10.41

Following the instruction of the problem, the resistance seen between the common base connection of Q_3 and Q_4 and ground is :

$R_f = (1 + A\beta)R_O = (1 + \beta_P)R_O$

Since the loop is broken, we have :

$$\frac{i}{\beta_P} + \frac{i}{\beta_P} \approx \frac{v_{icm}}{R_f} \Rightarrow$$

$$\frac{2i}{\beta_P} \approx \frac{v_{icm}}{(1+\beta_P)R_O} \Rightarrow i = \frac{\beta_P}{1+\beta_P}\frac{v_{icm}}{2R_O} \approx \frac{v_{icm}}{2R_O}$$

..ave

$$i_o = E_{mi} \Rightarrow G_{mcm} \equiv \frac{i_o}{v_{icm}} = \frac{E_{mi}}{v_{icm}} = \frac{E_m}{2R_o}$$

$$CMRR = \frac{G_{m1}}{G_{mcm}} = \frac{g_{m1}}{\frac{E_m}{2R_o}}$$

$$= \frac{2g_{m1}R_o}{E_m} = \frac{2g_{m1}(R_{09} \| R_{o10})}{E_m}$$

10.42

$R_{i2} = (\beta + 1)[r_{e16} + (R_{i17} \| R_9)]$

$r_{e16} = 1.54\ \text{k}\Omega$

$r_{e17} = 45.5\ \Omega$

$R_{i17} = 201(45.5 + 50) = 19.2\ \text{k}\Omega$

$\Rightarrow R_{i2} = 201[(1.54 + 19.2 \| 50)]\ \text{k}\Omega = 3.1\ \text{M}\Omega$

$$v_{b17} = \frac{(R_{i17} \| R_9)}{r_{e16} + (R_{i17} \| R_9)} v_{i2}$$

$= 0.9 v_{i2}$

$$i_{c17} = \frac{\alpha}{r_{e17} + R_8} 0.9 v_{i2}$$

$$\Rightarrow G_{m2} = \frac{\alpha(0.9)}{45.5 + 50} = 9.38\ \text{mA/V}$$

10.43

$R_{o17} = 787\ \text{k}\Omega$

$i_{c13B} = 550\ \mu\text{A}$

$g_{mB8} = 22\ \text{mA/V};$

$r_{\pi B8} = (\beta + 1)/g_m = 2.32\ \text{k}\Omega$

$$r_o = \frac{50}{550\mu\text{A}} = 90.9\ \text{k}\Omega$$

$R_{o13B} = r_o(1 + g_m(R_e \| r_\pi))$

$= 90.9[1 + 22(R_E \| 2.32)]$

$= 787$

$\Rightarrow R_E \| 2.32 = 0.348$

and $\frac{1}{R_E} = 2.44$ or $R_E = 0.410\ \text{k}\Omega$

$R_E = 410\ \Omega$

$$\text{Current}\frac{R_{E12}}{R_E} = \frac{550\ \mu\text{A}}{730\ \mu\text{A}} \Rightarrow R_{E12} = 309\ \Omega$$

$$\frac{R_{E13A}}{R_E} = \frac{550}{180} = 1.25\ \text{k}\Omega$$

10.44

$\hat{V}_o = V_{CC} - V_{CEsat18A} - V_{BE14}$

$= 4.2\ \text{V}$

$\hat{V}_o \approx -V_{EE} + V_{CEsat17} + V_{BE23} + V_{BE20}$

$= -5 + 0.2 + 0.6 + 0.6$

$= -3.6\ \text{V}$

10.45

With Q_{23} removed, current in Q_{17} increases to $730\mu\text{A}$. This changes G_{m2}

$$r_{e17} = \frac{V_T}{730\ \mu\text{A}} = 34.2\ \Omega$$

$$\Rightarrow G_{m2} \approx 0.923\frac{\alpha}{100 + 34.2} = 6.8\ \text{mA/V}$$

Because $r_{o17} >> r_{o13B}$ R_{o2} remains Virtually unchanged at $81\text{k}\Omega$

$R_{i3} = (\beta + 1)(R_L \| r_{o13A}) = 74\ \text{k}\Omega$

$$\Rightarrow A_v = -6.8(81)\frac{74}{74 + 81} = -263\ \text{V/V}$$

10.46

Ignore base current of Qs

$$180\ \mu\text{A} = I_{C15} + \frac{I}{\beta_{T1}},$$

where $I = I_{R6}$

$I_{C15} = I_S e^{V_{BE}/VT}$

where $V_{BE} = IR_6$

$$\text{Thus } I = \frac{V_T}{27}\ln\left[\frac{180\mu\text{A} - \frac{I}{201}}{I_S}\right]$$

$= 191{,}422\ \text{V/V}$

$= 105.6\text{dB}$

Output current is limited to ± 20 mA
(see problem 37 and 38)

$\Rightarrow |V_o| < 20\text{ mA}(200)$

$|V_o| < 4\text{ V}$

To obtain a seed solution, let $I = 0$ right hand side

$$\Rightarrow I = \frac{V_T}{27}\ln\frac{180\ \mu A}{10^{-14}} = 21.9\text{ mA}$$

Iterating $I = 21.0\text{ mA}$

10.47

Maximum output current of the 1st stage = 19μA

$\Rightarrow I_{C22} = 19\mu A \Rightarrow V_{BE22} = V_{BE24} = 534\text{ mV}$

$\Rightarrow I_{R11} = \frac{534}{50} = 10.7\mu A$

$\therefore I_{c21} = (19 + 10.7)\mu A = 29.7\ \mu A$

and $V_{BE21} = 545.3\text{ mV}$

$V_{BE21} = IR_7 \Rightarrow I = 20.2\text{ mA}$

A simple doubling of R_7

10.48

$$\frac{v_o}{v_i} = \frac{243{,}147}{0.97} = 250{,}667\ \text{V/V} = \underline{108\ \text{dB}}$$

$$\frac{R_L}{R_o + R_L} = 0.9 \Longrightarrow R_o = R_L\left(\frac{1}{0.97} - 1\right)$$

or $R_o = \underline{61.9\Omega}$

$$\left.\frac{v_o}{v_i}\right|_{R_L = 200} = 250{,}667 \cdot \frac{200}{200 + 61.9}$$

10.49

80° PM says that 2nd pole introduces 10° of phase shift at 1MHz

i.e. $\tan^{-1}\frac{f_t}{f_{P2}} = 10°$

or $f_{P2} = \underline{5.67\text{ MHz}}$

10.50

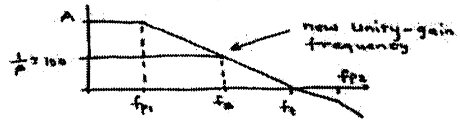
Each pole adds 5° of phase shift

$\tan^{-1}\frac{10^6}{f_{2,3}} = 5°$

$\Rightarrow f_{2,3} = \underline{11.4\text{ MHz}}$

10.51

Consider Bode plot



85° of closed-loop phase margin

$\Rightarrow \tan^{-1}\frac{f_B}{f_{P2}} = 5°$

or $f_B = \underline{437\text{ kHz}}$

Recalling the "broadbanding" effect of negative feedback, we get

$f_B = f_{P1}(1 + A\beta) \simeq f_{P1}A\beta$

Loop gain $A\beta = 2.43\times10^5 \cdot \frac{1}{100} = 2.43\times10^3$

$\Rightarrow f_{P1} = \underline{180\text{ Hz}}$

$$f_B = \frac{G_{m1}}{2\pi C_C} = 437\text{ kHz}$$

$$\Rightarrow C_C = \frac{1}{5.26\times10^{3}\,(2\pi)\,437\times10^3}$$

$= \underline{0.69\text{ pF}}$

10.52

dominant pole $f_P = \frac{1}{2\pi R(AC_C)}$; $A = 1000$

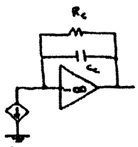
with single pole response

$A_o f_P = f_t \Rightarrow f_P = \frac{5\times10^6}{10^6} = 5\text{ Hz}$

$$\Rightarrow R = \frac{1}{2\pi(5)\,1000\,(50\text{pF})} = \underline{637\text{ k}\Omega}$$

10.53

DC, gain is



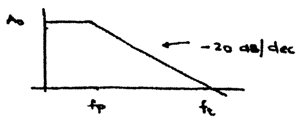
$A_0 = G_{m1} R_c$
$= 10 \times 10^{-3} \times 10^{8}$
$= 10^{6}$ V/V

$f_P = \frac{1}{2\pi R_c C_c} = \frac{1}{2\pi \times 10^{8} \times 50 \times 10^{-12}}$

$= 31.8$ Hz

$f_t = A_0 f_P$

$= 31.8$ MHz



$SR = \frac{2I}{C_c}$

$G_{m1} = \frac{I}{2V_T} \Rightarrow 2I = 4 G_{m1} V_T$

$SR = \frac{4 G_{m1} V_T}{C_c} = \frac{4(10 \times 10^{-3})(25 \times 10^{-3})}{50 \times 10^{-12}}$

$= 20$ V/μs

10.54

$I_{E1} = I_{E2} = 50\mu A = I_{E3} = I_{E4}$

$I_{E5} = 1$ mA ; $V_{BE5} = V_{BE6}$

$\therefore I_{E6} = 1$ mA $= I_{E7}$

$r_{e1} = r_{e2} = 500\Omega$

$r_{e5} = r_{e6} = r_{e7} = 25\Omega$

$G_{m1} = 2\left(\frac{1}{2r_{e1}}\right) = 2$ mA/V

$R_{o1} = (\beta+1)(r_{e5} \| r_{e6})$

$= 1.25$ kΩ

and $A_1 = G_{m1} R_{o1} = 2.5$ V/V

10.55

$I = 10\ \mu A$, $\frac{I_{S2}}{I_{S1}} = 2$,

$R_2 = 1.73$ kΩ, $R_3 = R_4 = 20$ kΩ,

$I_5 = 10\ \mu A$, $I_6 = 40\ \mu A$

From $V_{BE5} = V_{BE1} \Rightarrow V_T \ln \frac{I_5}{I_{S5}} = V_T \ln \frac{I_1}{I_{S1}}$

or $\frac{I_5}{I_{S5}} = \frac{I_1}{I_{S1}}$

Since $I_5 = 10\ \mu A = I_1$, then $\frac{I_{S5}}{I_{S1}} = 1$ or equivalently Q_1 and Q_5 have the same emitter area.

For Q_6: $I_6 = 40\ \mu A$ or $I_6 = 4I_1$. Similar to Q_5:

$V_{BE6} = V_{BE1}$, therefore: $\frac{I_{S6}}{I_{S1}} = 4$. If a resitor R_6 is connected to the emitter of Q_6 and the current I_6 is reduced to 10μA, then we can write:

$V_{BE1} - V_{BE6} = R_6 I_6$ or

$V_T \ln \frac{I_1}{I_{S1}} - V_T \ln \frac{I_6}{I_{S6}} = R_6 I_6$

$\Rightarrow V_T \ln \frac{I_1}{I_{S1}} \frac{I_{S6}}{I_6} = R_6 I_6$

$\therefore 25 \times 10^{-3} \times \ln 4 = R_6 \times 10 \times 10^{-6}$

$\Rightarrow R_6 = 3.5$ kΩ

The output resistance of Q_5 is simply r_{o5}:

$R_{o5} = r_o = \frac{V_A}{I_C} = \frac{30}{10\ \mu A} = 3$ MΩ

For Q6, the output resistance is increased by a factor of $(1 + g_m R'_E)$ where

$R'_E = R_6 \| r_{\pi 6}$ (See Eq. on page of the Text.)

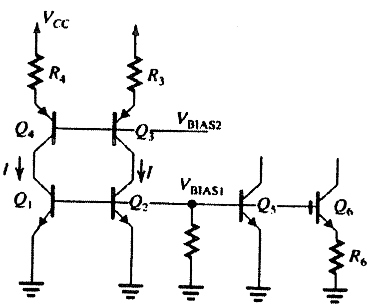
$R_{o6} = (1 + g_m R'_E) r_{o6}$

$g_{m6} = \frac{I_{C6}}{V_T} = \frac{10}{25} = 0.4$ mA/V

$r_\pi = \frac{\beta_n}{g_m} = \frac{40}{0.4} = 100$ kΩ, $r_{o6} = 3$ MΩ

$R_{o6} = (1 + 0.4 \times (3.5\ k\Omega \| 100\ k\Omega)) \times 3$ MΩ

$= 7$ MΩ



10.56

a) $V_{CC} = 3\text{ V}$ $V_{BIAS} = 2.3\text{ V}$

The minimum allowed value of V_{ICM} in the circuit of Fig. 12.40(a) is limited by the need to keep Q_1 in the active mode. Since the collector of Q_1 is at a voltage $V_{BE3} \simeq 0.7\text{V}$, we see that the voltage applied to the base of Q_1 cannot go lower than 0.1 V. Thus $V_{ICMIN} = 0.1\text{ V}$

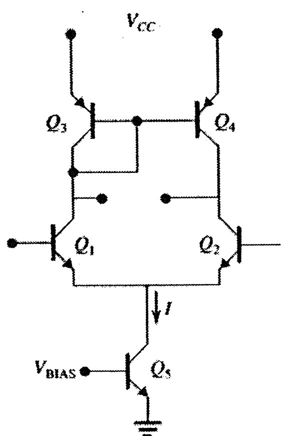
For V_{ICMmax}, see Eq. on page of the Text:

$V_{ICMmax} = V_{CC} - 0.8\text{V} = 3\text{V} - 0.8\text{V} = 2.2\text{V}$

b) Similarly, V_{ICMmax} is limited by the need to keep Q_1 in the active mode. $V_{ICMmax} = V_{CC} - 0.1\text{V} = 2.9\text{V}$

The lower end of the input common-mode range is achieved when the voltage across Q_5, V_{CE5}, does not fall below 0.1V. Hence:

$V_{ICMmin} = 0.1 + 0.7 = 0.8\text{V}$



10.57

$V_{CC} = 3\text{ V}, V_{BIAS} = 2.3\text{ V},$

$I = 20\ \mu\text{A}, R_C = 20\text{ k}\Omega$

From Equations we have :

$$V_{ICM\,min} = V_{RC} - 0.6\text{ V} = \frac{I}{2} \times R_C - 0.6\text{ V}$$

$$= \frac{20}{2} \times 10^{-6} \times 20 \times 10^3 - 0.6 = -0.4\text{ V}$$

$$V_{ICM\,max} = V_{CC} - 0.8\text{ V} = 2.2\text{ V}$$

$$-0.4\text{V} \le V_{ICM} \le 2.2\text{V}$$

$$\frac{v_o}{v_{id}} = -g_{m1,2}R_C = -\frac{I/2}{V_T}R_C$$

$$= -\frac{V_{RC}}{V_T} = -\frac{20/2 \times 10^{-6} \times 20 \times 10^3}{25 \times 10^{-3}} = 8\text{ V/V}$$

10.58

$$\frac{v_o}{v_{id}} = -g_{m3,4}R_C \cdot$$

With $A_V = 10\text{V/V}$ and $I_6 = 10\ \mu\text{A}$, we have :

$$g_{m3} = \frac{I_{C3}}{V_T} = \frac{10/2 \times 10^{-6}}{25 \times 10^{-3}}$$

$$\Rightarrow g_{m3,4} = 0.2\text{ mA/V}$$

Then $10\text{V/V} = -0.2R_C \Rightarrow R_C = 50\text{ k}\Omega$

$$R_{id} = 2r_{\pi 3,4} = 2\frac{\beta_n}{g_{m3}} = 2 \times \frac{40}{0.2} = 400\text{ k}\Omega$$

To find the common-input range, we calculate

$V_{RC}: V_{RC} = 50\text{ k}\Omega \times \frac{10}{2}\ \mu\text{A} = 0.25\text{ V}$ From discussion on page of the Text :

$$V_{ICMmin} = 0.1 + 0.7 = 0.8\text{V},$$

$$V_{ICMmax} = V_{CC} - V_{RC} - 0.1 + 0.7$$

$$= 3 - 0.25 - 0.1 + 0.7$$

$$= 3.35\text{ V}$$

$$0.8\text{ V} \le V_{ICM} \le 3.35\text{V}$$

As we can see, in the circuit in Fig. 12.41, the input common-mode range is extended above V_{CC}.

10.59

$$R_{id} = 2r_{\pi 1} = 2p_P/g_{m1} = 2 \times 10 \times \frac{25 \times 10^{-3}}{\frac{6}{2} \times 10^{-6}}$$

$= 167\text{ k}\Omega$

To find the short-circuit trans conductance, we short the output to ground as shown in Fig. 12.43(b) on the same page of the Text:

$$G_{m1} = \frac{i_{c7}}{V_{id}/2} \text{ (see Example 12.5)}$$

$$r_{o1} = \frac{|V_{AP}|}{I_{C1}} = \frac{20}{3} = 6.7\text{ M}\Omega$$

$$r_{o7} = \frac{|V_{An}|}{I_{C7}} = \frac{30}{3} = 10\text{ M}\Omega$$

$$r_{e7} \approx \frac{1}{g_{m7}} = \frac{V_t}{I_{C7}} = \frac{25}{3} = 8.3\text{ k}\Omega$$

$R_7 = 22\text{ k}\Omega$ and if we neglect r_{o1} and r_{o7} as they are large, we can write :

$$i_{c7} = g_{m1}\left(\frac{v_{id}}{2}\right)\frac{R_7}{r_{e7} + R_7} = \frac{3 \times 10^{-6}}{25 \times 10^{-3}}$$

$$\left(\frac{v_{id}}{2}\right)\frac{22\text{ k}\Omega}{8.3\text{ k}\Omega + 22\text{ k}\Omega} = 87.1 \times 10^{-6}\frac{v_{id}}{2}$$

$$G_{m1} = \frac{i_{c7}}{v_{id}/2} = 0.087 \text{ mA/V}$$

Now to calculate $R_o : R_o = \left(R_{o9} \| R_{o7} \| \frac{R_L}{2}\right)$

$R_{o9} = r_{o9} + (R_9 \| r_{\pi 9})(1 + g_{m9} r_{o9})$,

$r_{o9} = \frac{V_{AP}}{I_{en}} = \frac{20}{6} = 6.7 \text{ M}\Omega$,

$r_{M9} = \frac{\beta_P}{g_{m9}} = \frac{10}{6/25} = 41.7 \text{ k}\Omega$

$R_{o9} = 6.7 \text{ k}\Omega + (33 \text{ k}\Omega \| 41.7 \text{ k}\Omega)$

$\left(1 + \frac{6 \times 10^{-6}}{25 \times 10^{-3}} \times 6.7 \times 10^6\right)$

$\Rightarrow R_{o9} = 36.3 \text{ M}\Omega$

$R_{o7} = r_{o7} + (R_7 \| r_{\pi 7})(1 + g_{m7} r_{o7})$,

$r_{o7} = \frac{V_{AN}}{I_{C7}} = \frac{30}{3} = 10 \text{ M}\Omega$,

$r_{\pi 7} = \frac{30}{6/25} = 5 \text{ k}\Omega$

$g_{m7} = \frac{6}{25} = 0.24 \text{ mA/V}$

$R_{o7} = 10 \text{ k}\Omega + (22 \text{ k}\Omega \| 5 \text{ k}\Omega)$

$(1 + 0.24 \times 10^{-3} \times 10 \times 10^6)$

$\Rightarrow R_{o7} = 19.8 \text{ M}\Omega$

$\therefore R = 36.3 \| 19.8 \| 1.3/2 = 0.62 \text{ M}\Omega$

$A_d = G_{m1} R = 0.087 \times 10^{-3} \times 0.62 \times 10^6$

$= 54 \text{ V/V}$

$A_d = 54 \frac{\text{V}}{\text{V}}$

10.60

$A_{open} = G_{m1} \cdot R_{out}$

$G_{m1} = \dfrac{i_O}{V_{id}/2}$ where

$i_O \approx i_{c7} = g_{m1} = \left(\dfrac{V_{id}}{2}\right)\left[\dfrac{R_7}{R_7 + r_{e7}}\right]$

$R_7 = \dfrac{0.2\text{ V}}{(2I + I)} = \dfrac{0.2\text{ V}}{3I}$ and

$r_{e7} \approx \dfrac{1}{g_{m7}} = \dfrac{V_T}{I_{C7}} = \dfrac{25\text{ m}}{2I}$

$$\therefore G_{m1} = \left[\frac{I_{C1}}{V_T}\right]\left[\frac{\left(\frac{0.2}{3I}\right)}{\left(\frac{25\text{ m}}{2I}\right) + \left(\frac{0.2}{3I}\right)}\right]$$

$$= \frac{I}{25\text{ m}}\left[\frac{\frac{0.2}{3I}}{\frac{75\text{ m}}{6I} + \frac{0.4}{6I}}\right] = \frac{I}{25\text{ m}}\left[\frac{0.2 \times 6I}{0.475 \times 3I}\right]$$

$G_{m1} = 33.7I$

$R_{out} = R_{O7} \| R_{O9}$

$R_{O7} = r_{O7} + (R_7 \| r_{\pi 7})(1 + g_{m7} r_{O7})$

using $\beta_n = 40,\ V_{An} = 30$:

$r_{07} = \dfrac{V_{An}}{I_{C7}} = \dfrac{30}{2I}$

$r_{\pi 7} = \dfrac{\beta_n}{g_{m7}}$

$g_{m7} = \dfrac{I_{C7}}{V_T} = \dfrac{2I}{25\text{ m}}$

$r_{\pi 7} = \dfrac{40}{2I} \times 25\text{ m} = \dfrac{1}{2I}$

$$R_{O7} = \frac{30}{2I} + \frac{\left[\frac{0.2}{3I} \times \frac{1}{2I}\right]}{\left(\frac{0.2}{3I} + \frac{1}{2I}\right)} \times \left[1 + \frac{2I}{25\text{ m}} \times \frac{30}{2I}\right]$$

$$R_{O7} = \frac{30}{2I} + \frac{\left[\frac{0.2}{6I}\right]}{\left[\frac{0.4 + 3}{6I}\right]} \times [1 + 1,200]$$

$$= \frac{30}{2I} + \frac{0.2}{3.4I} \times 1,201$$

$R_{07} = \dfrac{102 + 240.2}{6.8I} = \dfrac{50.3}{I}$

$R_{O9} = r_{O9} + (R_9 \| r_{\pi 9})(1 + g_{m9} r_{O9})$ where

$R_9 = \dfrac{0.3}{2I}$

using $\beta_p = 10$ and $|V_{A_p}| = 20$ V

$r_{O9} = \dfrac{|V_{A_p}|}{I_{C9}} = \dfrac{20}{2I}$

$g_{m9} = \dfrac{I_{C9}}{V_T} = \dfrac{2I}{25\text{ m}}$

$r_{\pi 9} = \dfrac{\beta_P}{g_{m9}} = \dfrac{10 \times 25\text{ m}}{2I}$

$$R_{O9} = \frac{20}{2I} + \frac{\left[\frac{0.3}{2I} \times \frac{0.25}{2I}\right]}{\frac{(0.3 + 0.25)}{2I}} \times \left[1 + \frac{2I}{25\text{ m}} \times \frac{20}{2I}\right]$$

$$= \frac{20}{2I} + \frac{0.075}{(0.55)2I} \times [801]$$

$R_{O9} = \dfrac{(20 + 109.2)}{2I} = \dfrac{129.2}{2I} =$

$$R_{out} = R_{O7} \| R_{O9} = \frac{\frac{50.3}{I} \times \frac{64.6}{I}}{\left(\frac{50.3 + 64.6}{I}\right)} = \frac{3,249.4}{114.9I}$$

$$= \frac{28.3}{I}$$

$A_{open} = 33.7I \times \dfrac{28.3}{I} = 0.84$ V/V

$$R_{out} = R_{O7} \| R_{O9} \| \left(\frac{R_L}{2}\right) = \frac{\frac{28.3}{I} \cdot \frac{R_L}{2}}{\frac{28.3(2) + IR_L}{2I}}$$

$$= \frac{28.3(R_L)}{56.6 + IR_L}$$

$A_d = 33.7I \times \dfrac{28.3(R_L)}{56.6 + IR_L} = \dfrac{953.7IR_L}{56.6 + IR_L}$

For $A_d = 160$ V/V $= \dfrac{953.7 \times 2 \times 10^6 \times I}{56.6 + I \times 2 \times 10^6}$

$160(56.6) + [I \times 2 \times 10^6 \times 160 - I \times 953.7 \times 2 \times 10^6] = 0$

$I(320 \times 10^6 - 1,907 \times 10^6) = -9056$

$I = \dfrac{-9056}{-1,587\text{ M}} = 5.7\ \mu\text{A}$

For $A_d = 320$ V/V $= \dfrac{1,907\text{ M} \times I}{56.6 + I(2\text{ M})}$

$320(56.6) = [1,907\text{ M} - 320(2\text{ M})]I$

$I = \dfrac{18,112}{1,267\text{ M}} = 14.3\ \mu\text{A}$

10.61

(a)To find the loopgain of the common-mode feedback loop for the circuit in Fig. 12.44 of the Text, we set the input voltage to zero (that is, I_1, and I_2 are zero), break the loop at the input of the common-mode feedback circuit block, apply a test voltage v_t to the input of common-mode feedback circuit and find the output voltage v_o at the output of the amplifier (where the loop was broken), and then $A\beta = -\frac{v_o}{v_t}$. Looking at half circuit and assuming that r_{o7} is relatively large, we have : $i_{07} \approx \frac{v_t}{(\beta+1)(r_{e7}+R_7)}$. Note that $(\beta+1)(r_{e7}+R_7)$ is the small-signal input resistance seen at the base of Q_7.

Thus, $i_o = i_{c7} = \beta i_{b7} \approx \frac{\beta v_t}{(\beta+1)(r_{e7}+R_7)}$

$$= \frac{v_t}{\frac{\beta+1}{\beta}(r_{e7}+R_7)} \approx \frac{v_t}{r_{e7}+R_7}$$

$$v_o = -i_o \times (R_{o7} \| R_{o9}) \approx -\frac{v_t(R_{o7}\|R_{o9})}{r_{e7}+R_7}$$

$$\Rightarrow A\beta = -\frac{v_o}{v_t} \approx \frac{R_{o7}\|R_{o9}}{r_{e7}+R_7}$$

(b) From Example 12.6 of the Text, we have :

$R_{o7} = 23\ \text{M}\Omega$ and

$R_{o9} = 12.9\ \text{M}\Omega \Rightarrow R_{o7}\|R_{o9} = 8.3\ \text{M}\Omega$

$r_{e7} = \frac{25\text{ mV}}{10\ \mu\text{A}} = 2.5\ \text{k}\Omega$ and $R_7 = 20\ \text{k}\Omega$

Thus, $A\beta \approx \frac{R_{o7}\|R_{o9}}{r_{e7}+R_7} = \frac{8.3\ \text{M}\Omega}{2.5\ \text{k}\Omega + 20\ \text{k}\Omega}$

$= 368.9 \Rightarrow A\beta \approx 368.9$

(c) With the CMF present, we have

$$\Delta V_{CM} = \frac{\Delta V_{CM}\text{ when CMF is absent}}{1+A\beta}$$

$$\Rightarrow \Delta V_{CM} = \frac{2.5\text{V}}{1+368.9} = 6.76\text{ mV}$$

Note that the corresponding value for ΔV_{CM} found by a different approach in Example 12.6 is 6.75mV which is only 0.1% off from the calculated value in this problem.

10.62

$I_Q = 0.4$ mA$i_{LMax} = 10$ mA

a) The output voltage v_o can swing as low as 0.1V when Q_P is in active, and Q_N supplies the load current : $v_{omin} = 0.1\text{V}$

v_o can go up as high as $V_{CC} - 0.1$ V when Q_N is inactive and Q_P supplies the load current :

$0.1\text{V} \le v_o \le V_{CC} - 0.1$ V

b) $i_L = 0$ $R_o = R_{oN} \| R_{op}$

$i_L = 0 \Rightarrow i_p = i_N = I_Q = 0.4$ mA

$$r_{oN} = \frac{V_{AN}}{I_N} = \frac{30}{0.4} = 75\ \text{k}\Omega$$

$$r_{op} = \frac{V_{AP}}{I_P} = \frac{20}{0.4} = 50\ \text{k}\Omega$$

$R_O = 30\ \text{k}\Omega$

c) $R_{op} = \frac{R_o}{1+A\beta} = \frac{30\ \text{k}\Omega}{1+10^5\times 1} = 0.3\ \Omega$

d) $i_L = 10$mA Since i_L is at its max, then Q_N is inactive mode. Hence: $i_N = \frac{I_Q}{2} = 0.2$ mA and since we have: $i_p = i_N + i_L \Rightarrow i_p = 10.2$ mA

e) $i_L = -10$mA then Q_P is inactive and $i_P = 0.2$mA.

ForQ_N:$i_P - i_L$ $i_N = 0.2 - (-10) = 102$ mA

10.63

$$v_{B7} = v_{BEN} = V_T \ln\frac{i_N}{I_{SN}}$$

$v_{B6} = R_5 i_4 + v_{BE5} = R_5 i_4 + V_T \ln\frac{i_4}{I_{S5}}$ Note that $i_4 \approx i_5$

IF we substitute for $i_4 = \frac{v_{BEP} - v_{BE4}}{R_4}$, then :

$v_{B6} = \frac{R_5}{R_4}(v_{BEP} - v_{BE4}) + V_T \ln\frac{i_4}{I_{S5}}$ Note that $R_5 = R_4$

$$v_{B6} = V_T \ln\frac{i_P}{I_{SP}} - V_T \ln\frac{i_4}{I_{S5}} + V_T \ln\frac{i_4}{I_{S5}}$$

$$V_T \ln\frac{i_P}{I_{SP}} \times \frac{I_{S4}}{i_4} \times \frac{i_4}{I_{S5}}$$

Now substitute for $I_{S5} = I_{S4}\frac{I_{SN}}{I_{SP}}$, then we have:

$$v_{B6} = V_T \ln\frac{i_P}{I_{SP}}\frac{I_{S4}}{I_{S4}}\frac{I_{SP}}{I_{SN}} = V_T \ln\frac{i_P}{I_{SN}}$$

Now if we consider V_E and write :

$$v_E = v_{B6} + v_{BE6} = v_{B7} + v_{BE7}$$

$$\Rightarrow V_T \ln\frac{i_P}{I_{SN}} + v_T \ln\frac{i_6}{I_{S6}} = V_T \ln\frac{i_N}{I_{SN}} + V_T \ln\frac{i_7}{I_{S7}}$$

$$\Rightarrow \frac{i_P}{I_{SN}} \times \frac{i_6}{I_{S6}} = \frac{i_N}{I_{SN}} \times \frac{i_7}{I_{S7}}$$

$\Rightarrow i_7 = \frac{I_{S7}}{I_{S6}}\frac{i_P}{i_N} \times i_6$ Note that $I_{S7} = I_{S6}$, hence

$i_7 = \frac{i_P}{i_N} i_6$

We can write : $i_6 + i_7 = I$,hence :

$$i_6 + \frac{i_P}{i_N}i_6 = I \Rightarrow i_6 = \frac{I i_N}{i_P + i_N}$$

$$i_7 = \frac{i_P}{i_N} \times I\frac{i_N}{i_N + i_P} \Rightarrow i_7 = I\frac{i_P}{i_P + i_N}$$

10.64

For Q_7 we can write : $v_{B7} = v_{BEN} = V_T \ln\frac{i_N}{I_{SN}}$

At node E, we have :

$$v_E = v_{EB7} + v_{B7} = V_T \ln\frac{i_7}{I_{S7}} + V_T \ln\frac{i_N}{I_{SN}}$$

$$= V_T \ln\frac{i_7 i_N}{I_{S7} I_{SN}}$$

$$i_{C7} = I\frac{i_P}{i_P + i_N}$$

$$\therefore v_E = V_T \ln\left[\frac{i_P i_N}{i_N + i_P}\frac{I}{I_{SN} I_{S7}}\right]$$

10.65

$I_Q = 0.36$ mA , $I = 10$ μA , $I_{SN} = 8I_{s10}$,

$I_{S7} = 4I_{S11}$

From Eq. 12.138, we have :

$$I_Q = 2\left(\frac{I_{REF}^2}{I}\right)\frac{I_{SN}}{I_{S10}}\frac{I_{S7}}{I_{S11}} \Rightarrow 0.36 \times 10^{-3}$$

$$= 2\left(\frac{I_{REF}^2}{10 \times 10^{-6}} \times 8 \times 4\right)$$

$$I_{REF}^2 = \frac{0.36 \times 10^{-8}}{64} \Rightarrow I_{REF} = 7.5\ \mu\text{A}$$

The minimum current in the inactive output transistors, Q_N and Q_P, is $\frac{1}{2}I_Q$ or 0.18 mA.

11.1

$$T(s) = \frac{1}{(s+1)(s^2+s+1)}$$

$$= \frac{1}{s^3+2s^2+2s+1}$$

$$T(j\omega) = [j(2\omega-\omega^3)+(1-2\omega^2)]$$

$$|T(j\omega)| = [(2\omega-\omega^3)^2+(1-2\omega^2)^2]^{-1}$$

$$= [4\omega^2-4\omega^4+\omega^6+1-4\omega^2+4\omega^4]$$

$$= [1+\omega^6]^{-\frac{1}{2}}$$

$$= \frac{1}{\sqrt{1+\omega^6}}$$

For phase Angle:

$$\phi(\omega) = \tan^{-1}\left[\frac{Im[T(j\omega)]}{Re[T(j\omega)]}\right]$$

$$= -\tan^{-1}\left[\frac{2\omega-\omega^3}{1-2\omega^2}\right]$$

For $w = 0.1$:

$$|T(j\omega)| = (1+0.1^6)^{-1/2} \cong 1$$

$$\phi(\omega) = -11.5° = -0.20 \text{ rad}$$

For $\omega = 1$ rad/s:

$$|T(j\omega)| = (1+1^6)^{-1/2} = 1/\sqrt{2} = 0.707$$

$$\phi = -\tan^{-1}\left(\frac{1}{-1}\right) = -135° = 2.356 \text{ rad}$$

Note: $G = -3$ dB

Also: $\tan^{-1}(-1) = -45°$ or $-135°$

$$\tan^{-1}\left(\frac{-1}{1}\right) = -45°$$

$$\tan^{-1}\left(\frac{1}{-1}\right) = -135°$$

For $w = 10$ rad/s:

$$|T(j\omega)| = (1+10^6)^{-1/2} = 0.001$$

$$\phi = -\tan^{-1}\left[\frac{2(10)-10^3}{1-2(10^2)}\right]$$

$$= -\tan^{-1}\left[\frac{-980}{-199}\right]$$

$$= -\left[180° + \tan^{-1}\left(\frac{980}{199}\right)\right]$$

$$= -258.5°$$

$$= 4.512 \text{ rad}$$

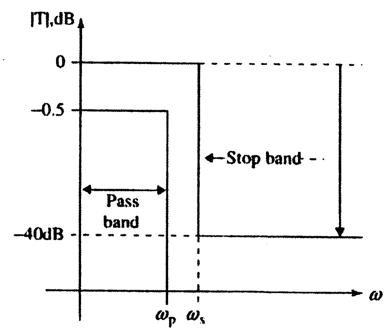
Now consider an input of $A \sin wt$ to $T(s)$. The output is then given by:

$$A|T(j\omega)|\sin(\omega t + \phi(\omega))$$

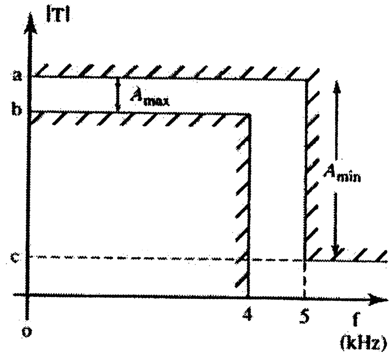
Using this result, the output to each of the following inputs will be:

INPUT	OUTPUT
$2\sin(0.1t)$	$2\sin(0.1t - 0.2)$ ie $2 \times 1 = 2$
$2\sin(1t)$	$\sqrt{2}\sin(t - 2.356)$ i.e. $2 \times 1/\sqrt{2} = \sqrt{2}$
$2\sin(10t)$	$2 \times 10^{-3}\sin(10t - 4.512)$

11.2



11.3



Note $|T|$ is shown in a linear scale but A_{max} and A_{min} are in dB

From the problem

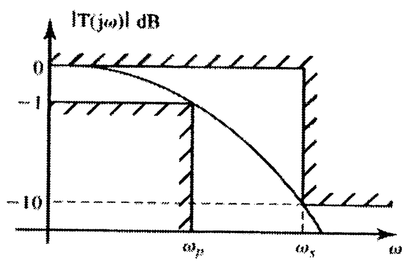
$$\frac{a}{b} = 1.1,\ C = 0.1\%\ a \text{ or } \frac{C}{a} = 0.001$$

$$A_{max} = 20\log_{10}a - 20\log_{10}b$$
$$= 20\log_{10}a/b$$
$$= 20\log_{10}(1.1)$$
$$= 0.83\text{ dB}$$

$$A_{min} = 20\log_{10}a - 20\log_{10}c$$
$$= 20\log_{10}\left(\frac{a}{c}\right)$$
$$= 20\log_{10}(0.001)$$
$$= 60\text{ dB}$$

$$\text{Selectivity} = \frac{w_s}{w_p} = \frac{2\pi 5}{2\pi 4} = 1.25$$

11.4



$T(s) = \frac{k}{1+s\tau}$ If $\tau = 1\text{s}$ & the DC gain $= 1$

$$= \frac{1}{1+1s}$$

then $k = 1$

$$|T(j\omega)| = \frac{1}{\sqrt{1+\omega^2}}$$

At the passband edge:

$$|T(j\omega_p)| = \frac{1}{\sqrt{1+\omega_p^2}} = 10^{-1/20}$$

$\therefore \omega_p = 0.5088$ rad/s

At the stopband edge:

$$|T(j\omega_s)| = \frac{1}{\sqrt{1+\omega_s^2}} = 10^{-10/20}$$

$\therefore \omega_s = 3$ rad/s

$$\text{Selectivity} = \frac{\omega_s}{\omega_p} = \frac{3}{0.5088} = 5.9$$

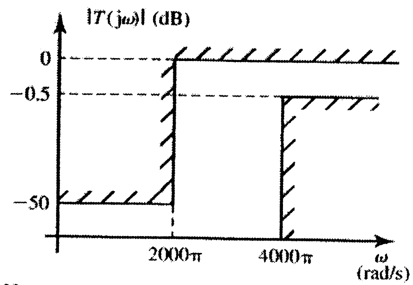
11.5

Passband is defined by: $f \geq 2$ kHz

$\Rightarrow w_p = 2\pi(2000)$ rad/s

Stopband is defined by: $f \leq 1$ kHz

$\Rightarrow w_s = 2\pi(1000)$ rad/s



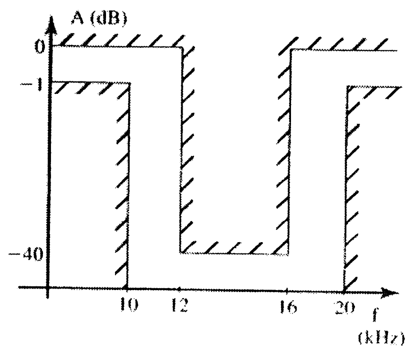
Note we assumed a maximum transmission of 0 dB.

11.6

Passband: $f \in \{[0, 10\text{ kHz}] \cup [20\text{ kHz}, \infty]\}$

Stopband: $f \in [12\text{ kHz}, 16\text{ kHz}]$

$A_{max} = 1$ dB, $A_{min} = 40$ dB



11.7

Poles at -1 and $-0.5 \pm j0.8$ gives a denominator:

$D(s) = (s+1)(s+0.5-j0.8)(s+0.5+j0.8)$

$=(s+1)(s^2+2(0.5)s+0.5^2+0.8^2)$

$=(s+1)(s^2+s+0.89)$

Zeros at ∞ and $\pm jz$ give a numerator:

$N(s) = k(s+jz)(s-jz) = k(s^2+4)$

Note there is one zero at ∞ because Degree $(D(s))$ − Degree $(N(s))$ − 1

$$T(s) = \frac{k(s^2+4)}{(s+1)(s^2+s+0.89)}$$

$|T(jo)| = \frac{k(4)}{0.89} = 1 \quad \therefore \text{DC gain} = 1$

$\Rightarrow k = 0.225$

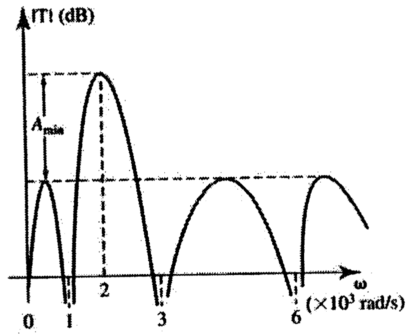
$$\therefore T(s) = \frac{0.2225(s^2+4)}{(s+1)(s^2+s+0.89)}$$

$G_{m1} = 1$ mA/V, $G_{m2} = 2$ mA/V,

$R = 500\ \Omega$

$$f_t \approx \frac{G_{m1}}{2\pi C_C} \Rightarrow C_C = \frac{G_{m1}}{2\pi ft}$$

11.8



Numerator is given by

$a_7 (s - 0)(s^2 + (10^3)^2)(s^2 + (3 \times 10^3)^2)3$

$(s^2 + (6 \times 10^3)^2)$

$= a_7 s(s^2 + 10^6)(s^2 + 9 \times 10^6)(s^2 + 36 \times 10^6)$

Degree of Numerator $\triangleq m = 7$

Degree of Denominator $\triangleq N$

Given that there is one zero at ∞:

$N - M = 1 \Rightarrow N = 8$

$$\therefore T(s) = \frac{a_7 s(s^2 + 10^6)(s^2 + 9 \times 10^6)(s^2 + 36 \times 10^6)}{s^8 + b_2 s^7 + b_6 s^6 + \ldots + b_0}$$

From circuit : current drawn from V_{DD} rail = 2 IB = current returnen to V_{SS} rail

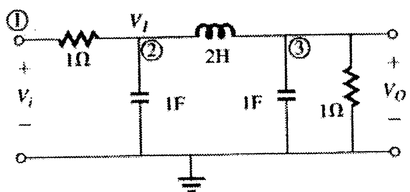
$\therefore Power = (V_{DD} + V_{SS}) \times 2 I_B \Rightarrow$

$1 \text{ mW} = (1.65 + 1.65) \times 2 I_B$

$$\therefore I_B = \frac{1 \text{ mW}}{4 \times 1.65 \text{ V}} = 151.5 \ \mu\text{A}$$

$\Rightarrow I = I_B / 1.2 = 126.3 \ \mu\text{A}$

11.9



The easiest way to solve the circuit is to use nodal analysis at nodes (1), (2), (3)

At node (3) $\Sigma I = 0$

$$\frac{V_O}{1} + \frac{V_O}{1/s} + \frac{V_O - V_1}{2s} = 0$$

$\therefore V_1 = V_O(2s^2 + 2s + 1)$ Eq. (a)

At node (2) $\Sigma I = 0$

$$\frac{V_1 - V_i}{1} + \frac{V_1}{1/s} + \frac{V_1 - V_O}{2s} = 0$$

$\therefore V_1(2s^2 + 2s + 1) = V_O + 2sV_i$ Eq. (b)

(a) → (b)

$V_O(2s^2 + 2s + 1)^2 = V_O + 2sV_i$

$V_O(4s^4 + s^3(4 + 4) + s^2(2 + 4 + 2) + s(2 + 2) + 1)$

$= V_O + 2sV_i$

$$\frac{V_O(s)}{V_i(s)} \triangleq T(s) \frac{2s}{4s^4 + 8s^3 + 8s^2 + 4s}$$

$$T(s) = \frac{0.5}{s^3 + 2s^2 + 2s + 1}$$

Poles are given by:

$s^3 + 2s^2 + 2s + 1 = 0$

$(s + 1)(s^2 + s + 1) = 0$

$\therefore$ Poles are $s = -1$ and $s = -\frac{1}{2} \pm j\sqrt{3}/2$

11.10

$A_{max} = 1$ dB, $A_{min} = 20$ dB, $w/w_p = 1.3$

Using: $A(\omega_s) = 10 \log\left[1 + \epsilon^2\left(\frac{\omega_s}{\omega_p}\right)^{2N}\right]$

$= A_{min}$

$\epsilon = [10^{1/10} - 1]^{1/2} = 0.5088$

$A_{min} = 10 \log\left[1 + \epsilon^2\left(\frac{\omega_s}{\omega_p}\right)^{2N}\right]$

$10^{A_{min}/10} - 1 = \epsilon^2 \left(\frac{\omega_s}{\omega_p}\right)^{2N}$

$\log(10^{A_{min}/10} - 1) = \log\left(\epsilon^2\left(\frac{\omega_s}{\omega_p}\right)^{2N}\right)$

$$N = \frac{\log[(10^{A_{min}/10} - 1)/\epsilon^2]}{2\log(\omega_s/\omega_p)}$$

$= 11.3 \Rightarrow$ choose $N = 12$

The actual value of stopband attenuation can be calculated using the integer value of N:

$A(\omega_s) = 10\log\left[1 + \epsilon^2\left(\frac{\omega_s}{\omega_p}\right)^{2N}\right]; N = 12$

= 27.35 dB actual attenuation

If the stopband specs are to be met exactly we need to find A_{max}.

$$\epsilon^2 = \frac{10^{A_{min}/10} - 1}{(\omega_s/\omega_p)^{2N}}$$

$A_{min} = 20$

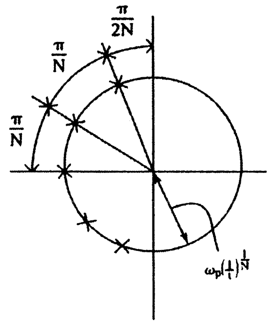
$N = 12$

$= 0.1824$

$\therefore A_{max} = 10\log(1 + \epsilon^2)$

$= 0.73$ dB

11.11



$\omega_p = 10^3 \text{ rad/s} \quad N = 5$

$A_{max} = 1 \text{ dB} \Rightarrow \varepsilon = 0.5088$

find solution graphically

$P_1 = \omega_p\left(\frac{1}{\varepsilon}\right)^{1/N} \measuredangle\left(\frac{\pi}{2} + \frac{\pi}{2N}\right)$

$= 873.59 \measuredangle\left(\frac{6\pi}{10}\right)$

$= 873.59\left[\cos\left(\frac{6\pi}{10}\right) \pm j\sin\left(\frac{6\pi}{10}\right)\right]$

$= -269.96 \pm j830.84$

$P_2 = 873.59 \measuredangle\left[\frac{\pi}{2} + \frac{\pi}{2N} + \frac{\pi}{N}\right]$

$= -706.75 \pm j\,513.49$

$P_3 = 873.59 \measuredangle \pi = -873.59$

11.12

$f_p = 10 \text{ kHz} \qquad \frac{\omega_s}{\omega_p} = 1.5 \qquad A_{min} = 15 \text{ dB}$

$f_s = 15 \text{ kHz} \qquad A_{max} = 2 \text{ dB}$

$\varepsilon^2 = 10^{A_{max}/10} - 1 \Rightarrow \varepsilon = 0.76478$

Manipulation Eq (16.15) we get:

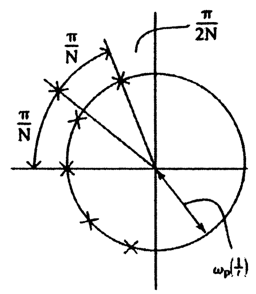
$N \doteq \dfrac{\log[(10^{A_{min}/10} - 1)/\varepsilon^2]}{2\log(\omega_s/\omega_p)} = 4.88$

$\therefore$ use $N = 5$

finding natural modes graphically:-

radius $= \omega_p\left(\frac{1}{\varepsilon}\right)^{1/N} \Delta\omega_O$

$\omega_O = 6.629 \times 10^4$



$P_1 = \omega_O \measuredangle\left(\frac{\pi}{2} + \frac{\pi}{2N}\right) = \omega_O \measuredangle\left(\frac{6\pi}{10}\right)$

$= \omega_O\left(\cos\left(\frac{6\pi}{10}\right) \pm j\sin\left(\frac{6\pi}{10}\right)\right)$

$= \omega_O(-0.309 \pm j0.951)$

$P_2 = \omega_O\left(\cos\frac{8\pi}{10} \pm j\sin\frac{8\pi}{10}\right)$

$= \omega_O(-0.809 \pm j0.588)$

$P_3 = \omega_O(\cos\pi \pm j\sin\pi) = -\omega_O$

Given a natural mode $-\alpha \pm j\beta$, the following term results

$(s + \alpha + j\beta)(s + \alpha - j\beta)$

$= s^2 + 2\alpha s + \alpha^2 + \beta^2$

$= s^2 + 2Re[P]s + |P|^2$

Also, note that for a Butterworth, all natural modes have a magnitude of w_o.

P_1 yields: $s^2 + 0.618\omega_O s + \omega_O^2$

P_2 yields: $s^2 + 1.618\omega_O s + \omega_O^2$

P_3 yields: $s + \omega_O$

$\therefore T(s) = \dfrac{k}{(s + \omega_O)(s^2 + 0.618\omega_O s + \omega_O^2)} \times \dfrac{1}{s^2 + 1.618\omega_O s + \omega_O^2}$

for unity dc gain

$|T(jo)| = \dfrac{k}{\omega_O^5} = 1 \Rightarrow k = \omega_O^5$

$\therefore T(s) = \dfrac{\omega_O^5}{(s + \omega_O)(s^2 + 0.618\omega_O s + \omega_O^2)} = \dfrac{1}{(s^2 + 1.618\omega_O s + \omega_O^2)}$

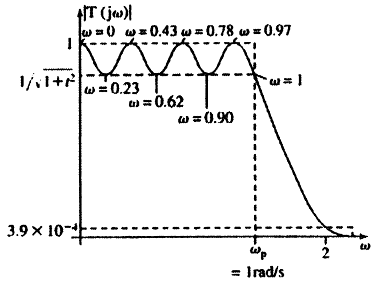
for attenuation at 20 kHz

$\frac{\omega_s}{\omega_p} = \frac{20}{10} = 2$

$A(\omega_s) = 10\log\left[1 + \epsilon^2\left(\frac{\omega_s}{\omega_p}\right)^{2N}\right]$

$= 27.8$ dB

11.13



Given $A_{max} = 1\text{dB} \Rightarrow \epsilon = 0.5088$

$|T(j\omega)| = \left[1 + \epsilon^2\text{Cos}^2\left(N\text{Cos}^{-1}\left(\frac{\omega}{\omega_p}\right)\right)\right]^{-1/2}$

for $\omega \le \omega_p$

If $|T(j\omega)| = 1$

$1 = 1 + \epsilon^2\text{Cos}^2(N\text{Cos}^{-1}(\omega/\omega_p))$

$\omega_p = 1$

$N\text{Cos}^{-1}(\omega/1) = \text{Cos}^{-1}(0)$

$\text{Cos}^{-1}(\omega) = \frac{2i+1}{2N}\pi$

$\therefore \omega_i = \text{Cos}\left[\frac{2i+1}{2N}\pi\right]$

ω's repeat after this value $i = 0, 1, \ldots, \frac{N-1}{2}$

$\omega_0 = 0.9749$

$\omega_1 = 0.7818$

$\omega_2 = 0.4339$

$\omega_3 = 0$

ω values at which $|T| = 1$

note $\omega_4 = -0.4339$

$= -\omega_2!$

If $|T| = 1/\sqrt{1+\epsilon^2}$, then

$1/\sqrt{1+\epsilon^2} = [1 + \epsilon^2\text{Cos}^2(N\text{Cos}^{-1}(\omega/\omega_p))]^{-1/2}$

$1 = \text{Cos}\left(N\text{Cos}^{-1}\left(\frac{\omega}{\omega_p}\right)\right)$

$N\text{Cos}^{-1}(\omega) = \text{Cos}^{-1}(0)$

$= i\pi \qquad i = 0, 1, 2\ldots$

$\omega_i = \text{Cos}\left[\frac{i\pi}{N}\right] \qquad i = 0, 1, 2, \ldots\frac{N}{2}$

$\omega_0 = 1.0$

$\omega_1 = 0.9010$

$\omega_2 = 0.6235$

$\omega_3 = 0.2252$

ω values at which

$|T| = (1+\epsilon^2)^{-1/2}$

Note $\omega_4 = 0.2252$

$= -\omega_3!$

To find $|T(jz)|$ since $\omega > \omega_p$

$|T(j\omega)| = \left[1 + \epsilon^2\text{Cosh}^2\left(N\text{Cosh}^{-1}\left(\frac{\omega}{\omega_p}\right)\right)\right]^{\frac{-1}{2}}$

$= [1 + 0.5088^2\text{Cosh}^2(7\text{Cosh}^{-1}2)]^{\frac{-1}{2}}$

$= 3.898 \times 10^{-4}$ V/V

$|T|_{ab} = -68.2$ dB

For roll-off consider

$T(s) = \frac{k}{s^7 + b_6 s^6 + \ldots b_0}$

for $\omega \gg \omega_p$ $|T(j\omega)| \cong \frac{k}{\omega^7}$

$\therefore$ Roll-off is $\frac{1}{2^7}$ or $20\log\left(\frac{1}{2^7}\right)$

per octave $= -42$ dB / octave.

11.14

$\omega_s/\omega_p = 2$

$A_{max} = 1\text{dB} \Rightarrow \epsilon = \sqrt{10^{\frac{A_{max}}{10}} - 1}$

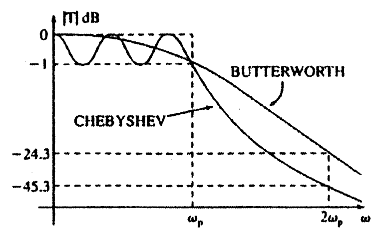
$= 0.5088$

$|T_B| = [1 + \epsilon^2(\omega_s/\omega_p)^{2N}]^{-1/2}$

$T_C| = \left[1 + \epsilon^2\text{Cosh}^2\left(N\text{Cosh}^{-1}\left[\frac{\omega_s}{\omega_p}\right]\right)\right]^{-1/2}$

$|T_B| = 6.13 \times 10^{-2} \Rightarrow -24.3$ dB

$|T_C| = 5.43 \times 10^{-3} \Rightarrow -45.3$ dB



11.15

(a) $f_P = 3.4$ kHz

$A_{max} = 1 \text{ dB} \Rightarrow \epsilon = 0.5088$

$f_S = 4$ kHz $A_{min} = 35$ dB

$\omega_S / \omega_P = 1.176$

Using Eq (16.22):

$$A(\omega_S) = 10\log\left[1 + \epsilon^2 \text{Cosh}^2\left(N\text{Cosh}^{-1}\left(\frac{\omega_S}{\omega_P}\right)\right)\right]$$

& trying different values for N

N	$A(\omega_s)$
8	28.8 dB
9	33.9 dB
10	38.98 dB

∴ Use N = 10

Excess attenuation = 39 − 35 = 4dB

(b) Poles are given by:

$$P_K = -\omega_P \text{Sin}\left(\frac{2k-1}{N}\cdot\frac{\pi}{2}\right)\text{Sinh}\left(\frac{1}{N}\left(\text{Sinh}^{-1}\left(\frac{1}{\epsilon}\right)\right)\right)$$

$$+ j\omega_P \text{Cos}\left(\frac{2k-1}{N}\cdot\frac{\pi}{2}\right)\text{Cosh}\left(\frac{1}{N}\text{Sinh}^{-1}\left(\frac{1}{\epsilon}\right)\right)$$

for k = 1, 2, ..., N.

Since $\epsilon = 0.5088$ and $N = 10$

$\text{Sinh}(1/N\text{Sinh}^{-1}(1/\epsilon)) = 0.1433$

$\text{Cosh}(1/N\text{Sinh}^{-1}(1/\epsilon)) = 1.010$

$$\therefore P_1 = \omega_P\left[-0.1433\text{Sin}\left(\frac{\pi}{20}\right) + j1.010\text{Cos}\left(\frac{\pi}{20}\right)\right]$$

$= \omega_P(-0.0224 + j0.9978)$

$P_2 = \omega_P(-0.0650 + j0.900)$

$P_3 = \omega_P(-0.1013 + j0.7143)$

$P_4 = \omega_P(-0.1277 + j0.4586)$

$P_5 = \omega_P(-0.1415 + j0.1580)$

Now it should be realized that the remaining poles are complex conjugates of the above.

pole-pair P_1 & P_1^* give a factor:

$S^2 + 2(0.0224)\omega_P S + \omega_P^2(0.0224^2 + 0.9978^2)$

$= S^2 + 0.0448\omega_P S + 1.023\omega_P^2$

i.e. this factor is from $(S - P_1)(S - P_1^*)$

$P_{2\text{yields}}$: $S^2 + 0.130\omega_P S + 0.902\omega_P^2$

$P_{3\text{yields}}$: $S^2 + 0.203\omega_P S + 0.721\omega_P^2$

$P_{4\text{yields}}$: $S^2 + 0.255\omega_P S + 0.476\omega_P^2$

$P_{5\text{yields}}$: $S^2 + 0.283\omega_P S + 0.212\omega_P^2$

Now $T(S)$ is given by

$$T(S) = \frac{k\omega_P^{10}}{E2^9(S - P_1)(S - P_1^*)\ldots(S - P_5)(S - P_5^*)}$$

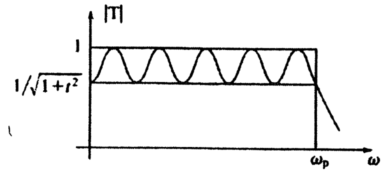
where the second order terms of the denominator are given above.

k is the dc gain

∴ we want the dc gain to be

$$k = \frac{1}{1 + \epsilon^2} = 0.8913$$

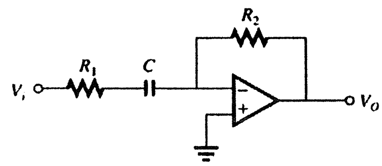
$\omega_P = 2\pi \times 3400$



11.16

$f_O = 100$ KHz $R_i(\infty) = 100$ kΩ

$|T(\infty)| = 1$



$R_i(\infty) = R_1 = 100$ kΩ

$|T(\infty)| = R_2/R_1 = 1$

$R_2 = R_1 = 100$ kΩ

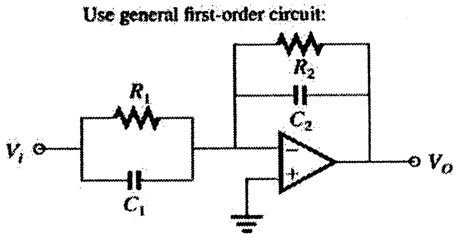
$CR_1 = 1/W_O$

$$C = \frac{1}{W_O R_1} = \frac{1}{2\pi\, 100 \times 10^3 \times 100 \times 10^3}$$

$= 15.9$ nF

11.17

Use general first-order circuit:



–Zero at 1 kHz; Pole at 100 kHz

$-|T(O)| = 1$; $R_i(O) = 1\ \text{k}\Omega$

Thus: $R_i(\text{DC}) = R_1 = 1\ \text{k}\Omega$

$T(\text{DC}) = -R_2/R_1 = -1$

$R_2 = R_1 = 1\ \text{k}\Omega$

For a pole at 100 kHz

$$C_2R_2 = \frac{1}{W_O} \Rightarrow C_2 = \frac{1}{2\pi f_O R_2}$$

$= 1.59$ nF

For the circuit $T(S) = \dfrac{a_1S + a_O}{S + W_O}$

Thus the Zero at $-a_O/a_1 = -2\pi\ 10^3$

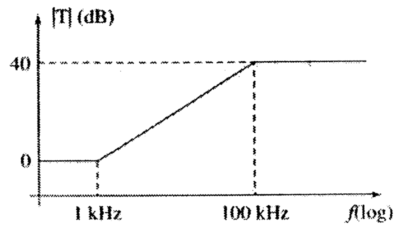
$C_1R_1 = a_1/a_O$

$$C_1 = \frac{1}{2\pi\ 10^3\ R_1}$$

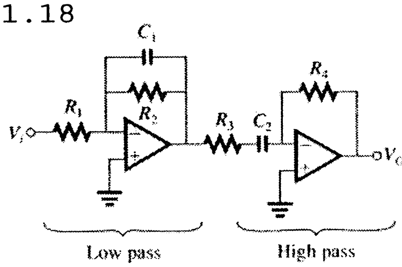
$= 159$ nF

High freq gain $= \dfrac{-C_1}{C_2} = -100$

$= 40$ dB



11.18



gain $= 10^{12/20} = 3.98 \approx 4$

want $R_i = R_1$ large

$\therefore R_1 = 100\ \text{k}\Omega$

Total gain $= A_{LP}A_{HP} = 4$

$A_{LP} = -R_2/R_1 \Rightarrow R_2 = -A_{LP}R_1$ and

$R_2 \leq 100\ \text{k}\Omega$

$\therefore$ make $A_{LP} = -1 A_{HP} = -4$

$R_2 = 100\text{k}\Omega$

$$R_2C_1 = \frac{1}{W_{O,\ LP}}$$

$$C_1 = \frac{1}{2\pi f_{O,\ LP}R_2} = \frac{1}{2\pi(10 \times 10^3)100 \times 10^3}$$

$= 0.159$ nF

$$A_{HP} = \left.\begin{matrix} -R_4/R_3 = -4 \\ R_4 = 4R_3 \end{matrix}\right\}\text{make}\ \begin{matrix} R_4 = 100\ \text{k}\Omega \\ R_3 = 25\ \text{k}\Omega \end{matrix}$$

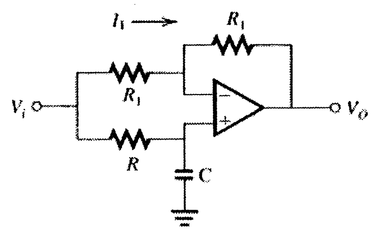
Now $R_3C_2 = 1/W_{O,\ HP}$

$$C_2 = \frac{1}{2\pi f_{O,\ HP}R_3}$$

$$= \frac{1}{2\pi(100 \times 10^3)25 \times 10^3}$$

$= 63.7$ nF

11.19



At +ve terminal

$$V_1 = \frac{1/SC}{1/SC + R_1}V_i$$

$$= \frac{1}{1 + S\tau}V_i\ \ \tau = R_C$$

V– = V+ due to virtual short between terminals

$$\therefore I_1 = \left(V_i - \frac{1}{1 + S\tau}V_i\right)\frac{1}{R_1}$$

$$V_O = V_2 - I_1R_1$$

$$= \frac{V_i}{1 + S\tau} - \left(V_i - \frac{V_i}{1 + S\tau}\right)\frac{R_1}{R_1}$$

$$\frac{V_O}{V_i} = \frac{1 - (1 + S\tau) + 1}{1 + S\tau} = \frac{1 - S\tau}{1 + S\tau}$$

$= \frac{\omega_O - S}{\omega_O + S} \quad \omega_O = \frac{1}{\tau}$

$= -\frac{S - \omega_O}{S + \omega_O} = T(S)$

$T(jW) = \frac{j\omega - \omega_O}{j\omega + \omega_O}$

$\phi(W) = 180° + \tan\left(\frac{\omega}{-\omega_O}\right) - \tan^{-1}\left(\frac{\omega}{\omega_O}\right)$

$= 360° - 2\tan^{-1}\left(\frac{\omega}{\omega_O}\right) \because \tan^{-1}\left(\frac{\omega}{\omega_O}\right)$

$= -2\tan^{-1}(\omega/\omega_O) \quad = 180 - \tan^{-1}\left(\frac{\omega}{\omega_O}\right)$

Now this equation can be rearranged:

$\frac{W}{W_O} = \tan(-\phi/2) \Leftarrow \omega_O = \frac{1}{2} = \frac{1}{R_C}$

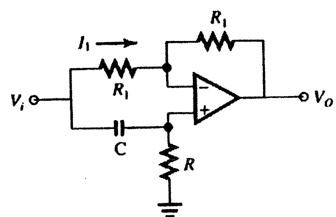
$R_C W = \tan(-\phi/2)$

$\therefore R = \frac{\tan(-\phi/2)}{CW} = 10^4 \tan(-\phi/2)$

$\phi = -30°, -60°, -90°, -120°, -150°$

$R = 2.68\ k\Omega,\ 5.77\ k\Omega,\ 10\ k\Omega,\ 17.32\ k\Omega,\ 37.32\ k\Omega$

11.20



$V+ = \frac{R}{R + 1/SC} V_i = \frac{S}{S + \omega_O} V_i$

Where $\omega_O = \frac{1}{R_C}$

$I_1 = \frac{V_i - (S/S + \omega_O)V_i}{R_1}$

$V_O = \frac{S}{S + \omega_O} V_i - I_1 R_1$

$= \frac{S}{S + \omega_O} V_i - V_i\left(1 - \frac{S}{S + \omega_O}\right)$

$\frac{V_O}{V_i} = \frac{2S - S - \omega_O}{S + \omega_O} = \frac{S - \omega_O}{S + \omega_O}$

Now :

$\phi(W) = \tan^{-1}\left(\frac{\omega}{-\omega_O}\right) - \tan^{-1}\left(\frac{\omega}{\omega_O}\right)$

Note $\tan^{-1}\frac{\omega}{-\omega_O} = 180° - \tan^{-1}\frac{\omega}{-\omega_O}$

$= 180 - \tan^{-1}\left(\frac{\omega}{\omega_O}\right) - \tan^{-1}\left(\frac{\omega}{\omega_O}\right)$

$= 180 - 2\tan^{-1}\left(\frac{\omega}{\omega_O}\right)$

Clearly $\phi(O) = 180°$ & $\phi(\omega \to \infty) = 0°$

11.21 Low pass $\omega_O = 10^3$ rad/s

$Q = 1$

DC gain = 1

$T(S) = \frac{a_O}{S^2 + S\frac{\omega_O}{Q} + \omega_O^2}$

$T(O) = a_O \omega_O^2 = 1$

$a_O = W_O^2 = 10^6$

$\therefore T(S) = \frac{10^6}{S^2 + 10^3 S + 10^6}$

$\omega_{max} = \omega_O\sqrt{1 - 1/2Q^2}$

$= \frac{\omega_O}{\sqrt{2}}$

$= 0.707$ rad/s

$|T_{max}| = \frac{|a_O|Q}{\omega_O^2\sqrt{1 - \frac{1}{4Q^2}}} \Leftarrow a_O = \omega_O^2$

$= \frac{|\omega_O^2|}{\omega_O^2\sqrt{3/4}}$

$= 2/\sqrt{3}$

$= 1.15$ V/V

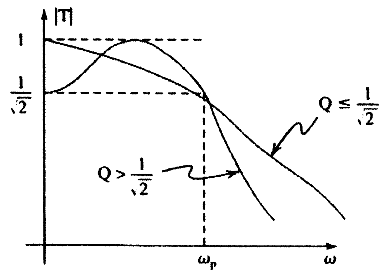
$= 1.21$ dB

11.22 $W_P = 1$ rad/s

$A_{max} = 3$ dB

$10^{-3/20} = 0.708 \cong \frac{1}{\sqrt{2}}$

There are many Q-values which may be used



$Q \le 1/\sqrt{2}$ - no peaking

$Q > 1/\sqrt{2}$ - peaking

Solution 1 $Q \le 1/\sqrt{2}$

For $Q = 1/\sqrt{2}$ the response is maximally flat.

Because this is desirable, use: $Q = \frac{1}{\sqrt{2}}$

$$T(S) = \frac{a_O}{S^2 + SW_O\sqrt{2} + W_O^2}$$

$$|T(O)| = \frac{a_O}{W_O^2} = 1$$

$$a_O = W_O^2$$

$$|T(j1)|^2 = \frac{W_O^2}{(W_O^2 - 1) + 2W_O^2} = \left(\frac{1}{\sqrt{2}}\right)^2$$

$W_O = 1$ rad/s

$$\therefore T_1(S) = \frac{1}{S^2 + \sqrt{2}S + 1}$$

Solution 2 $Q > 1/\sqrt{2}$

From the figure:

$$|T(O)| = 1/\sqrt{2} = \frac{a_O}{W_O^2}$$

$$\therefore a_O = \omega_O^2/\sqrt{2}$$

$$\text{Now}\,|T|_{max} = \frac{|a_O|}{W_O^2\sqrt{1 - 1/4Q^2}} = 1$$

$$\frac{Q}{\sqrt{2}\sqrt{1 - 1/4Q^2}} = 1$$

$$Q = \sqrt{2}\sqrt{1 - 1/4Q^2}$$

$$\therefore Q^2 = 2\left(1 - \frac{1}{4Q^2}\right)$$

$$= 2 - \frac{1}{2Q^2}$$

$$Q^4 - 2Q^2 + \frac{1}{2} = 0$$

Solving for Q^2 gives:

$$Q^2 = 1 \pm \sqrt{2}$$

ASIDE:

$\because Q > 1/\sqrt{2}$

$Q^2 > 1/2$

$4Q^2 > 2$

$$\frac{1}{4Q^2} < \frac{1}{2}$$

$$\therefore 1 - \frac{1}{4Q^2} > 1/2$$

$$\therefore \left|1 - \frac{1}{4Q^2}\right| = 1 - \frac{1}{4Q^2}$$

$\Rightarrow Q = 0.5412$ or 1.3066

$\because Q > \frac{1}{\sqrt{2}}$ use $Q = 1.3066$

Now at the passband edge

$$|T(j1)| = 1/\sqrt{2}$$

$$|T(j1)|^2 = \frac{(W_O^2/\sqrt{2})^2}{(W_O^2 - 1)^2 + \frac{W_O^2}{Q^2}} = \frac{1}{2}$$

$$\frac{W_O^4}{2} = \frac{1}{2}\left[W_O^4 - 2W_O^2 + 1 + \frac{W_O^2}{Q^2}\right]$$

$$W_O^2(2 - 1/Q^2) = 1$$

$$W_O = 0.841$$

$$\therefore T_2(S) = \frac{W_O^2/\sqrt{2}}{S^2 + W_O/QS + W_O^2}$$

$$= \frac{0.5}{S^2 + 0.644s + 0.707}$$

If $W_S = 2$

$|T_1(j2)| = 0.242$ $|T_2(j2)| = 0.1414$

$\therefore A_{min,1} = -12.3$ dB $A_{min,2} = -17$ dB

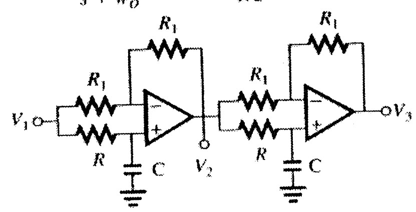
11.23

V_2 lags V_1 by 120°

V_3 lags V_2 by 120°

$w = 2\pi 60$ & $C = 1\ \mu F$

$$T(s) = \frac{s - w_O}{s + w_O} \qquad w_O = \frac{1}{RC}$$



$$\phi(\omega) = 180^\circ + \tan^{-1}\left(\frac{\omega}{-\omega_O}\right) - \tan^{-1}\left(\frac{\omega}{\omega_O}\right)$$

sub: $\tan\left(\frac{\omega}{-\omega_O}\right) = 180 - \tan^{-1}\left(\frac{\omega}{\omega_O}\right)$

$\Rightarrow \phi(\omega) = -2\tan(\omega/\omega_O)$

Now $\phi = -120^\circ$ at $\omega = 2\pi 60$

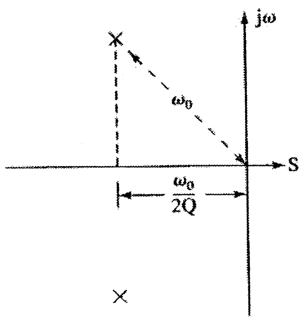
$-120^\circ = -2\tan(WRC)$

$-60 = -\tan^{-1}(2\pi 60 \times R \times 10^{-6})$

$R = 4.59\ \text{k}\Omega$

R_i can be arbitrarily chosen use $R_1 = 10\ \text{k}\Omega$

11.24



Natural Modes:

$$-\frac{1}{2} \pm j\frac{\sqrt{3}}{2}$$

$$w_O = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}\quad 51.0$$

$$\frac{w_O}{2Q} = \frac{1}{2} \Rightarrow \frac{\omega_O}{Q} = 1$$

$$T(s) = \frac{a_2 s^2}{s^2 + s\frac{\omega_O}{Q} + \omega_O^2} = \frac{a_2 s^2}{s^2 + s + 1}$$

$$|T(j\infty)| = a_2 = 1$$

$$\therefore T(s) = \frac{s^2}{s^2 + s + 1}$$

11.25

For a 2nd-order bandpass

$$T(S) = \frac{a_1 S}{S^2 + S\frac{\omega_O}{Q} + \omega_O^2}$$

$$T(j\omega) = \frac{j\omega a_1}{(\omega_O^2 - w^2) + \frac{j\omega\omega_O}{Q}}$$

$$|T(j\omega)| = \frac{aW}{\left[(\omega_O^2 - \omega^2)^2 + \frac{\omega^2\omega_O^2}{Q^2}\right]^{1/2}}$$

Part (a):

$$|T(j\omega_1)| = |T(j\omega_2)|$$

$$\frac{aW_1}{\sqrt{(\omega_O^2 - \omega_1^2)^2 + \left(\frac{\omega_1\omega_O}{Q}\right)^2}} = \frac{aW_2}{\sqrt{(\omega_O^2 - \omega_2^2)^2 + \left(\frac{\omega_2\omega_O}{Q}\right)^2}}$$

$$\omega_1^2\left[(\omega_O^2 - \omega_2^2)^2 + \left(\frac{\omega_2\omega_O}{Q}\right)^2\right] = \omega_2^2\left[(\omega_O^2 - \omega_1^2)^2 + \left(\frac{\omega_1\omega_O}{Q}\right)^2\right]$$

$$\omega_1^2(\omega_O^4 - 2\omega_O^2\omega_2^2 + \omega_2^4) = \omega_2^2(\omega_O^4 - 2\omega_O^2\omega_1^2 + \omega_1^4)$$

$$\omega_1^2\omega_O^4 + \omega_1^2\omega_2^4 = \omega_2^2\omega_O^4 + \omega_2^2\omega_1^4$$

$$\omega_O^4(\omega_1^2 - \omega_2^2) = \omega_2^2\omega_1^4 - \omega_1^2\omega_2^4$$

$$\omega_O^4(\omega_1^2 - \omega_2^2) = \omega_2^2\omega_1^2(\omega_1^2 - \omega_2^2)$$

$$\omega_O^4 = \omega_1^2\omega_2^2$$

$\omega_O^2 = \omega_1\omega_2$ Q.E.D.

(b) For Fig. 16.4:

$\omega_{P1} = 8100$ rad/s

$\omega_{P2} = 10000$ rad/s

$A_{max} = 1\text{dB}$

$\omega_O^2 = (8100)(10000)$

$\omega_O = 9000$ rad/s

$$|T(jw_{P1})| = |T(j\omega_{P2})| = 10^{-1/20} = 0.8913$$

$$|T(j\omega_O)| = \frac{\omega_O a_1}{\sqrt{(\omega_O^2 - \omega_O^2)^2 + \left(\frac{\omega_O^2}{Q}\right)^2}} = 1$$

$$\Rightarrow \frac{\omega_O a_1}{\omega_O^2/Q} = 1$$

$$\therefore \frac{Qa_1}{\omega_O} = 1 \Rightarrow a_1 = \frac{\omega_O}{Q}$$

$$|T(j\omega_{P1})|^2 = |T(j0.9\omega_O)|^2 = 0.8913^2$$

$$\frac{(\omega_O/Q)^2(0.9\omega_O)^2}{(\omega_O^2-(0.9\omega_O)^2)^2+\left(\frac{0.9\omega_O}{Q}\right)^2} = 0.8913^2$$

$$\left(\frac{\omega_O}{Q}(.9\omega_O)\right)^2$$

$$= 0.8913^2\left[(\omega_O^2-(0.9\omega_O)^2)^2+\left(\frac{0.9\omega_O}{Q}\right)^2\right]$$

$$\frac{0.81\omega_O^4}{Q^2} = 0.8913^2\left[W_O^4(1-0.81)^2+\frac{0.81\omega_O^4}{Q^2}\right]$$

$$\frac{0.81\omega_O^4}{Q^2}(1-0.8913^2) = 0.8913^2\omega_O^4\times(1-0.81)^2$$

SUB $\omega_O = 9000$ gives

$Q = 2.41$

Now $Q_1 = \frac{\omega_O}{Q} = 0.415\omega_O$

$$\therefore T(S) = \frac{0.415\omega_O S}{S^2+0.415\omega_O S+\omega_O^2}$$

IF $WS_1 = 3000$ rad/s

$$|T(j3000)| = \frac{0.415\omega_O(3000)}{\sqrt{(W_O^2-3000^2)^2+(\omega_O 3000\times .415)^2}}$$

$= 0.1537$

$\therefore A_{min} = -20\log(0.1537)$

$= 16.3\text{dB}$

Now ω_{S1} and ω_{S2} are geometrically symmetrical about ω_O :

$\omega_{S1}\omega_{S2} = \omega_O^2$

$$\omega_{S2} = \frac{9000^2}{3000}$$

$= 27000$ rad/s

11.26

$$Q = \frac{\omega_O}{BW\sqrt{10^{A/10}-1}}$$

$\omega_O = 2\pi(60)$ $BW = 2\pi 6$ $A = 20\text{dB}$

$= 1.005$

$$T(S) = a_2\frac{S^2+\omega_O^2}{S^2+S\frac{\omega_O}{Q}+\omega_O^2}$$

$$|T(0)| = \frac{a_2\omega_O^2}{\omega_O^2} = 1 \leftarrow \text{DC Gain}$$

$Q_2 = 1$

$$T(S) = \frac{S^2+(2\pi 60)^2}{S^2+S\frac{2\pi 60}{1.005}+(2\pi 60)^2}$$

$$T(S) = \frac{S+1.421\times 10^5}{S^2+375.1s+1.421\times 10^5}$$

11.27

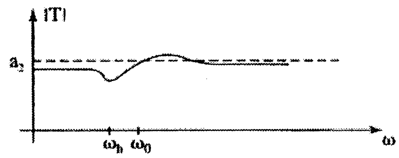
FOR ALL PASS:

$$T(S) = Q_2\frac{S^2-SW_O/Q+\omega_O^2}{S^2+SW_O/Q+\omega_O^2}$$

If Zero frequency < pole frequency

$$T(S) = Q_2\frac{S^2-SW_n/Q+\omega_n^2}{S^2+SW_O/Q+\omega_O^2}\quad \omega_n<\omega_O$$

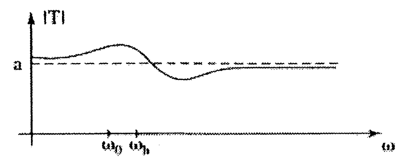
At DC: $|T| = a\frac{\omega_n^2}{\omega_O^2}$ where $\frac{\omega_n^2}{\omega_O^2}<1$



If Zero frequency > pole frequency

then $\omega_n > \omega_O$

At DC: $|T| = Q_2\frac{\omega_n^2}{\omega_O^2}$ where $\frac{\omega_n^2}{\omega_O^2}>1$



11.28

$$T(S) = \frac{S^2-SW_O/Q^1+\omega_O^2}{S^2+SW_O/Q_O+\omega_O^2}a$$

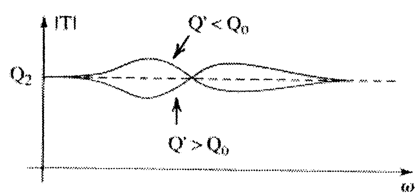
Zero Q < pole $Q \Rightarrow Q^1 < Q_O$

At $W = W_O$:

$$|T| = \frac{a_2\omega_O^2/Q'}{\omega_O^2/Q_O} = \frac{a_2 Q_O}{Q'} > a_2$$

If $Q^1 > Q_O$

$$|T(j\omega_O)| = \frac{Q_2 Q_O}{Q_1} < Q_2$$



11.29

$W_O = \frac{1}{\sqrt{LC}}$

If $L' = 1.01L$

$W_O' = (1.01LC)^{-1/2}$

$= 0.9950\frac{1}{\sqrt{LC}}$

$= 0.9950W_O$

$\therefore \Delta W_O = -0.5\%$

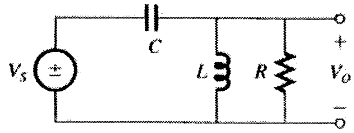
If $C^1 = 1.01C$

$W_O' = 0.9950W_O$

$\Delta W_O' = -0.5\%$

Changing R has no effect on W_O

11.30



Use voltage divider rule:

$$V_O = \frac{Z_{R\parallel L}}{Z_{R\parallel L} + Z_C}V_S$$

$$\frac{V_O}{V_S} = \frac{\left(\frac{1}{R} + \frac{1}{SL}\right)^{-1}}{\left(\frac{1}{R} + \frac{1}{SL}\right)^{-1} + \frac{1}{SC}}$$

$$= \frac{SC}{\left(\frac{1}{SL} + \frac{1}{R}\right) + SC}$$

$$\therefore T(S) = \frac{V_O(S)}{V_S(S)} = \frac{S^2}{S^2 + S/RC + \frac{1}{LC}}$$

11.31

Low Pass: $\omega_0 = 10^5$, $C = 0.1\mu F$

$Q = 1/\sqrt{2}$

$Q = \omega_0 CR$

$R = \frac{Q}{\omega_0 C}$

$= \frac{1}{\sqrt{2} \times 10^5 \times 0.1 \times 10^{-6}}$

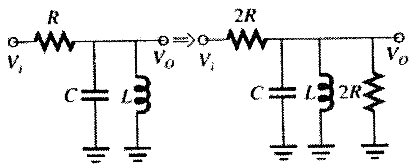
$= \underline{\underline{70.7\Omega}}$

$\omega_0 = 1/\sqrt{LC}$

$L = \frac{1}{\omega_0^2 C}$

$= \underline{\underline{1mH}}$

11.32



$\omega_O = \frac{1}{\sqrt{LC}}$

$Q = \omega w_O CR$

$A_{mid} = 1$

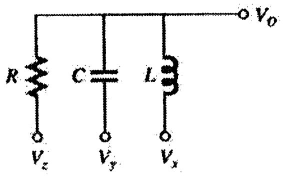
$\omega_O = \frac{1}{\sqrt{LC}}$

$Q = \omega_O C(2R \parallel 2R)$

$= \omega_O CR$

$A_{mid} = \frac{2R}{2R + 2R} = \frac{1}{2}$

11.33



$V_O/V_z|_{V_y = V_x = 0} = T_{BP}(S)$

$V_O/V_y|_{V_x = V_z = 0} = T_{HP}(S)$

$\frac{V_O}{V_x}\Big|_{V_y = V_z} = T_{LP}(S)$

Using superposition

$$V_O = \frac{V_O}{V_x}V_x + \frac{V_O}{V_y}V_y + \frac{V_O}{V_z}V_z$$

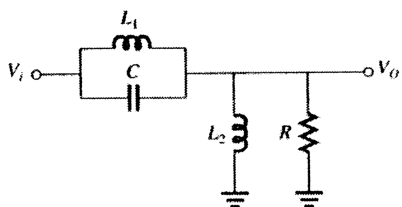
$$= T_{LP}V_x + T_{HP}V_y + T_{BP}V_z$$

$$\frac{\frac{1}{LC}V_x + S^2V_y + \frac{S}{RC}V_z}{S^2 + S/RC + \frac{1}{LC}}$$

$$\therefore V_O = V_x\frac{y_{LC}}{S^2 + S/R_C + 1/L_C}$$

$$V_y\frac{S^2}{S^2 + S/R_C + 1/L_C} + V_z\frac{S/R_C}{S^2 + S/R_C + 1/L_C}$$

11.34



From Eq 16.46

$$T(S) = \frac{S^2 + 1/L_1C}{S^2 + S(1/CR) + \frac{1}{(4\| L_2)C}}$$

Required notch $\omega_n^2 = \frac{1}{L_1C} = (0.9\omega_O)^2$

but:

$$\omega_O^2 = \frac{1}{(L_1\| L_2)C} \text{ where}$$

$$L_1\| L_2 = \frac{1}{4^{-1} + L_2^{-1}} = \frac{L_1L_2}{L_1 + L_2}$$

$$= \frac{L_1 + L_2}{L_1L_2C}$$

$$= \frac{L_1 + L_2}{L_2}(.9\,\omega_O)^2$$

$$1 = \left(\frac{L_1}{L_2} + 1\right)0.9^2$$

$$\therefore L_1/L_2 = \frac{1}{0.9^2} - 1 = 0.2346$$

For $\omega << \omega_o$:-

$$|T| \cong \frac{1/L_1C}{1/(L_1\| L_2)C} = \frac{L_2}{L_1 + L_2}$$

i.e. inductors dominate!

For $\omega >> \omega_o$, L_1 & L_2 are "open" C is shorted

$|T| \cong 1$

11.35

$L = C_4 R_1 R_3 R_5/R_2$

Choose $R_1 = R_2 = R_3 = R_5 = 10\text{ k}\Omega$

$\therefore L = C_4 \times 10^8$

For:

$L = 10\text{ H} = C_4 \times 10^8 \Rightarrow C_4 = 100\text{ nF}$

$L = 1\text{ H} \Rightarrow C_4 = 10\text{ nF}$

$L = 0.1\text{ H} \Rightarrow C_4 = 1\text{ nF}$

11.36

$A_{max} = 10\log(1 + \varepsilon^2) = 3\text{ dB}$

$\therefore \varepsilon = 0.998 \cong 1$

$\omega_o = \omega_p = 10^4$

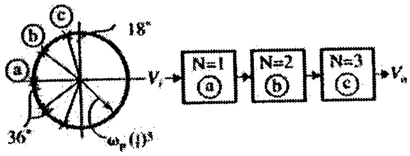
For circuit Q use fig 16.13 (a)

DC Grain

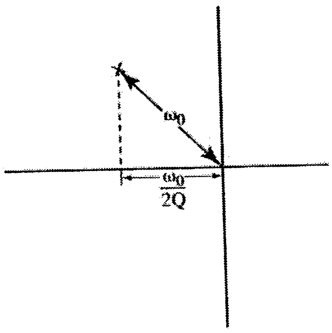
$= 1 = R_2/R_1 \Rightarrow R_1 = R_2 = 10\text{ k}\Omega$

$CR_2 = 1/\omega_o \Rightarrow C = 1/R_2\,\omega_o = \dfrac{1}{10^4\ 10^4}$

$= 10\text{ nf}$



For circuit (b)



$\omega_o = 10^{4\ \text{rad/s}}$

$\dfrac{\omega_o}{2Q} = \omega_o \cos 36^\circ$

$Q = \dfrac{1}{2\,\text{Cos}\,36^\circ} = 0.618$

$$T(S) = \frac{\dfrac{k\,R_2}{C_4\,C_6\,R_1\,R_3\,R_5}}{S^2 + S/C_6\,R_6 + \dfrac{R_2}{C_4\,C_6\,R_1\,R_3\,R_5}}$$

$\omega_o^2 = \dfrac{R_2}{C_4\,C_6\,R_1\,R_3\,R_5}$

Let $= R_1 = R_3 = R_5 = R_2 = R$

$C_4 = C_6 = C$

$\omega_o^2 = \dfrac{1}{R^2\,C^2}$

USE $C_4 = C_6 = 100$ nF

$\therefore R = \dfrac{1}{\omega_o C} \Rightarrow R_1 = R_2 = R_3 = R_5 = 1\text{ k}\Omega$

Now using:

$\dfrac{\omega_o}{Q} = \dfrac{1}{C_6\,R_6}$ & $Q = 0.618$

$R_6 = \dfrac{Q}{C_6\,\omega_o} = 618\ \Omega$

For circuit (c) use

$\omega_o = 10^4$ which is the same as for circuit (b).

$\therefore C_4 = C_6 = 100$ nF

$R_1 = R_2 = R_3 = R_5 = 1\text{ k}\Omega$

Now: $Q = \dfrac{1}{2\cos 72^\circ} = 1.618$

$R_6 = Q/\omega_o C_6 = 1.618\text{ k}\Omega$

11.37

$f_O = 4\text{kHz}$ $\quad f_N = 5\text{kHz}$ $\quad Q = 10$

now $C_4 = 10\text{nF}$ and $k = 1 \equiv$ dc gain

$W_O = [C_4(C_{61} + C_{62})R_1R_3R_5/R_2]^{-1/2}$

$C_{61} + C_{62} = C_6$

Choose $C_6 = C_4 = 10\text{nF}$

$R_1 = R_3 = R_5 = R_2 = R$

$\therefore \omega_O = (C_4C_6R^2)^{-1/2}$

$R = \dfrac{1}{\omega_O C_4}$

$\Rightarrow R_1 = R_3 = R_5 = R_2 = 3.979\text{ k}\Omega$

$\omega_n = (C_4C_6R^2)^{-1/2}$

$C_{61} = \dfrac{1}{\omega_n^2 R^2 C_4} \Rightarrow C_{61} = 6.4\text{ nF}$

$C_{62} = 3.6\text{ nF}$

$Q = R_6\sqrt{\dfrac{C_{61}+C_{62}}{C_4}\cdot\dfrac{R_2}{R_1R_3R_5}}$

$= R_6\sqrt{\dfrac{1}{R_1^2}} = R_6/R_1 \Rightarrow R_6 = 39.79\text{ k}\Omega$

11.38

$\theta = 180^\circ$ at f_o!

$\therefore$ Use $f_O = 1\text{kHz}$ $\quad Q = 1$

$W_O^2 = \dfrac{R_2}{C_4C_6R_1R_3R_5}$

Let $C = C_4 = C_6 = 1\text{nF}$

$R_1 = R_3R_5 = R_2 = R$

$= 1/C^2R^2$

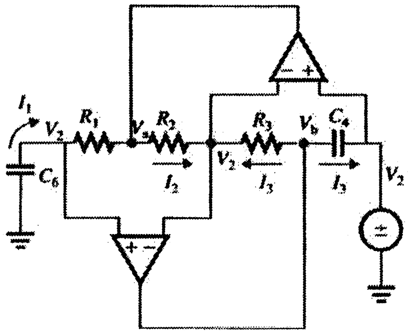
$R = \dfrac{1}{W_O C} = 159.16\text{ k}\Omega = R_1 = R_3 = R_5 = R_2$

$\dfrac{W_O}{Q} = \dfrac{1}{R_6C_6} \Rightarrow R_6 = \dfrac{Q}{C_6\omega_O}$

$= \dfrac{1}{10^{-9}2\pi 10^3}$

$\therefore R_6 = 159.16\text{k}\Omega$

11.39



Because of virtual short at opamp input terminals all modes are at V_2!

$I_1 = -SC_6V_2$

Since no current goes into the opamp input terminals we have:

$$V_a = V_2 - I_1R_1$$
$$= V_2(1 + SC_6R_1)$$
$$I_2 = (V_a - V_2)\frac{1}{R_2}$$
$$= \frac{SC_6R_1}{R_2}$$
$$V_b = V_2 - I_2R_3$$
$$= V_2 - \frac{SC_6R_1R_3}{R_2}$$
$$I_3 = (V_b - V_2)SC_4$$
$$= -\frac{S^2C_4C_6R_1R_3}{R_2}V_2$$

Now the voltage source sees an input impedance given by:

$$Z_{in} = -\frac{V_2}{I_3} = \frac{R_2}{S^2C_4C_6R_1R_3}$$

As required

for $S = j\omega \Rightarrow S^2 = -\omega^2$

$$Z_{in}(j\omega) = \frac{-R_2}{C_4C_6R_1R_3}\cdot\frac{1}{\omega^2}$$
$$= -R(W)$$

ie. A PURE NEGATIVE RESISTANCE!

11.40

$$T(s) = k\,\frac{C_{61}}{C_{61}+C_{62}}\cdot\frac{S^2 + \dfrac{R_2}{C_4\,C_{61}\,R_1\,R_3\,R_5}}{S^2 + S/C_6\,R_6 + \dfrac{R_2}{C_4(C_{61}+C_{62})\,R_1\,R_3\,R_5}}$$

At DC $\Rightarrow s = 0$

$$T(o) = k\,\frac{C_{61}}{C_{61}+C_{62}}\cdot\frac{R_2/C_4\,C_6\,R_1\,R_3\,R_5}{R_2/C_4(C_{61}+C_{62})\;R_1\,R_3\,R_5}$$

$\Rightarrow T(o) = k \triangleq$ DC Gain!

Note that $C_{61} + C_{62}$ is the total capacitance across R_6

$$\omega_o^2 = R_2/C_4\,C_6\,R_1\,R_3\,R_5$$
$$\Rightarrow \omega_n = \sqrt{\frac{R_2}{C_4\,C_{61}\,R_1\,R_3\,R_5}}$$
$$\therefore C_6 = C_{61} + C_{62}$$
$$\frac{\omega_n^2}{\omega_o^2} = \frac{R_2/C_4\,C_{61}\,R_1\,R_3\,R_5}{R_2/C_4(C_{61}+C_{62})\;R_1\,R_3\,R_5}$$
$$= \frac{C_{61}}{C_{61}+C_{62}}$$
$$\frac{\omega_n^2}{\omega_o^2} = \frac{C_{61}}{C_6}$$
$$\therefore C_{61} = C_6\left(\frac{\omega_n}{\omega_o}\right)^2 = C\left(\frac{\omega_n}{\omega_o}\right)^2$$

Clearly from T(S) above:

$$\omega_n^2 = R_2/C_4\,C_6\,R_1\,R_3\,R_5$$
$$\Rightarrow \omega_o = \sqrt{\frac{R_2}{C_4\,(C_{61}+C_{62})\,R_1\,R_3\,R_5}}$$

11.41

$T(s)$

$$= k\frac{S^2 + (R_2/C_4\,C_6\,R_1\,R_3\,R_{51})}{S^2 + S/C_6\,R_6 + \left(\dfrac{R_2}{C_4\,C_6\,R_1\,R_3}\right)\left(\dfrac{1}{R_{51}} + \dfrac{1}{R_{52}}\right)}$$

clearly: $w_n = \sqrt{\dfrac{R_2}{C_4\,C_6\,R_1\,R_3\,R_{51}}}$

$$w_o = \sqrt{\frac{R_2}{C_4\,C_6\,R_1\,R_3}\left(\frac{1}{R_{51}} + \frac{1}{R_{52}}\right)}$$

At high frequencies $S \to \infty$

$T(\infty) = k \triangleq$ high freq gain

Observe that the equivalent resistance at the tue terminal of A_1 is:

$$\frac{1}{R_5} = \frac{1}{R_{51}} + \frac{1}{R_{52}} \text{ AND}$$

For the resonator (table 16.1)

$$R_5 = 1/\omega_o c \Rightarrow \frac{1}{R_5} = w_o c$$

$$\frac{\omega_o^2}{\omega_n^2} = \frac{R_2/C_4\,C_6\,R_1\,R_3\,R_5}{R_2/C_4\,C_6\,R_1\,R_3\,R_{51}} \Rightarrow R_{51} = R_5\frac{\omega_o^2}{\omega_n^2}$$

Now $\frac{1}{R_5} = \frac{1}{R_5 \frac{\omega_o^2}{\omega_n^2}} + \frac{1}{R_{52}}$

$$\frac{1}{R_{52}} = \frac{1}{R_5}\left[1 - \frac{\omega_n^2}{\omega_o^2}\right]$$

$$R_{52} = \frac{R_5}{1 - \omega_n^2 / \omega_o^2}$$

11.42

$T(s)$

$$= \frac{0.4508\,(S^2 + 1.6996)}{(S + 0.7294)\,(S^2 + 0.2786s + 1.0504)}$$

Part (a) Replace s with s/ω_p

$T(s)$

$$= \frac{0.4508\,(S^2/\omega_p^2 + 1.6996)}{\left(\frac{S}{\omega_p} + 0.7294\right)\left(\frac{S^2}{\omega_p^2} + \frac{0.27865}{\omega_p} + 1.0504\right)}$$

$T(s)$

$$= \frac{0.4508\,\omega_p\,(S^2 + 1.6996\,\omega_p^2)}{(S + 0.7294\,\omega_p)\,(S^2 + 0.2786\,\omega_p S + 1.0504\,\omega_p^2)}$$

Sub $\omega_p = 10^4$ rad / s

$T(s)$

$$= \frac{4508\,(S^2 + 1.6996 \times 10^8)}{(S + 7294)\,(S^2 + 2786s + 1.0504 \times 10^8)}$$

Part (b)

First decompose T(s) into 1st and 2nd- order sectionswith unity DC gain!

$T_1(s) = \frac{k_1}{S + 7294}$ $T_1(o) = \frac{k_1}{7294} = 1$

$\Rightarrow k_1 = 7294$

Now $k_1\,k_2 = 4508 \Rightarrow k_2 = 0.6180$

$$\therefore T_2(s) = \frac{0.6180\,(S^2 + 1.6996 \times 10^8)}{S^2 + 2786s + 1.0504 \times 10^8}$$

As a check:

$$T_2(o) = \frac{0.6180\,(1.6996 \times 10^8)}{1.0504 \times 10^8} = 1.000$$

AS EXPECTED!

$\therefore T(s) = T_1(s) \cdot T_2(s)$

$w_o = 7294$ rad / s DC Gain = 1

Let $C = 10$ nF

$R_1 = R_2 = \frac{1}{\omega_o C} \Rightarrow R_1 = R_2 = 13.71\ \text{k}\Omega$

11.43

$R_L = R_H = R_B/Q \Rightarrow R_B = QR_H$

$R_L = R_H$

$$\frac{V_O}{V_i} = -K\frac{\frac{R_F}{R_H}S^2 - S\left(\frac{R_F}{R_B}\right)\omega_o + \left(\frac{R_F}{R_L}\right)\omega_o^2}{S^2 + S\frac{\omega_o}{Q} + \omega_o^2}$$

$$= -K\,\frac{R_F}{R_H}\,\frac{S^2 - \frac{\omega_o}{Q}S + \omega_o^2}{S^2 + \frac{\omega_o}{Q}S + \omega_o^2}$$

Flat Gain = $-K\ R_F / R_H$

Part (b)

$-\ \omega_o = 10^4$ rad / s $Q = 2$ Flat Gain = 10

Choose C = 10 nF $\Rightarrow R = \frac{1}{\omega_o C} = 10\ \text{k}\Omega$

Choose $R_f = R_1 = 10\ \text{k}\Omega$

$\frac{R_3}{R_2} = 2Q - 1 = 3 \Rightarrow R_2 = 10\ \text{k}\Omega$

$R_3 = 30\ \text{k}\Omega$

Now $K = 2 - 1/Q = 1.5$

$\therefore$ Flat Gain $= 10 = (1.5)\,\frac{R_F}{R_H}$

$\therefore \frac{R_H}{R_F} = 0.15$

Choose $R_F = 100\ \text{k}\Omega$

$R_H = R_L = 15\ \text{k}\Omega$

$R_B = QR_H = 30\ \text{k}\Omega$

11.44

Note ω_n does not depend on R or C From

$$\frac{R_H}{R_L} = \left(\frac{\omega_n}{\omega_o}\right)^2$$

$\therefore \omega_n = \omega_o\sqrt{\frac{R_H}{R_L}}$ Nominally $R_H = R_L \pm 1\%$

Thus:

$\omega_n' = \omega_o\sqrt{\frac{1.01}{0.99}}$ $\omega_n'' = \omega_o\sqrt{\frac{0.99}{1.01}}$

$= 1.01\,\omega_o$ $= 0.99\,\omega_o$

$\therefore \omega_n$ can deviate from ω_o by $\pm 1\%$

11.45

Use Tow Thomas to realize a LPN

$\omega_o = 10^4 \quad \omega_n = 1.2\omega_o \quad Q = 10$

DC Gain = 1

C = 10 nF r = 20 kΩ

$$R = \frac{1}{\omega_o C} = 10\ \text{k}\Omega$$

From 16.16 (e):

$$\text{DC Gain} = a_2 \frac{\omega_n^2}{\omega_o^2} = 1$$

$$a_2 \frac{1.2^2\,\omega_o^2}{\omega_o^2} = 1$$

$$a_2 \frac{1}{1.2^2} = \text{HF} \cdot \text{Gain}$$

$$C_1 = Ca_2 = \frac{10 \times 10^{-9}}{1.2^2} = 6.94\ \text{nF}$$

$$R_2 = \frac{R(\omega_o/\omega_n)^2}{\text{HF Gain}} = R\left(\frac{1}{1.2}\right)^2 \times (1.2)^2$$

$$= R = 10\ \text{k}\Omega$$

$$R_1 = R_3 = \infty$$

11.46

For all pass:

$T(S)$

$$-g^2\left(\frac{C_1}{C}\right) + s\frac{1}{C}\left(\frac{1}{R_1} - \frac{r}{RR_3}\right) + \frac{1}{C^2RR_2}$$

$$\omega_z^2 = \frac{1}{C^2RR_2} \cdot \frac{C}{C_1} \Rightarrow \omega_Z = \frac{1}{C\sqrt{RR_2}} \cdot \sqrt{\frac{C}{C_1}}$$

$$Q_Z = \frac{\omega_Z}{\frac{1}{C}\left(\frac{1}{R_1} - \frac{r}{RR_3}\right)\frac{C}{C_1}}$$

$$Q_Z = \frac{\sqrt{\frac{1}{C^2RR_2}\frac{C}{C_1}}}{\frac{1}{C}\left(\frac{1}{R_1} - \frac{r}{RR_3}\right)\left(\frac{C}{C_1}\right)}$$

$$\frac{1}{\sqrt{RR_2}\left(\frac{1}{R_1} - \frac{r}{RR_3}\right)\sqrt{\frac{C}{C_1}}}$$

For All Pass $R_1 \Rightarrow \infty$

To adjust Q_3, trim r or R_2 (independent of W_z !)

Now $\omega_o = \frac{1}{CR}$ so do not trim R or C!

Note if we trim R_2 or C_1 to adjust W_z This will also affect Q_z. So the options are:

For W_z: (a) trim R_2 and (r or R_3) to maintain the value of Q_z

OR

(b) trim C_1, and r or R_3

Prefer not to trim a capacitor so use (a) !

11.47

$T(s)$

$$= \frac{0.4508\ (S^2 + 1.6996)}{(S + 0.7294)\ (S^2 + 0.27865 + 1.0504)}$$

Part (a) Replace s with s/ω_p

$\omega_p = 10^4\text{rad/s}$.

$T(s)$

$$= \frac{0.4508\ (S^2/\omega_p^2 + 1.6996)}{\left(\frac{S}{\omega_p} + 0.7294\right)\left(\frac{S^2}{\omega_p^2} + \frac{0.2786s}{\omega_p} + 1.0504\right)}$$

$T(s)$

$$= \frac{0.4508\ \omega_p\ (S^2 + 1.6996\ \omega_p^2)}{(S + 0.7294\ \omega_p)\ (S^2 + 0.27865\ \omega_p S + 1.0504\ \omega_p^2)}$$

$$= \frac{4508\ (S^2 + 1.6996 \times 10^8)}{(S + 7294)\ (S^2 + 27865 + 1.0504 \times 10^8)}$$

For FIRST ORDER SECTION

$\omega_o = 7294 \qquad$ DC Gain = 1

Choose C = 10 nF

$$R_1 = R_2 = \frac{1}{\omega_o C} \Rightarrow R_1 = R_2 = 13.71\ \text{k}\Omega$$

For SECOND ORDER SECTION

$$\omega_n^2 = 1.6996 \times 10^8 \Rightarrow \omega_n = 13.037 \times 10^3$$

$$\omega_o^2 = 1.0504 \times 10^8 \Rightarrow \omega_o = 10.249 \times 10^3$$

$$\frac{\omega_o}{Q} = 2786 \Rightarrow Q = 3.6787$$

DC gain = 1

For Tow Thomas LPN

Choose C = 10 nF

$$R = \frac{1}{\omega_o C} = \frac{1}{10.249 \times 10^3 \times 10 \times 10^{-9}}$$

$$= 9.757\ \text{k}\Omega$$

Choose r = 20 kΩ

(e):

$$T(o) = a_Z \frac{\omega_n^2}{\omega_o^2} = 1 \Rightarrow a_Z \frac{\omega_o^2}{\omega_n^2} = 0.618$$

$\therefore$ HF gain $= a_Z = 0.618$

$$C_1 = C \times \text{HF gain} \Rightarrow C_1 = 0.618\ \text{nF}$$

$$R_2 = R\ (\omega_o/\omega_n)^2$$

$$R_2 = 0.618\ R \Rightarrow R_2 = 6.03\ \text{k}\Omega$$

$$R_1 = R_3 = \infty \qquad QR = 35.89\ \text{k}\Omega$$

11.48

$$t(S) = \frac{S^2 + S\left(\frac{1}{C_1} + \frac{1}{C_2}\right)\frac{1}{R_3} + \frac{1}{C_1C_2R_3R_4}}{S^2 + S\left(\frac{1}{C_1R_3} + \frac{1}{C_1R_4} + \frac{1}{C_2R_3}\right) + \frac{1}{C_1C_2R_3R_4}}$$

But $C_1 = C_2 = C$ & $R_3 = R_4 = R$, $RC = \tau$

$$\therefore t(S) = \frac{S^2 + S^2/RC + 1/R^2C^2}{S^2 + S\,3/RC + \frac{1}{R^2C^2}}$$

$$= \frac{S^2 + S^2/\tau + 1/\tau^2}{S^2 + S\frac{3}{\tau} + 1/\tau^2}$$

Zeros defined by $w_z = 1/\tau$

$Q_z = \frac{1}{2}$

$\Rightarrow$ Double Root at $S = -1/\tau$

Poles of t(S) are given by the quadratic formula:

$$S = \frac{-3}{2\tau} \pm \frac{\sqrt{5}}{2\tau} = \frac{-3 \pm \sqrt{5}}{2\tau}$$

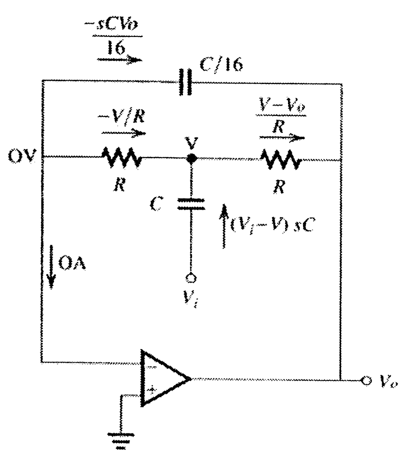
i.e. two roots on the negative real axis

If the network is placed in the negative feedback path of an ideal amplifier ($A = \infty$) then the poles are given by the zeros of $t(S)$:

Closed loop poles:

$S = -1/\tau$ (multiplicity = 2)

11.49



Note first $\frac{-sCV_O}{16} = \frac{-V}{R}$

$$V = -\frac{sCRV_O}{16}$$

ΣI at V

$$-\frac{V}{R} + sC(V_i - V) - \frac{V - V_O}{R} = 0$$

$$\frac{sCV_O\not{R}}{\not{R}\;16} + sCV_i + \frac{s^2C^2RV_O}{16} + \frac{sCV_O}{16} + \frac{V_O}{R} = 0$$

mult by: 16 R and let $RC = \tau$

$$s\tau V_o + 16\tau V_i s + s^2\tau^2 V_o + s\tau V_o + 16V_o = 0$$

$$V_o[s^2\tau^2 + s \times 2^\tau + 16] = -16 s^\tau V_i$$

$$\therefore \frac{V_O}{V_i} = -\frac{16s\tau}{s^2\tau^2 + 2\tau s + 16}$$

$$\therefore T(s) = \frac{s\,16/RC}{s^2 + s^2/RC + 16/R^2C^2}$$

Let $\omega_O^2 = \frac{16}{(RC)^2} \Rightarrow \omega_O = \frac{4}{RC}$

$$\frac{\omega_O}{Q} = \frac{2}{RC} \Rightarrow Q = \frac{RC\omega_O}{2} = 2$$

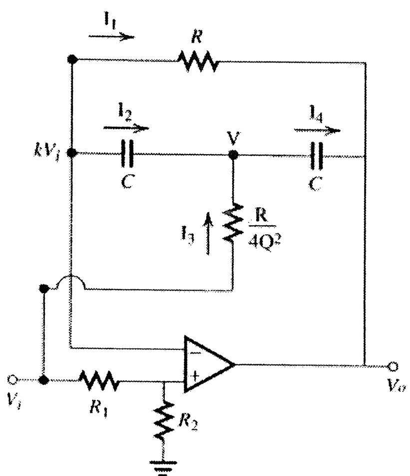
$$\frac{\omega_O}{Q} = \frac{2}{RC} \Rightarrow Q = \frac{RC\omega_O}{2} = 2$$

$$\therefore \frac{V_O}{V_i} = \frac{-4\omega_O s}{s^2 + s\frac{\omega_O}{Q} + \omega_O^2}$$

$|T|_{s=0} = 0$
$|T|_{s \to \infty} = 0$ } Bandpass

$|T(j\omega_O)| = 4/1/2 = 8\frac{\text{V}}{\text{V}}$ CENTER FREQ GAIN

11.50



$RC = 2Q/w_o$

$$k = \frac{R_2}{R_1 + R_2}$$

$V_+ = V_{-ve} = kV_i$ due to virtual short

$I_1 = -I_2$

$$\frac{kV_i - V_O}{R} = \frac{V - kV_i}{1}(SC)$$

$$V = \frac{1}{SCR}(kV_i - V_O + SCRKV_i)$$

ΣI at $V = 0$

$I_2 + I_3 - I_4 = 0$

$$SC(kV_i - V) + \frac{4Q^2}{R}(V_i - V) - SC(V - V_o) = 0$$

$$SC\left(kV_i - \frac{kV_i}{SCR} 1 \frac{V_O}{SCR} - kV_i\right) + \frac{4Q^2}{R}\left(V_i - \frac{kV_i}{SCR} + \frac{V_O}{SCR} - kV_i\right)$$

$$-SC\left(\frac{kV_i}{SCR} - \frac{V_O}{SCR} + kV_i - V_O\right) = O$$

$$\Rightarrow -\frac{kV_i}{R} + \frac{V_O}{R} + \frac{4Q^2}{R}\left(V_i - \frac{kV_i}{SCR} + \frac{V_O}{SCR} - kV_i\right)$$

$$-\frac{SC}{R}\left(\frac{kV_i}{SC} - \frac{V_O}{SC} + kRV_i - V_OR\right) = O$$

$$\Rightarrow \text{SUB CR} = \frac{2Q}{\omega_O} \ \& \ R = \frac{2Q}{C\omega_O}$$

$$-kV_i + V_o + 4Q^2V_i - \frac{{}^2 4Q^2kV_i\omega_O}{S2Q} + \frac{V_O\omega_O 4^2Q^2}{S2Q} - 4Q^2kV_i$$

$$-kV_i + V_O - Sk2Q/w_OV_i + SV_O\frac{2Q}{w_O} = O$$

$$V_O\left[1 + \frac{2Q\omega_O}{S} + 1 + \frac{2QS}{\omega_O}\right] = V_i\left[k - 4Q^2 + \frac{2kQ\omega_O}{S} + 4Q^2k + k + \frac{2kQS}{w_O}\right]$$

$$\Rightarrow V_O\left[S^2\frac{2Q}{\omega_O} + 2S + 2Q\omega_O\right] = V_i\left[S^2\frac{2kQ}{\omega_O} + S(4Q^2k - 4Q^2 + 2K) + 2kQw_O\right]$$

$$\Rightarrow \frac{V_O}{V_i} = \frac{S^2\frac{2kQ}{\omega_O} + S(4Q^2k - 4Q^2 + 2k) + 2kQ\omega_O}{S^2\frac{2Q}{\omega_O} + 2S + 2Q\omega_O}$$

$$= k\frac{S^2 + S\frac{\omega_O}{Q}\left(2Q^2 - \frac{2Q^2}{k} + 1\right) + \omega_O^2}{S^2 + S\frac{\omega_O}{Q} + \omega_O^2}$$

Recall $k = \frac{R_2}{R_1 + R_2}$ and $\frac{1}{k} = 1 + \frac{R_1}{R_2}$

$$\Rightarrow \frac{V_O}{V_i} = \left(\frac{R_L}{R_1 + R_2}\right)\frac{S^2 + S\frac{\omega_O}{Q}\left(1 - \frac{R_L}{R_2}.2Q^2\right) + \omega_O^2}{S^2 + S\frac{\omega_O}{Q} + \omega_O^2}$$

$$\therefore T(S) = \frac{R_2}{R_1 + R_2}$$

$$\frac{S^2 + S\frac{\omega_O}{Q}\left(1 - \frac{2Q^2R_1}{R_2}\right) + \omega_O^2}{S^2 + S\frac{\omega_O}{Q} + \omega_O^2}$$

For All Pass

we want $T(S)\alpha\frac{S^2 + \frac{\omega_O}{Q}(-1)S + \omega_O^2}{S^2 + S\frac{\omega_O}{Q} + \omega_O^2}$

$$\Rightarrow 1 - \frac{2Q^2R_1}{R_2} = -1$$

$$2Q^2\frac{R_1}{R_2} = 2$$

$$\frac{R_1}{R_2} = \frac{1}{Q^2}$$

$$\therefore \frac{R_2}{R_1} = Q^2 \ \& \ \frac{R_2}{R_1 + R_2} = \frac{R_2/R_1}{1 + R_2/R_1} = \frac{Q^2}{1 + Q^2}$$

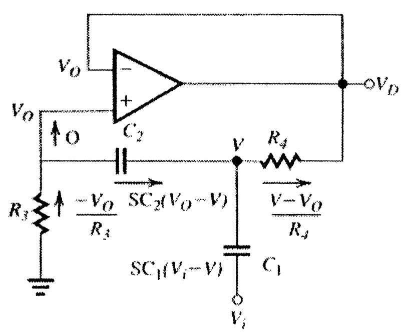
For Notch:

$$1 - 2Q^2\frac{R_1}{R_2} = O$$

$$\frac{R_1}{R_2} = \frac{1}{2Q^2}$$

$$\frac{R_2}{R_1} = 2Q^2 \ \& \ \frac{R_2}{R_1 + R_2} = \frac{2Q^2}{1 + 2Q^2}$$

11.51



∴ No current can't flow into the terminal

$$-\frac{V_O}{R_3} = SC_2(V_O - V)$$

$$V = V_O\left(1 + \frac{1}{SC_2R_3}\right)$$

$$\Sigma I \; @ \; V = O$$

$$-\frac{V_O}{R_3} + \frac{V_i - V}{1}SC_1 = \frac{V - V_O}{R_4}$$

$$V_O\left[-\frac{1}{R_3} + \frac{1}{R_4}\right] + V\left[-SC_1 - \frac{1}{R_4}\right] = -SC_1V_i$$

$$V_o[R_4 - R_3] + V[SC_1R_3R_4 + R_3] = V_iSC_1R_3R_4$$

$$V_O(R_4 - R_3) + V_O\left(1 + \frac{1}{SC_2R_3}\right)(SC_1R_3R_4 + R_3)$$

$$= SC_1R_3R_4V_i$$

$$V_O\left(R_4 - \cancel{R}_3 + SC_1R_3R_4 + \cancel{R}_3 + \frac{C_1}{C_2}R_4 + \frac{1}{SC_2}\right)$$

$$= SC_1R_3R_4V_i$$

$$V_o(S^2C_1C_2R_3R_4 + SC_1R_4 + SC_2R_4 + 1)$$

$$= S^2C_1R_3R_4C_2V_i$$

$$\therefore \frac{V_O}{V_i} = \frac{S^2C_1C_2R_3R_4}{S^2C_1C_2R_3R_4 + SC_1R_4 + SC_2R_4 + 1}$$

$$= \frac{S^2}{S^2 + S\left(\frac{1}{C_2R_3} + \frac{1}{C_1R_3}\right) + \frac{1}{C_1C_2R_3R_4}}$$

Note $\left.\begin{matrix}|T(O)| = 0 \\ |T(\infty)| = 1\end{matrix}\right\}$ ∴ High Pass, High Freq Gain $= 1\frac{V}{V}$

3 dB freq = 10^3 rad/s, $Q = \frac{1}{\sqrt{2}}$ for max flat.

$\therefore w_O = 10^3 \frac{rad}{s}$ $C_1 = C_2 = 10$ nF

clearly $w_O^2 = \frac{1}{C_1C_2R_3R_4}$ and

$$\frac{w_O}{Q} = \frac{1}{C_2R_3} + \frac{1}{C_1R_3} = \frac{C_1 + C_2}{C_1C_2R_3}$$

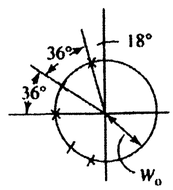
$$= \frac{2C}{C^2R_3} = \frac{2}{CR_3} = \sqrt{2} \times 10^3$$

$$R_3 = \frac{2}{10 \times 10^{-9} \times 10^3 \times \sqrt{2}}$$

$R_3 = 141.4$ kΩ

$$R_4 = \frac{1}{w_O^2C_1C_2R_3} \Rightarrow R_4 = 70.7 \text{ k}\Omega$$

11.52



$A_{max} = 3$ dB

$$\epsilon = (10^{3/10} - 1)^{-1/2} \cong 1$$

$$\omega_O = \omega_p\left(\frac{1}{\epsilon}\right)^{1/N} = \omega_p = 2\pi 5000 = 10^4\pi$$

$$Q_1 = \frac{1}{2\text{Cos}36} = 0.618$$

$$Q_2 = \frac{1}{2\text{Cos}72} = 1.618$$

For first order section:

$\omega_o = 10^4\pi$ dc gain = 1

From 16.13 (a)

$R_1 = R_2 = 10$ kΩ

$$C = \frac{1}{\omega_O R_2} = \frac{1}{10^4\pi 10^4} = 3.18 \text{ nF}$$

Second order section $Q = 0.618$:

From 16.34 (c) $m = 4Q^2 = 1.528$

$RC = \frac{2Q}{\omega_O}$ let $R_1 = R_2 = 10$ kΩ

$$C = \frac{2Q}{\omega_O R} \Rightarrow C_4 = C = 3.93 \text{ nF}$$

$$C_3 = \frac{C}{m} = 2.57 \text{ nF}$$

Second Order Section $Q = 1.618$:

$$C = \frac{2Q}{\omega_O R} \quad m = 4Q^2 = 10.472$$

$= 10.3$ nF $\Rightarrow R_1 = R_2 = 10$ kΩ

$C_4 = C = 10.3$ nF

$$C_3 = \frac{C}{m} = 0.984 \text{ nF}$$

11.53

For a bandpass filter

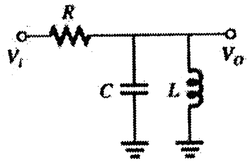
$$t(s) = \frac{\omega_O / Qs}{s^2 + s\omega_O / Q + \omega_O^2}$$

center freq. gain = 1

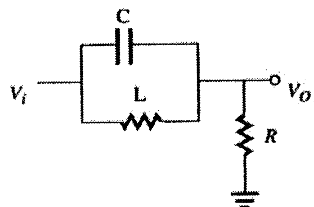
complementary transfer function :

$$t' = 1 - t$$

$$= \frac{s^2 + \omega_O^2}{s^2 + s\omega_O / Q + \omega_O^2} \equiv \text{NOTCH!}$$



⇒ INTERCHANGE V_i & gnd to get:



11.54

$$T(S) = \frac{s/RC}{s^2 + s/RC + 1/LC}$$

$$\omega_O = \frac{1}{\sqrt{LC}} \quad Q = R\sqrt{\frac{C}{L}}$$

For w_o

$$\frac{\partial \omega_O}{\partial L} = \frac{\partial (LC)^{-1/2}}{\partial L} = -\frac{1}{2}L^{-3/2}C^{-1/2} = \frac{\omega_O}{2L}$$

$$\frac{\partial \omega_O}{\partial C} = -\frac{\omega_O}{2C}$$

$$\frac{\partial \omega_O}{\partial R} = 0$$

$$\therefore S_L^{\omega_O} = \frac{\partial \omega_O}{\partial L}\frac{L}{\omega_O} = -1/2$$

$$S_C^{\omega_O} = \frac{\partial \omega_O}{\partial C} \times \frac{C}{\omega_O} = -1/2$$

$$S_R^{\omega_O} = \frac{\partial \omega_O}{\partial R}\frac{R}{\omega_O} = 0$$

For Q

$$\frac{\partial Q}{\partial L} = \frac{R\sqrt{C}}{L\sqrt{L}}\left(\frac{-1}{2}\right) = \frac{-Q}{2L}$$

$$\frac{\partial Q}{\partial C} = \frac{1}{2}\frac{R}{\sqrt{LC}} = \frac{1}{2}\frac{R\sqrt{C}}{C\sqrt{L}} = \frac{Q}{2C}$$

$$\frac{\partial Q}{\partial R} = \sqrt{C/L} = \frac{R}{R}\sqrt{C/L} = Q/R$$

$$S_L^Q = -\frac{Q}{2C} \times \frac{L}{Q} = -\frac{1}{2}$$

$$S_C^Q = \frac{Q}{2C} \times \frac{C}{Q} = \frac{1}{2}$$

$$S_R^Q = -\frac{Q}{R} \cdot \frac{R}{Q} = 1$$

11.55

$$S^2 + S\frac{\omega_O}{Q}\left[1 + \frac{2Q^2}{A+1}\right] + \omega_O^2 = 0$$

Now the actual w_O and Q are given by:

$$\omega_{O,a} = \omega_O \text{ and } Q_a = \frac{Q}{1 + \frac{2Q^2}{(A+1)}}$$

$$S_A^{\omega_{O,a}} = 0$$

$$S_A^{Q_a} = \frac{A}{A+1}\frac{2Q^2(A+1)}{1 + 2Q^2/(A+1)}$$

$$\therefore S_A^{Q_a} \cong \frac{2Q^2}{A}$$

11.56

$R_1 = R_2$,

$$W_O = \frac{1}{\sqrt{C_3C_4R_1R_2}}$$

$$Q = \frac{1}{\sqrt{C_3C_4R_1R_2\left(\frac{1}{C_4}\right)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)}}$$

$$\frac{\partial \omega_O}{\partial C_3} = \frac{-1}{2C_3\sqrt{C_3C_4R_1R_2}}$$

$$\frac{\partial \omega_O}{\partial C_3} = \frac{-1}{2C_3\sqrt{C_3C_4R_1R_2}} = \frac{-\omega_O}{2C_3}$$

$$S_{C_3}^{\omega_O} = \frac{\partial \omega_O}{\partial C_3}\frac{C_3}{\omega_O} = -\frac{1}{2}$$

clearly $S_{C_3}^{\omega_O} = S_{C_4}^{\omega_O} = S_{R_1}^{\omega_O} = S_{R_2}^{\omega_O} = -\frac{1}{2}$

$$\frac{\partial Q}{\partial C_3} = \frac{-1}{2C_3\sqrt{C_3C_4R_1R_2\left(\frac{1}{C_4}\right)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)}} = \frac{-Q}{2C_3}$$

$$\therefore S_{C_3}^Q = -\frac{1}{2}$$

$$\frac{\partial Q}{\partial C_4} = \frac{Q}{2C_4} \Rightarrow S_{C_4}^Q = +\frac{1}{2}$$

$$\frac{\partial Q}{\partial R_1} = \frac{1/\sqrt{R_1} - \sqrt{R_1}/R_2}{R_1\left(\frac{1}{\sqrt{R_1}} + \sqrt{\frac{R_1}{R_2}}\right)} \cdot \frac{Q}{2}$$

$$= \frac{\sqrt{R_2/R_1} - \sqrt{R_1/R_2}}{R_1\left(\sqrt{\frac{R_2}{R_1}} + \sqrt{\frac{R_1}{R_2}}\right)} \cdot \frac{Q}{2}$$

$$\therefore S_{R_1}^{Q} = \frac{\sqrt{R_2/R_1} - \sqrt{R_1/R_2}}{\sqrt{R_2/R_1} + \sqrt{R_1/R_2}}$$

If $R_1 = R_2 \Rightarrow S_{R_1}^{Q} = 0$

$S_{R_2}^{Q} = 0$

11.57

$$\omega_O = \frac{1}{\sqrt{C_4 C_6 R_1 R_3 R_5 / R_2}}$$

$$Q = R_6 \sqrt{\frac{C_6}{C_4} \frac{R_2}{R_1 R_3 R_5}}$$

$$\frac{\partial \omega_O}{\partial C_4} = \frac{-\omega_O}{2C_4}$$

$$\therefore S_{C_4}^{\omega_o} = \frac{-\omega_O}{2C_4} \times \frac{C_4}{\omega_O} = -\frac{1}{2}$$

Similarly $S_{C_6}^{\omega_o} = S_{R_1}^{\omega_o} = S_{R_3}^{\omega_o} = S_{R_5}^{\omega_o} = \frac{1}{2}$

$$\frac{\partial \omega}{\partial R_2} = \frac{\omega_O}{2R_2} \Rightarrow S_{R_2}^{\omega_o} = \frac{1}{2}$$

Now for Q:

$$\frac{\partial Q}{\partial R_6} = \frac{Q}{R_6} \Rightarrow S_{R_6}^{Q} = \frac{\partial Q}{\partial R_6}\frac{R_6}{Q} = +1$$

$$\frac{\partial Q}{\partial C_6} = \frac{Q}{2C_6} \Rightarrow S_{C_6}^{Q} = S_{R_2}^{Q} = +\frac{1}{2}$$

$$\frac{\partial Q}{\partial C_4} = -\frac{Q}{2C_4} \Rightarrow S_{C_4}^{Q} = S_{R_1,R_2,R_3}^{Q} = -\frac{1}{2}$$

11.58

change transferred $\Rightarrow Q = CV$

$= 10^{-12}(1)$

$= 1$ pC

For $f_O = 100$ kHz, average current is given by:

$$I_{AVE} = \frac{Q}{T} = 1\text{ pC} \times \frac{1}{100 \times 10^3}$$

$= 0.1\ \mu$A

For each clock cycle, the output will change by the same amount as the change in voltage across C_2!

$$\therefore \Delta V = Q/C_2 = \frac{1\text{ pC}}{10\text{ pF}} = 0.1\text{ V}$$

For $\Delta V = 0.1$ V for each clock cycle, the amplifier will saturate in

$$\text{vcycles} = \frac{10\text{ V}}{0.1\text{ V}} = 100\text{ cycles}$$

$$\text{slope} = \frac{\Delta V}{\Delta t} = \frac{10\text{ V}}{(100\text{ cycles})(1/100 \times 10^3)}$$

$$= 10^4 \frac{\text{V}}{\text{s}}$$

11.59

$f_C = 400$ kHz $f_O = 10$ kHz $Q = 20$

$C_1 = C_2 = 20\text{ pF} = C$

$C_3 = C_4 = \omega_O T_C C$

$$= 2\pi(10^4)\frac{1}{400 \times 10^3} 20 \times 10^{-12}$$

$= 3.14$ pF

$$C_5 = \frac{\omega_O T_C C}{Q}$$

$$= \frac{C_3}{Q} = 0.157\text{ pF}$$

$$C_6 = \frac{\omega_O T_C C}{Q} \times \text{centre frequency gain}$$

$= 0.157$ pF

Note that the clock frequency has doubled. Hence the period, T_C, is halved. Therefore, for the same integratity capacitors, the resistors (switched capacitors) will change by the factor of z. so compensate for this by changing the switched caps by a factor of 1/2.

11.60

for $Q = 40$

$f_C = 200$ kHz $f_O = 10$ kHz

$C_1 = C_2 = 20\text{ pF} = C$

$C3 = C4 = \omega_O T_C C$

$$= 2\pi(10^4)\left(\frac{1}{200 \times 10^3}\right) 20 \times 10^{-12}$$

$= 6.28$ pF

$$C_5 = \frac{\omega_O T_C C}{Q} = \frac{C_3}{Q} = 0.157\text{ pF}$$

$$C_6 = \frac{\omega_O T_C C}{Q} = C_5 = 0.157\text{ pF}$$

11.61

$\omega_O = 10^4, Q = 1/\sqrt{2}, f_C = 100$ kHz

$$\text{DC gain} \Rightarrow \frac{R_4}{R_6} \Rightarrow \frac{C_6}{C_4} = 1$$

$C_1 = C_2 = 10$ pF

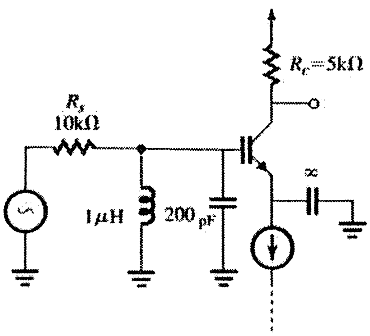
$C_3 = C_4 = C_6 = \omega_O T_C C$

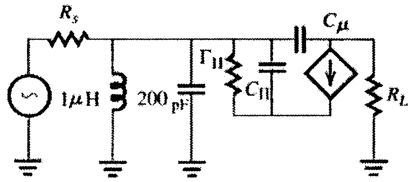
$$= 10^4\left(\frac{1}{100 \times 10^3}\right) 10 \times 10^{-12}$$

$= 1\text{ pF}$

$C_5 = C_4/Q = 1.41\text{ pF}$

11.62





$r_e = 25\ \Omega,\ C_\mu = 1\text{ pF},\ C_\pi = 10\text{ pF},$

$\beta = 200$

$r_\pi = (\beta + 1)\sqrt{2} = 5.025\text{ k}\Omega$

From base to collector

$\frac{V_C}{V_b} = -\frac{\beta}{\beta+1}\cdot\frac{R_2}{r_e} = -199 = \text{k}$

Total capacitance at base

$C_T = C_\pi + 200\text{ p} + C_\mu(1 - \text{k})$ Miller Effect

$= 10 + 200\text{ p} + 1(1 + 199)$

$= 410\text{ pF}$

$\therefore \omega_O = \frac{1}{\sqrt{LC}}$

$= \frac{1}{\sqrt{10^{-6} \times 410 \times 10^{-12}}}$

$= 49.4 \times 10^6\text{ rad/s}$

centre frequency gain $= \frac{r_\pi}{R_s + r_\pi}\cdot\text{k}$

$= \frac{5.025}{10 + 5.025} \times -199$

$= -66.6\text{ V/V}$

$BW = \frac{1}{RC}$

$= \frac{1}{(R_S \| r_\pi)410\text{ pF}}$

$= 729 \times 10^3\text{ rad/s}$

$Q = \frac{\omega_O}{BW}$

$= 49.4/0.7293$

$= 67.7$

11.63

$Q_O = \frac{R_p}{\omega_O L} \Rightarrow R_p = Q_O\omega_O L$

$= 200(2\pi 10^6)(10 \times 10^{-6})$

$= 12.57\text{ k}\Omega$

$\omega_O = \frac{1}{\sqrt{LC}} \Rightarrow C = \frac{1}{\omega_O^2 L}$

$= \frac{1}{(2\pi 10^6)^2\ 10 \times 10^{-6}}$

$= 2.533\text{ nF}$

$B = \frac{1}{RC}$

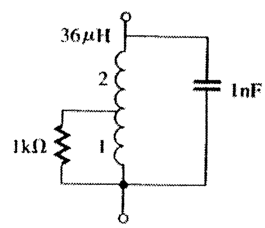
$R_r = \frac{1}{(2\pi \times 10 \times 10^3)(2.533 \times 10^{-9})}$

$= 6.283\text{ k}\Omega$

$\therefore \frac{1}{R_1 + R_p + R_r}$

$\Rightarrow R_1 = 12.57\text{ k}\Omega$ ie. $R_1 \| R_p = R_r$

11.64



$f_O = \frac{1}{2\pi\sqrt{LC}}$

$= (2\pi(36 \times 10^{-6})10^{-9})^{-1}$

$= 838.8\text{ kHz}$

$R_p = n^2 R$

$= 9\ (1\text{ k}\Omega)$

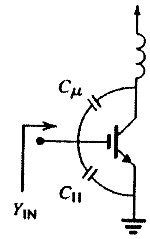
$= 9\text{ k}\Omega$

$Q = R_p\omega_O L$

$$= \frac{9 \times 10^3}{2\pi\, 838.8 \times 10^3 \times 36 \times 10^{-6}}$$

$= 47.4$

11.65



for $\omega C_\mu << \frac{1}{WL}$

$\therefore \omega^2 << \frac{1}{LC_\mu}$

ie well below resonance

$\therefore$ gain $= -g_m(jwL)$

$$\therefore y_{in} = \frac{1}{\sqrt{r_\pi}} + j\omega C_\pi + j\omega C_\mu(1 + g_m j\omega L)$$

$$= \left(\frac{1}{r_\pi} - \omega^2 g_m C_\mu\right) + j\omega(C_\pi + C_\mu)$$

AS REQUIRED!

11.66

$$T(S) = \frac{a_1 s}{s^2 + s\frac{\omega_O}{Q} + \omega_O^2}$$

$$T(j\omega) = \frac{ja_1\omega}{\omega_O^2 - \omega^2 + \frac{j\omega\omega_O}{Q}}$$

$$T(j\omega_O) = \frac{ja_1\omega_O}{j\omega_O^2/Q} = \frac{a_1 Q}{\omega_O}$$

$$|T(j\omega)| = a_1\omega\left[(\omega_O^2 - \omega^2)^2 + \left(\frac{\omega\omega_O}{Q}\right)^2\right]^{-\frac{1}{2}}$$

$$= \frac{a_1\omega Q/\omega\omega_O}{\sqrt{1 + Q^2\left(\frac{\omega_O^2 - \omega^2}{\omega_O\omega}\right)^2}}$$

Now $\omega = \omega_O + \delta\omega$, $\frac{\delta\omega}{\omega_O} << 1$

and $\omega^2 \simeq \omega_O^2\left(1 + \frac{2\delta\omega}{\omega_O}\right)$

so $\omega_O^2 - \omega^2 = -2\delta\omega\omega_O$

$$\therefore |T(j\omega)| \cong \frac{a_1 Q/\omega_O}{\sqrt{1 + Q^2\left(\frac{2\delta\omega}{\omega}\right)^2}}$$

for $Q >> 1: Q^2\left(\frac{2\delta\omega}{\omega}\right)^2 \cong Q^2\left(\frac{2\delta\omega}{\omega_O}\right)$

$\because \omega \approx \omega_O$!

$$\Rightarrow |T(j\omega)| \cong \frac{|T(j\omega_O)|}{\sqrt{1 + Q^2\left(\frac{2\delta\omega}{\omega_O}\right)^2}}$$

$$= \frac{|T(j\omega_O)|}{\sqrt{1 + 4Q^2\left(\frac{\partial\omega}{\omega_O}\right)^2}}$$

For N bandpass sections, synchronously tuned in cascade, half power is given by:

$$\left(\frac{1}{\sqrt{1 + 4Q^2\left(\frac{\delta\omega}{\omega_O}\right)^2}}\right)^N = \frac{1}{\sqrt{2}}$$

$$\left(1 + 4Q^2\left(\frac{\delta\omega}{\omega_O}\right)^2\right)^N = 2$$

$$4Q^2\left(\frac{\delta\omega}{\omega_O}\right)^2 = 2^{1/N} - 1$$

$$\delta w = \frac{\omega_O}{2Q}\sqrt{2^{1/N} - 1}$$

$\therefore$ Bandwidth:

$$B = 2\delta\omega = \frac{\omega_O}{Q}\sqrt{2^{1/N} - 1}$$

11.67

For first order lowpass:

$$T(S) = \frac{\omega_O}{S + \omega_O} \quad |T(jw)| = \frac{\omega_O}{\sqrt{\omega^2 + \omega_O^2}}$$

for a bandpass response arount w_O with

$$\omega_O' = \frac{\omega_O}{2Q}:$$

$$|T(j\omega)| \cong \frac{\omega_O/2Q}{(\delta\omega)^2 + \left(\frac{\omega_O}{2Q}\right)^2}$$

$$= \frac{\omega_O/2Q}{\frac{\omega_O}{2Q}\sqrt{\left(\frac{2Q}{\omega_O}\right)^2(\partial W)^2 + 1}}$$

$$= \frac{1}{\sqrt{1 + 4Q^2(\delta\omega/w_O)^2}}$$

Now at $\omega = \omega_O$ or $\delta\omega = 0$

$|r(j\omega_O)| = 1$, then

$$T(j\omega) \approx \frac{|T(j\omega_O)|}{\sqrt{1 + 4Q^2\left(\frac{\delta\omega}{\omega_O}\right)^2}}$$

Part (b)

For N synchronously tuned sections in cascade;

3 dB bandwidth is given by:

$$(|T|/|T_D|)^N = \frac{1}{\sqrt{2}}$$

$$(|T|/|T_D|)^2 = \frac{1}{2^{1/N}} \text{ OR}$$

$$1 + 4Q^2\left(\frac{\delta\omega}{\omega_O}\right)^2 = 2^{1/N} \text{ OR}$$

$$2\delta\omega = \frac{\omega_O}{Q}\sqrt{2^{1/N} - 1} \qquad (16.110)$$

Thus: $|T(j\omega)|_{overall} = |T(j\omega)|^N$

$$= \frac{|T(j\omega)|_{overall}}{\left[1 + 4Q^2\left(\frac{\delta\omega}{\omega_O}\right)^2\right]^{N/2}}$$

NOTE

$$Q = \frac{\omega_O}{Q}\sqrt{2^{1/N} - 1}$$

$$= \frac{|T(j\omega_O)|_{overall}}{\left[1 + 4\frac{\omega_O^2}{B^2}(2^{1/N} - 1)\left(\frac{\delta\omega}{\omega_O}\right)^2\right]^{\frac{N}{2}}}$$

$$= \frac{|T(j\omega_O)|_{overall}}{\left[1 + 4(2^{1/N} - 1)\left(\frac{\delta\omega}{B}\right)^2\right]^{\frac{N}{2}}}$$

Part (c)(i)

for bandwidth = $2B$, ie $\delta w = \pm B$

$$Att = -20\log(1 + (2^{1/N} - 1)(1))^{-N/2}$$

$$= -10N \log(1 + 2^{2+1/N} - 4)$$

$$= 10N \log(2^{2+1/N} - 3)$$

N	1	2	3	4	2
Att(dB)	6.70	8.49	9.28	9.79	10.13

Part (ii)

3 dB bandwidth $\delta\omega = \pm B/2$

30 dB bandwidth $\frac{\delta\omega}{B} = x$

$$-30 = -20\frac{N}{2}\log(1 + 4(2^{1/N} - 1)x^2)$$

$$3 = N\log(1 + 4(2^{1/N} - 1)x^2)$$

$$x = \left[\frac{10^{3/N} - 1}{4(2^{1/N} - 1)}\right]^{1/2}$$

Ratio of 30 dB to 3 dB

$$BW = \frac{2Bx}{B} = 2x$$

N	1	2	3	4	5
Ratio	31.6	8.6	5.7		4.5

11.68

(a) For the narrowband approximation variation of Ω around O is equivalent to sw around w_O. Thus, a low-pass maximally flat filter of bandwidth $B/2$ and order N for which

$|T| = [(1 + \Omega/B/2)^{2N}]^{\frac{-1}{2}}$ is transformed to a band-pass maximally flat filter of bandwidth $B/2$ and order 2N, and centre frequency ω_O for which:

$$|T| = \left(1 + \left(\frac{\delta\omega}{B/2}\right)^{2N}\right)^{-1/2}$$

(b) For bandwidth $2B$, $\delta\omega = B$ &

$$|T| = \left(1 + \left(\frac{B}{B/2}\right)^{2N}\right)^{-1/2}$$

$$= (1 + 2^{2N})^{-1/2} \text{ thus:}$$

N	1	2	3	4	5
$\lvert T\rvert$	0.447	0.242	0.124	0.062	0.031
$\lvert T\rvert_{dB}$	−6.99	−16.3	−18.1	−24.1	−30.1

For 30 dB bandwidth,

$$-30 = 20\log x \Rightarrow x = 10^{-3/2}$$

$$= \frac{1}{31.6}$$

$$\therefore 1 + \left(\frac{\delta\omega}{B/2}\right)^{2N} = (31.6)^2$$

$$\left(\frac{\delta\omega}{B/2}\right)^{2N} = 999 - 1 = 998$$

Now the ratio of 30 dB to 3 dB bandwidth is

$$\text{ratio} = \frac{2\delta\omega}{B} = \frac{\delta\omega}{B/2} = 998^{\frac{1}{2N}}$$

N	1	2	3	4	5
ratio	31.6	5.62	3.16	2.37	1.99

11.69

$A_{max} = 3\text{ dB} \Rightarrow \epsilon = \sqrt{10^{A_{max}/10} - 1} \approx 1$

Poles of lowpass prototype are given by

Poles: $-\omega_p, \omega_p(-1/2 \pm j\sqrt{3}/2)$

Make $w_p = B/2$

$$\Rightarrow \text{poles:}\left\{\frac{-B}{2}, \frac{+B}{2}\left(-\frac{1}{2} \pm j\frac{\sqrt{3}}{2}\right)\right\}$$

Using the low-pass to bandpass transformation:

Poles of the bandpass filter:

$$\frac{-B}{2} \pm j\omega_O$$

$$\frac{-B}{4} \pm j\left(\frac{\sqrt{3}}{4}B + \omega_O\right) \text{ and}$$

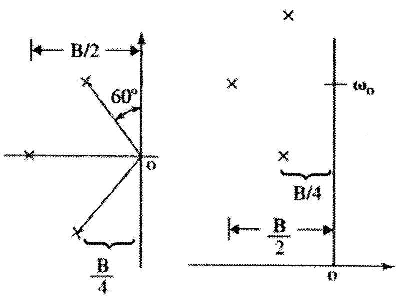
$$\frac{-B}{4} \pm j\left(\frac{\sqrt{3}}{4}B - \omega_O\right)$$

For the three circuits:

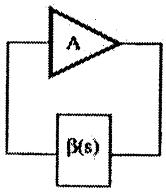
(1) $\omega_{O1} = \omega_O \; B_1 = B \; Q_1 = \omega_O/B$

(2) $\omega_{O2} \cong \frac{\sqrt{3}}{4}B + \omega_O \; B_2 = \frac{B}{2} \; Q_2 \cong \frac{2\omega_O}{B}$

(3) $\omega_{O3} \cong \frac{\sqrt{3}}{4}B - \omega_O \; B_3 = \frac{B}{2} \; Q_3 \cong \frac{2\omega_O}{B}$



12.1



$A = A_O > 0$

$$\beta(s) = \frac{K\frac{\omega_O}{Q}S}{S^2 + s\frac{\omega_O}{Q} + \omega_O^2}$$

(a) for oscillations $1 - A\beta(s) = 0$

$$A_O K\frac{\omega_O}{Q}S = S^2 + s\frac{\omega_O}{Q} + \omega_O^2$$

$$\omega_O^2 - \omega^2 = j\omega\left(\frac{\omega_O}{Q}\right)(A_O K - 1)$$

at the freq. of oscillation, both Real & imaginary parts are O.

$\therefore \omega = \omega_O$ & $A_O K = 1$

(b)

$$L(j\omega) \triangleq A\beta(j\omega) = \frac{AK\frac{\omega_O}{Q}j\omega}{(\omega_O^2 - \omega^2) + j\omega\left(\frac{\omega_O}{Q}\right)}$$

$$\therefore \phi(\omega) = 90^\circ - \tan^{-1}\left(\frac{\omega\omega_O/Q}{\omega_O^2 - \omega^2}\right)$$

$$\text{Now } \frac{\partial}{\partial x}\tan^{-1}v = \frac{1}{1+v^2}\cdot\frac{\partial u}{\partial x}$$

$$\therefore \frac{\partial\phi}{\partial\omega} = \frac{1}{1 + \left(\frac{\omega\omega_O/Q}{\omega_O^2 - \omega^2}\right)^2}\cdot\frac{\partial}{\partial\omega}\left(\frac{\omega\omega_O/Q}{\omega_O^2 - \omega^2}\right)$$

$$= \frac{-(\omega_O^2 - \omega^2)^2}{(\omega_O^2 - \omega^2) + \left(\frac{\omega\omega_O}{Q}\right)^2}\cdot\left[\frac{\frac{\omega_O}{Q}(\omega_O^2 - \omega^2) - 2\omega\frac{\omega\omega_O}{Q}}{(\omega_O^2 - \omega^2)^2}\right]$$

$$\left.\frac{d\phi}{\partial\omega}\right|_{\omega = \omega_O} = \frac{-1}{\omega_O^4/Q^2}\cdot\frac{2\omega_O^3}{Q}$$

$$= -\frac{2Q}{\omega_O}$$

$$\text{(c) } \Delta\omega_O = \frac{\Delta\phi}{\partial\phi/\partial\omega} = \frac{\Delta\phi}{-2Q/\omega_O}$$

$$= \frac{-\Delta\phi\omega_O}{2Q}$$

$\therefore$ Per unit change in ω_O is given by

$$\frac{\Delta\omega_O}{\omega_O} = \frac{-\Delta\phi}{2Q}$$

12.2

For the circuit of problem .1, the poles, which are the zeros of the characteristic equation, are given by:

$1 - L(S) = 0$

$L(S) = 1$

$$\frac{AK\left(\frac{\omega_O}{Q}\right)S}{S^2 + S\frac{\omega_O}{Q} + \omega_O^2} = 1$$

$$S^2 + S\frac{\omega_O}{Q}(1 - AK) + \omega_O^2 = 0$$

$\therefore$ Poles are at :

$$S = \frac{-\frac{\omega_O}{Q}(1 - AK) \pm \sqrt{\left(\frac{\omega_O}{Q}\right)^2(1 - AK)^2 4\omega_O^2}}{2}$$

$$= -\omega_O\left[\frac{1 - AK}{2Q} \pm \sqrt{\left(\frac{1 - AK}{2Q}\right)^2 - 1}\right]$$

$$= -\omega_O\left(\frac{1 - AK}{2Q}\right)\left[1 \pm j\sqrt{\left(\frac{2Q}{1 - AK}\right)^2 - 1}\right]$$

Radial distance of $\omega_O \Rightarrow$

$$|S^2| = \omega_O^2\left(\frac{1 - AK}{2Q}\right)^2\left[1 + \left(\frac{2Q}{1 - AK}\right)^2 - 1\right]$$

$$= \omega_O^2$$

$\therefore |S| = \omega_O$ independent of A or K !

(a) For poles on jw- axis $\Rightarrow$ real part = 0

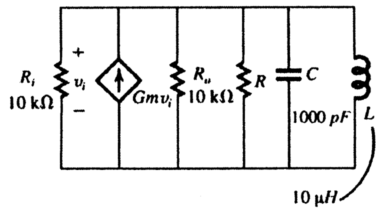
$\therefore -(1 - AK) = 0 \Rightarrow AK = 1$

(b) For poles in RHS $\Rightarrow$ Real Part > 0

$-(1 - AK) > 0$

$AK > 1$

12.3



For resonator: $\omega_O = \frac{1}{\sqrt{LC}} = 10^7 \frac{rad}{S}$

$\frac{\omega_O}{Q} = \frac{1}{RC}$

$R = \frac{Q}{\omega_O C} = \frac{100}{10^7 \times 1000 \times 10^{-12}} = 10\text{ k}\Omega$

Oscillation will occur at $\omega_O = 10^7 \frac{rad}{3}$

when $G_m = (R_i \parallel R_o \parallel R) = 1$ i.e. gain = 1

$\therefore G_m = \frac{1}{10\text{ K} \parallel 10\text{ K} \parallel 10\text{ K}} = \frac{3}{10^4} = 300 \frac{\mu A}{V}$

12.4

At ω_O $A\beta = 1$

If $\beta(\omega_O)$ is −20 dB with a phase shift of 180° then clearly A should have a gain of 20 dB (i.e. $A(\omega_O) = 10$) with a phase shift of ±180°

i.e. $A = -10$

12.5

$L_- = -V\frac{R_3}{R_2} - V_D\left(1 + \frac{R_3}{R_2}\right)$

$6 = 10\frac{R_3}{R_2} + 0.7\left(1 + \frac{R_3}{R_2}\right)$

$= 10.7\frac{R_3}{R_2} + 0.7$

$\frac{R_3}{R_2} = 0.495$ By symmetry $\frac{R_4}{R_5} = 0.495$

Use $R_2 = R_5 = 10\text{ k}\Omega$

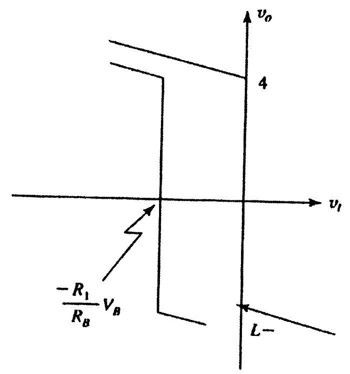
$\therefore R_3 = R_4 \cong 5\text{ k}\Omega$

Slope of limiting characteristic

$\frac{R_4}{R_5} = 0.1$

$\therefore R_1 = \frac{1}{0.1}R_4 = 50\text{ k}\Omega$

12.6



For V_B connected via R_B to the virtual ground, a current $= \frac{V_B}{R_B}$ flows into the node. To compensate, v_I must be moved by Δv_I, in a direction opposite to V_B to produce a current ⇒

$\frac{\Delta v_I}{R_1} = \frac{-v_B}{R_B}$

$\therefore \Delta v_I = -\frac{R_1}{R_B}V_B$

$v_D = 0$ ~ assumed

$L_- = -5 = -15\, R_3/R_2$

$\frac{R_3}{R_2} = \frac{1}{3} = R_4/R_5$

Given $R_{in} = 100\text{ k}\Omega \Rightarrow R_1 = 100\text{ k}\Omega$

Slope $= R_4/R_1 \le 0.05$

$R_4 \le R_1 \times 0.05$

$R_4 \le 5\ \text{k}\Omega \Rightarrow$ Let $R_4 = 4.3\ \text{k}\Omega$

$\because R_3 = R_4 \Rightarrow R_3 = 4.3\ \text{k}\Omega$

$R_2 = R_5 = 3R_4 = 12.9\ \text{k}\Omega$

For standard resistance values:

$R_2 = R_5 = 13\ \text{k}\Omega$

$\therefore\ L- = -15\frac{R_3}{R_2} = -15 \times \frac{4.3}{12.9}$

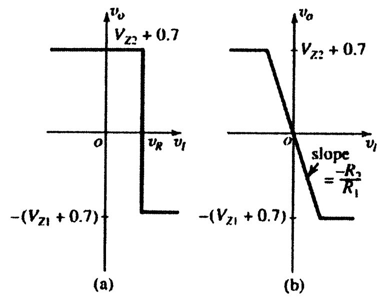
$= -4.96\ \text{V} \cong -5\ \text{V}$

Offset is $+5\ \text{V} \Rightarrow$ use $V_B = -15\ \text{V}$

and $5 = R_1/R_B\ 15$

$\therefore R_B = 3R_1 = 300\ \text{k}\Omega$

12.7



12.8

$$\frac{V_a}{V_o} = \frac{1/sc \parallel R}{1/sc \parallel R + 1/sc + R}$$

$$= \frac{(R/sc)/\left(\frac{1}{sc} + R\right)}{\frac{(R/sc)}{\left(\frac{1}{sc} + R\right)} + \frac{1}{sc} + R}$$

$$= \frac{\frac{R}{sc}}{\frac{R}{sc} + \left(\frac{1}{sc} + R\right)^2} \times \frac{S^2C^2}{S^2C^2}$$

$$= \frac{SCR}{SCR + (1 + SCR)^2}$$

$$= \frac{SCR}{SCR + (1 + 2SCR) + S^2C^2R^2}$$

$$= \frac{\frac{1}{RC}S}{S^2 + S\frac{3}{RC} + \frac{1}{R^2C^2}}$$

Note $\frac{V_a}{V_o}$ has zeros at O and ∞

i.e. A Band pass!

$$\omega_O^2 = \frac{1}{R^2C^2} \Rightarrow \omega_O = \frac{1}{RC}$$

$$\frac{\omega_O}{Q} = \frac{3}{RC} \Rightarrow Q = \frac{1}{3}$$

For centre frequency gain:

$S = j\omega_O = j/RC$

$$\therefore \left.\frac{V_a}{V_o}\right|_{S=(j/RC)} = \frac{\frac{1}{RC}\ j/RC}{-\frac{1}{R^2C^2} + \frac{3}{RC}\left(\frac{j}{RC}\right) + \frac{1}{R^2C^2}}$$

$$= \frac{1}{3} = \text{centre freq. gain}$$

12.9

$$L(j\omega) = \frac{1 + R_2/R_1}{3 + j\left(WCR - \frac{1}{WCR}\right)} \qquad \text{Eq(17.11)}$$

$$\phi(\omega) = -\tan^{-1}\left(\frac{WCR - \frac{1}{WCR}}{3}\right)$$

using $\frac{\partial \tan^{-1} v}{\partial x} = \frac{1}{1 + v^2}\frac{\partial v}{\partial x}$

$$\frac{\partial\phi}{\partial\omega} = \frac{-1}{1 + \left(\frac{WCR - \frac{1}{WCR}}{3}\right)^2} \cdot \frac{1}{3}\left(CR + \frac{1}{W^2CR}\right)$$

$$\left.\frac{\partial\phi}{\partial\omega}\right|_{\omega=\frac{1}{RC}} = \frac{-1}{3}(CR + CR) = \frac{-2}{3}CR$$

for $\Delta\phi = -0.1$ rad

$$\Delta\omega_O = \frac{\Delta\phi}{\partial\phi/\partial\omega} = \frac{-0.1}{-2/3\frac{1}{\omega_O}}$$

$= 0.15\ \omega_O$

$\therefore$ New frequency of oscillation

$= 1.15\ \omega_O = \frac{1.15}{RC}$

12.10

$$L(s) = \frac{1 + R_2/R_1}{3 + SCR + 1/SCR}$$

Poles of closed loop given by: $L(S) = 1$

$$1 + R_2/R_1 = 3 + SCR + \frac{1}{SCR}$$

$$O = S^2 + \frac{S}{RC}\left(2 - \frac{R_2}{R_1}\right) + \frac{1}{R^2C^2}$$

$$Q = \frac{1}{(2 - R_2/R_1)}$$

for $Q = \infty$ - poles on jω axis

$-R_2/R_1 = 2$

for poles in R.H.P $R_2/R_1 > 2$

12.11

assuming resistance of limiting network is very low

At positive peak

$$v_O = \left(\frac{1 + 20.3\text{ K}}{10\text{ K}}\right) v_O = 3.03 v_I \quad (1)$$

$$v_O - \left[\frac{R_5}{R_5 + R_6} \cdot (v_O - (-15))\right] - 0.7 = v_I \quad (2)$$

Now for $10V_{p-p}$ out

$\hat{v}_O = 5$ V

$$\hat{v}_I = \frac{5}{3.03} = 1.65 \text{ V}$$

using (2) $R_5 = 1$ kΩ

$$5 - \left(\frac{1}{1 + R_6} \cdot (V_o + 15)\right) - 0.7 = 1.65$$

$$\frac{20}{1 + R_6} = 2.65$$

$$R_6 = \frac{20}{2.65} - 1$$

$R_6 = 6.5$ kΩ $= R3$

If $R_3 = R_6 = \infty$ from (2)

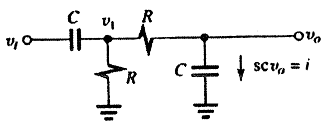
$$v_0 - \left(\frac{1}{1 + \infty}(V_0 + 15)\right) - 0.7 = \frac{v_0}{3.03}$$

$$v_0 - 0.7 = \frac{v_0}{3.3}$$

$v_0 = 1.04$ V

∴ oppoutput is $2\, v_0 = 2.08\ V_{p-p}$.

12.12



$$\frac{v_1 - v_0}{R} = SCv_0 \Rightarrow v_1 - v_0(1 + SCR)$$

ΣI at v_1

$$\frac{v_1}{R} + SC(v_1 - v_I) + SCv_0 = 0$$

$v_0(1 + SCR) + SCR(v_0 + v_0SCR) - SCRv_I + SCRv_0 = 0$

$$v_0(1 + SCR + SCR + S^2C^2R^2 + SCR) = SCRv_I$$

$$\beta(s) \triangleq \frac{v_0}{v_I} = \frac{SCR}{S^2C^2R^2 + 3SCR + 1}$$

$$= \frac{1}{3 + SCR + 1/(SCR)}$$

$$A = 1 + R_2/R_1$$

$$\beta(j\omega) = \frac{1}{3 + j\left(\omega CR - \frac{1}{\omega CR}\right)}$$

Zero phase when $\omega CR = \frac{1}{\omega CR}$

$$\omega = \frac{1}{CR}$$

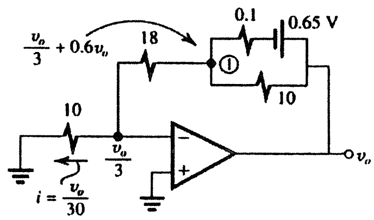
$$|\beta(W = 1/RC)| = \frac{1}{3}$$

for oscillations $1 + R_2/R_1 \geq 3 \Rightarrow \frac{R_2}{R_1} \geq 2$

$$L(s) = A\beta = \frac{1 + R_2/R_1}{3 + SCR + SCR}$$

$$L(jw) = \frac{1 + R_2/R_1}{3 + j\left(WCR - \frac{1}{WCR}\right)}$$

12.13



ΣI at node 1

$$\frac{v_O}{30} = \frac{v_O - \frac{v_O}{3} - 0.650}{10}$$

$$+ \frac{v_O - 0.65 - \frac{v_O}{3} - 0.65}{0.1}$$

$0.00666v_O + 0.666v_O - 0.65$

$v_O = 10.156$ V

∴ Max. ouput = 20.3 $Vp-p$

12.14

$$\omega_O = \frac{1}{RC} = 2\pi 10^4 \quad R = 10\text{ k}\Omega$$

$$C = \frac{1}{10^4 \times 2\pi \times 10^4} \Rightarrow C \cong 1.6 \text{ nF}$$

$$\beta(j\omega) = \left[3 + j\left(\omega CR - \frac{1}{\omega CR}\right)\right]^{-1}$$

$$\therefore \phi(\omega) = -\tan^{-1}\left(\frac{WCR - \frac{1}{WCR}}{3}\right)$$

using $\frac{\partial \tan^{-1} v}{\partial x} = \frac{1}{1+v^2}\frac{\partial v}{\partial x}$ we get

$$\frac{\partial\phi(\omega)}{\partial\omega} = \frac{-1}{1+\left(\frac{WCR - \frac{1}{WCR}}{3}\right)^2}\left[\frac{RC + 1/W^2RC}{3}\right]$$

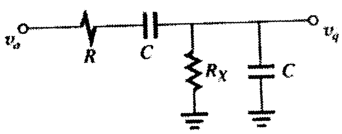
At $\omega = \omega_O = \frac{1}{RC}$ $\frac{\partial\phi(\omega)}{\partial\omega} = \frac{-2}{3}RC$

Now $5.7° \approx 0.1\text{rad}$ (lag = −0.1rad)

$$\therefore \Delta\omega_o = \frac{-0.1}{-2/3\ \text{RC}} = 0.15\ \omega_o = 1.5\ \text{kHz}$$

∴ New frequency of oscillation = 8.5 kHz

To restore Operation:



$$\beta(s) = \frac{Rx \| \frac{1}{SC}}{Rx \| \frac{1}{SC} + R + \frac{1}{SC}}$$

$$= \frac{\frac{Rx/SC}{Rx + 1/SC}}{\frac{Rx/SC}{Rx + \frac{1}{SC}} + R + \frac{1}{SC}}$$

$$= \frac{Rx/SC}{Rx/SC + RRx + \frac{R}{SC} + \frac{Rx}{SC} + \frac{1}{S^2C^2}}$$

$$\therefore \beta(s) = \frac{1}{2 + \frac{R}{Rx} + SCR + \frac{1}{SCRx}}$$

$$\phi = -\tan^{-1}\left(\frac{WCR - \frac{1}{WRxC}}{2 + R/Rx}\right)$$

Now it is required that $\phi = 5.7°$ at $\omega > \omega_O$!

where $\omega_O = 1/RC$

$$\therefore \omega_O RC - \frac{-1}{\omega_O RxC} = \left(2 + \frac{R}{Rx}\right)\tan^{(-5.7)}$$

$$1 - \frac{1}{\omega_O RxC} = (2 + R/Rx)(-0.1)$$

$$1 + 0.2 = \frac{1}{\omega_O RxC} - 0.1\frac{R}{Rx}$$

$$Rx = \frac{1/\omega_O C - 0.1\ R}{1.2}$$

given:

$\omega_O = 2\pi 10^4$

$C = 1.6 \times 10^{-9}$

$R = 10^4$

$Rx = 7.5\ \text{k}\Omega$

Now:

$$\beta(j\omega_O) = \frac{1}{2 + 10/7.5 + j(1 - 1/\omega_O CRx)}$$

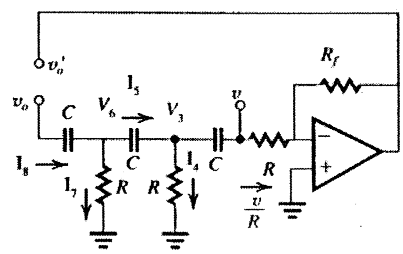
$$= (3.333 - j0.326)^{-1}$$

$$|\beta(j\omega_O)| = \frac{1}{3.35}$$

$\therefore 1 + R_2/R_1 = 3.35$ for oscillations

$\frac{R_2}{R_1} = 2.35$ (not 2 as before)

12.15



$$V_3 = v + \frac{1}{SCR}\frac{v}{} = v\left(1 + \frac{1}{SCR}\right)$$

$$I_4 = \frac{v\left(1 + \frac{1}{SCR}\right)}{R} = \frac{v}{R} + \frac{v}{SCR^2}$$

$$I_5 = I_4 + \frac{v}{R}i_s = \frac{2v}{R} + \frac{v}{SCR^2}$$

$$V_6 = V_3 + \frac{I_5}{sC}$$

$$= v + \frac{v}{sCR} + \frac{1}{sC}\left(2\frac{v}{R} + \frac{v}{sCR^2}\right)$$

$$V_6 = v + \frac{3v}{sCR} + \frac{v}{s^2C^2R^2}$$

$$I_7 = \frac{v}{R} + \frac{3v}{sCR^2} + \frac{v}{s^2C^2R^3}$$

12.16

$I_8 = I_5 + I_7$

$I_8 = \frac{3v}{R} + \frac{4v}{sCR^2} + \frac{v}{s^2C^2R^3}$

$V_o = V_6 + \frac{I_8}{sC}$

$v_o = v + \frac{3v}{sCR} + \frac{v}{s^2C^2R^2} + \frac{3v}{sCR} + \frac{4v}{s^2C^2R^2} + \frac{v}{s^3C^3R^3}$

$= v + \frac{6v}{sCR} + \frac{5v}{s^2C^2R^2} + \frac{v}{s^3C^3R^3}$

Now loop gain =

$L(S) = \frac{-v_o^1}{v_o}$

$v_o^1 = \frac{R_f}{R} \cdot v$

$\therefore L(S) = \frac{\frac{R_f}{R} \cdot v}{v\left(1 + \frac{6}{sCR} + \frac{5}{s^2C^2R^2} + \frac{1}{s^3C^3R^3}\right)}$

$= \frac{s^3 R_f / R}{s^3 + \frac{6s^2}{RC} + \frac{5s}{C^2R^2} + \frac{1}{C^3R^3}}$

$L(j\omega) = \frac{-j\omega^3 R_f / R}{\frac{1}{C^3R^3} - \frac{6\omega^2}{RC} + j\left(\frac{5\omega}{C^2R^2} - \omega^3\right)}$

$L(j\omega)$ is real if

$\frac{6\omega_O^2}{RC} = \frac{1}{R^3C^3}$

$\omega_O = \frac{1}{\sqrt{6}RC}$

$L(j\omega_O) = \frac{\omega_O^2 R_f / R}{-\omega_O^2 + 5/R^2C^2}$

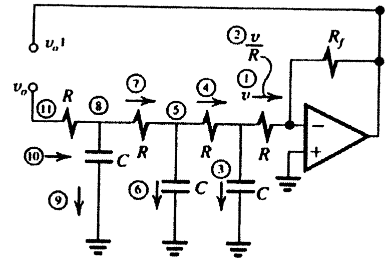
$= \frac{R_f / R\omega_O^2}{-w_O^2 + 30\omega_O^2}$

$= \frac{R_f / R}{29}$

Now loop Gain = 1 if $R_f = 29$ R

$\therefore$ Minimum value for $R_f = 29$ R

$f_O = \frac{0.065}{RC}$



(3) $i = scv$

(4) $scv = v/R$

(5) $v + \left(SCv + \frac{v}{R}\right)R = 2v + SCRv$

(6) $2SCv + S^2C^2Rv$

(7) = (6) + (4) = $3SCv + S^2C^2Rv + \frac{v}{R}$

(8) $2v + SCRv + v + 3SCRv + S^2C^2R^2v$

$= 3v + 4SCRv + S^2C^2R^2v$

(9) $3SCv + 4S^2C^2Rv + S^3C^3R^2v$

(10) = (7) + (9)

$= 6SCv + 5S^2C^2Rv + \frac{v}{R} + S^3C^3R^2v$

(11) = (8) + (10) $\times R$

$v_o = 4v + 10SCRv + 6S^2C^2R^2v + s^3c^3R^3v$

$L(s) = \frac{v_o^1}{v_o} = \frac{vR_f/R}{v(S^3C^3R^3 + 6S^2C^2R^2 + 10SCR + 4)}$

$= \frac{R_f/R}{S^3C^3R^3 + 6S^2C^2R^2 + 10SCR + 4}$

$L(j\omega) = \frac{R_f/R}{(4 - 6\omega^2C^2R^2) + j(10\omega CR + (\omega^3C^3R^3)}$

$L(j\omega)$ is purely real if

$10\omega_O CR = \omega_O^3C^3R^3$

$\omega_O = \frac{1}{\sqrt{10}}\frac{1}{RC}$

Given $R = 10$ kΩ, fo = 10 kHz

$C = \frac{1}{\sqrt{10} \times 10^4 \times 2\pi 10^4}$

= 0.503 nF

Now,

$|L(j\omega_O)| = \frac{R_f/R}{4 - 6\omega_O^2R^2C^2}$ sub for ω_o

$$= \frac{R_f / R}{4 - 6\frac{1}{10R^2C^2}R^2C^2}$$

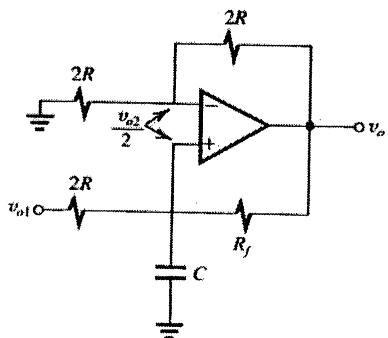
$$= \frac{R_f / R}{4 - 6/10} \geq 1$$

$\therefore R_f / R \geq 3.4$

$R_f \geq 34\ \text{k}\Omega$

12.17

for 2nd indicator



From the voltage divider around the upper branch:

$$v+ = v- = \frac{1}{2}v_{02}$$

$\Sigma I = 0$ at the input

$$\frac{\frac{1}{2}v_{02} - v_{01}}{2R} + SC\frac{v_{02}}{2} + \frac{\frac{v_{02}}{2} - v_{02}}{R_f} = 0$$

$$\frac{v_{02} - 2v_{01}}{2R} + SCv_{02} - \frac{v_{02}}{R_f} = 0 \quad R_f = \frac{2R}{H\Delta}$$

$$v_{02}\left(\frac{1}{2} + SC - \frac{H\Delta}{2R}\right) = \frac{v_{01}}{R}$$

$$v_{02}\left(SCR - \frac{\Delta}{2}\right) = v_{01}$$

$$\therefore \frac{v_{02}}{v_{01}} = \frac{1}{SCR - \Delta/2}$$

Now: $\frac{v_{01}}{v_s} = \frac{-1}{SCR}$

$$\therefore L(S) = \frac{-1/SCR}{SCR - \Delta/2}$$

Characteristic equation $L(s) = 1$

$$\therefore S^2C^2R^2 - \frac{SCR\Delta}{2} + 1 = 0$$

$\therefore$ Poles are

$$S_p = \frac{\frac{RC\Delta}{2} \pm \sqrt{\frac{R^2C^2\Delta^2}{4} - 4C^2R^2}}{2R^2C^2}$$

$$= \frac{\Delta/2 \pm 2j\sqrt{1 - (\Delta/4)^2}}{2RC}$$

for $\Delta << 1$ $\left(1 - \left(\frac{\Delta}{4}\right)^2\right)^{1/2} \approx \left(1 - \frac{1}{2}\left(\frac{\Delta}{4}\right)^2\right)$

$$\therefore S_p \cong \left[\Delta/2 \pm j2\left(1 - \frac{1}{2}\left(\frac{\Delta}{4}\right)^2\right)\right]\frac{1}{2RC}$$

$$= \frac{\frac{\Delta}{2} \pm j\left(2 - \left(\frac{\Delta}{4}\right)^2\right)}{2RC}$$

Now:

$R_e[S_p] > 0 \Rightarrow$ Poles in R.H.P.!

for $\Delta << 1$

$$S_p \cong \frac{\Delta/2 \pm j2}{2RC} = \frac{1}{RC}\left(\frac{\Delta}{4} \pm j\right) \text{ Q.E.D}$$

12.18

The transmission of the filter normalized to the centre frequency, w_o is:

$$|T(jw)| = \frac{\omega\omega_0/Q}{(\omega_0^2 - \omega^2)^2 + \frac{\omega^2\omega_0^2}{Q^2}}$$

$$= \frac{(1/Q)\left(\frac{\omega_0}{\omega}\right)}{\left(\left(\frac{\omega_0}{\omega}\right)^2 - 1\right)^2 + \frac{1}{Q^2}\left(\frac{\omega_0}{\omega}\right)^2}$$

Relative to the amplitude of the fundamental

(a) The second harmonic = 0

(b) The third harmonic

$$= \frac{1}{3}\frac{\frac{1}{20} \times \frac{1}{3}}{\left(\frac{1}{9} - 1\right)^2 + \left(\frac{1}{20}\right)^2\left(\frac{1}{9}\right)} = 6.25 \times 10^{-3}$$

(c) The fifth harmonic

$$= \frac{1}{5}\frac{\frac{1}{20} \times \frac{1}{5}}{\left(\frac{1}{25} - 1\right)^2 + \left(\frac{1}{20}\right)^2\left(\frac{1}{25}\right)} = 2.08 \times 10^{-3}$$

(d) The 4th harmonic = 6th = 10th = 0

7th harmonic $= 1.04 \times 10^{-3}$

9th harmonic $= 0.625 \times 10^{-3}$

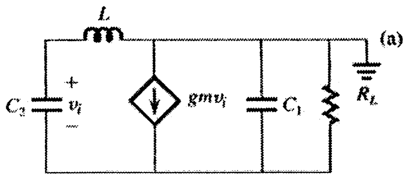
$\therefore$ $\frac{\text{RMS of 2}^{\text{nd}}\text{ to 10}^{\text{th}}\text{ harmonic}}{\text{RMS of fundamental}}$ is

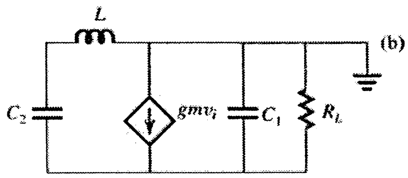
$$[6.25^2 + 2.08^2 + 1.04^2 + 0.625^2]^{1/2} \times 10^{-3}$$

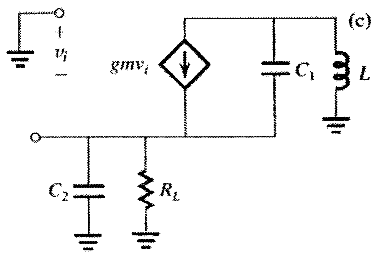
$= 6.7 \times 10^{-3}$ OR 0.7%

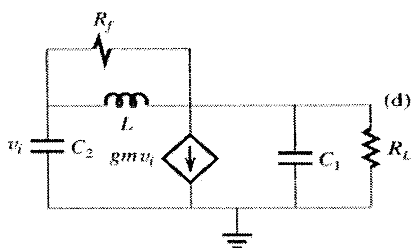
12.19

Consider the small signal models for each circuit.
Assume r_π very large:

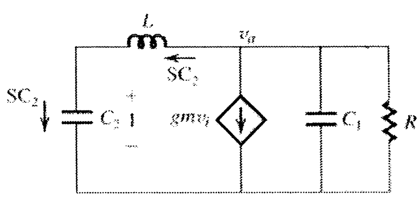








Given $R_f >> w_0 L$, circuits (a), (b) and (d) are the same except for the reference (ground) node.
For circuit (a), (b) & (d)



-Break the loop at v_i and assume unit return.

$$v_a = 1 + SC_2 sL$$
$$= 1 + S^2 C_2 sL$$

$\Sigma I = 0$ at v_a

$$g_m + SC_2 + SC_1(1 + S^2C_2sL) + \frac{(1 + S^2C_2sL)}{R} = 0$$

$$\therefore g_m + 1/R + S(C_1 + C_2) + \frac{S^2C_2L}{R} + S^3C_2sL = 0$$

This is the characteristic equation.
For $s = j\omega$:

$$g_m + \frac{1}{R} - \frac{\omega^2 C_2 L}{R} + j((C_1 + C_2)\omega - \omega^3(C_1C_2L)) = 0$$

IMAGINARY PART = 0:

$$C_1 + C_2 = \omega^2 C_1 + C_2 L$$

$$\omega = \sqrt{\frac{C_1 + C_2}{C_1C_2L}} \equiv \text{Frequency of Oscillation}$$

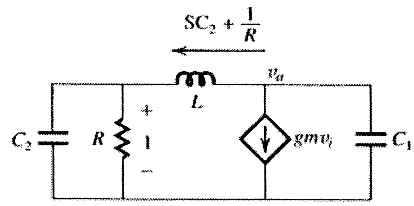
REAL PART = 0

$$g_m + \frac{1}{R} = \frac{\omega^2 C_2 L}{R}$$

$$g_m R = \left(\frac{C_1 + C_2}{C_1C_2L}\right)C_2L - 1$$

$$g_m R = \frac{C_2}{C_1} \equiv \text{LIMIT ON GAIN}$$

For Circuit (c)



$$v_a = \left(SC_2 + \frac{1}{R}\right)SL + 1$$

$\Sigma I = 0$ at v_a, $v_i = 1$

$$g_m + SC_2 + \frac{1}{R} + SC_1\left[SL\left(SC_2 + \frac{1}{R}\right) + 1\right] = 0$$

$$g_m + \frac{1}{R} + SC_2 + S^3C_1C_2L + \frac{S^2C_1L}{R} + SC_1 = 0$$

THE CHARACTRISTIC EQUATION $\equiv$

$$g_m + \frac{1}{R} + S(C_1 + C_2) + \frac{S^2C_2L}{R} + S^3C_1C_2L = 0$$

Note this is the same as above, with $C_1 \leftrightarrow C_2$

$$\therefore \omega_O = \sqrt{\frac{C_1 + C_2}{C_1C_2L}} \text{ and } g_m R = \frac{C_1}{C_2}$$

12.20

(a) frequency of oscillation $\omega_0 = \frac{1}{\sqrt{LC}}$

gain >> 1 gain $= \frac{RC}{2r_e} = \frac{RC}{2v_T/I/2}$

$= \frac{IRC}{4v_T}$

for $v_T = 0.025$ V then

$IRC \geq 4v_T$

$RC \geq 0.1/I$ for oscillations to start.

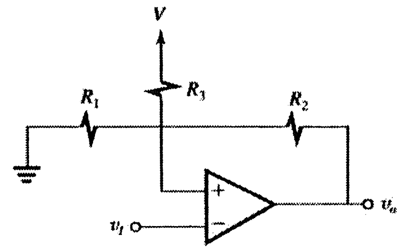
(b) For $RC = \frac{1}{I}$ (kΩ) we have

$$\text{gain} = \frac{1/I}{2\left(\frac{2v_T}{I}\right)} = \frac{1}{4 \times 0.025} = 10$$

Oscillations will start (10>1) and grow until Q1, Q2 go into cutoff. Output will go from V_{CC} to $V_{CC} - IRC = V_{CC} - 1$.

Therefore, output will be $1 V_{P-P}$. Fundamental has a P-P amplitude of $\frac{4}{\pi} = 1.27 V_{P-P}$

12.21



(a) ΣI at $+ v_L$ node:

$$\frac{V_{TH}}{R_1} = \frac{V - V_{TH}}{R_3} + \frac{L_T - V_{TH}}{R_2}$$

$$V_{TH}\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) = \frac{V}{R_3} + \frac{L_T}{R_2}$$

$$V_{TH} = (V/R_3 + L_T/R_2)\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)^{-1}$$

$$= \left(\frac{V}{R_3} + \frac{L_T}{R_2}\right)R_1 \parallel R_2 \parallel R_3$$

Similarly

$$V_{TL} = \left(\frac{V}{R_3} + \frac{L-}{R_2}\right)(R_1 \parallel R_2 \parallel R_3)$$

(b) Now

$$V_{TH} = 5.1 = \left(\frac{15}{R_3} + \frac{13}{R_2}\right)\left(\frac{1}{10} + \frac{1}{R_2} + \frac{1}{R_3}\right)^{-1}$$

$$\frac{5.1}{10} + \frac{5.1}{R_2} + \frac{5.1}{R_3} = \frac{15}{R_3} + \frac{13}{R_L}$$

$$0.51 = \frac{7.9}{R_2} + \frac{9.9}{R_3} \qquad (1)$$

AND

$$V_{TL} > 4.9 = \left(\frac{15}{R_3} - \frac{13}{R_2}\right)\left(\frac{1}{10} + \frac{1}{R_2} + \frac{1}{R_3}\right)^{-1}$$

$$0.49 = \frac{-17.9}{R_2} + \frac{10.1}{R_3} \qquad (2)$$

$$(1) \times \frac{10.1}{9.9} \Rightarrow 0.52 = \frac{8.06}{R_2} + \frac{10.1}{R_3}$$

$$(2) \Rightarrow 0.49 = \frac{-17.9}{R_2} + \frac{10.1}{R_3}$$

SUBTRACT TO GET ⇒

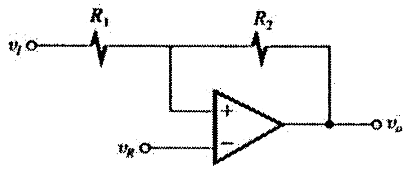
$$0.52 - 0.49 = \frac{8.06 + 17.9}{R_2}$$

$$R_2 = \frac{25.96}{0.0303} = 856.8 \text{ k}\Omega$$

$$\frac{10.1}{R_3} = 0.49 + \frac{17.9}{856.8}$$

$R_3 \cong 19.8$ kΩ

12.22



(a) for $v_I = v_{TL}$ and $v_O = L_+$ initially

$$\frac{L_+ - v_R}{R_2} = \frac{v_R - v_{TL}}{R_1}$$

$$v_{TL} = v_R - \frac{R_1}{R_2} v_R - \frac{R_1}{R_2} L_+$$

$$\therefore v_{TL} = v_R\left(1 - \frac{R_1}{R_2}\right) - \frac{R_1}{R_2} L_+$$

Similarly

$$\frac{L_- v_R}{R_2} = \frac{v_R - v_{TH}}{R_1}$$

$$v_{TH} = v_R(1 + R_2/R_1) - \frac{R_1}{R_2} L_-$$

(b) Given

$L_+ = -L_- = V$

$R_1 = 10\ \text{k}\Omega$

$V_{TL} = 0$

$V_{TH} = V/10$

Substituting these values we get:

$$O = V_R(1 + 10/R_2) - 10/R_2 V \qquad (1)$$

$$\frac{V}{10} = V_R(1 + 10/R_2) + 10/R_2 V \qquad (2)$$

$$(1) - (2) \; -\frac{V}{10} = -\frac{20}{R_2} v$$

$$R_2 = 200\ \text{k}\Omega$$

$$O = V_R(1 + 10/200) - \frac{10}{200} V$$

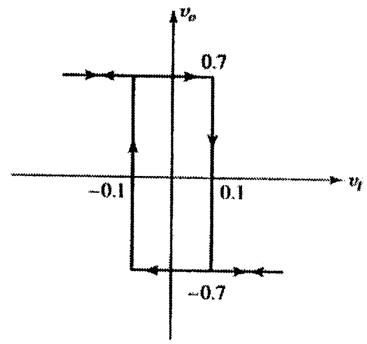
$$V_R = \frac{10/200\ \text{V}}{1 + 10/200} = 47.62\ \text{mV}$$

12.23

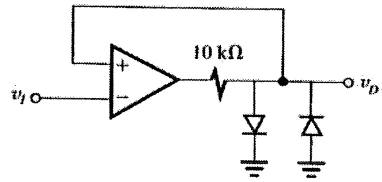
Output levels $= \pm 0.7$ V

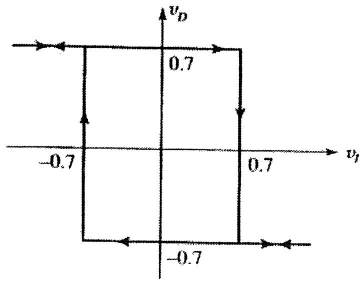
$$\text{Threshold levels} = \pm \frac{10}{10 + 60} \times 0.7 = 0.1\ \text{V}$$

$$i_{D,\max} = \frac{12 - 0.7}{10} - \frac{0.7}{10 + 60} = 1.12\ \text{mA}$$



12.24





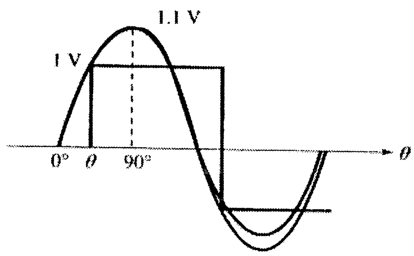
12.25

(a) A 0.5 V peak sine wave, is not large enough to change the state of the circuit. Hence, the output will be either +12 V or −12 V at DC.

(b) The 1.1 V peak will change the state when

$1.1 \sin\theta = 1$

$\theta = 65.40$



∴ The output is a symmetric square wave at frequency *f*, and lags the sine wave by an angle of 65.4°. The square wave has a swing of ±12 V. Since $v_{TH} - v_{TL} = 1$ V, if the average shifts by an amount so either the +ve or -ve swing is <1 V, then no change of state will occur. Clearly, if the shift is 0.1 V, the output will be a DC voltage.

12.26

For $L+ = -L- = 7.5$ V
$V_Z = 6.8$ V with $V_D = 0.7$ V.
For $V_{TH} = -V_{TL} = 7.5 \text{ V} \Rightarrow R_1 = R_2$

For $v_i = 0 \quad I_{R_2} = 0.1\text{mA} = \dfrac{7.5}{R_1 + R_2}$

$\Rightarrow R_1 = R_2 = 37.5 \text{ k}\Omega$

$I_D = 1\text{mA} = \dfrac{12 - 7.5}{R} - \dfrac{7.5}{2R_1}$

$1 = \dfrac{4.5}{R} + 0.1$

$R = 4.1 \text{ k}\Omega$

12.27

$T = 2\tau \ln \dfrac{1+\beta}{1-\beta} \quad \beta = \dfrac{R_1}{R_1 + R_2} = \dfrac{10}{26}$

$T = 2(10 \times 10^{-9})(62 \times 10^3) l_n\left(\dfrac{1 + 10/26}{1 - 10/26}\right)$

$T = 1.006\text{ms} \Rightarrow f = 994.5 \text{ Hz}$

12.28

for $\pm 5 V_{\text{outputs}}$

$V_z = 5 - 2V_{\text{DIODE}} = 5 - 1.4 = 3.6$ V

For $\pm 5 V_{\text{out}}$:

$R_1 = R_2 \; L_+ = -L_- = 5$ V

$V_{TH} = -V_{TL} = 5$ V

Max current in feedback network = 0.2 mA

$\therefore 0.2 = \dfrac{5}{R_1 + R_2} \Rightarrow R_1 = R_2 = 25 \text{ k}\Omega$

Max diode current = 1 mA

$\therefore \dfrac{13 - 5}{R_z} = (0.2 + 1)\text{mA}$

$R_z = \dfrac{8}{1.2} = 6.67 \text{ k}\Omega$

Now from Fig 17.25(c)

$\text{slope} = \dfrac{-L_-}{RC} = \dfrac{V_{TH} - V_{TL}}{T/2}$

for $f = 1$kHz
$T = 10^{-3}$ sec.
$C = 0.01\ \mu$F

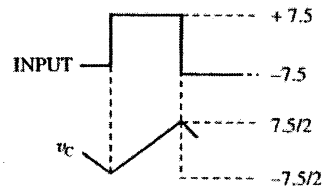
$\dfrac{5}{RC} = \dfrac{10}{10^{-3}/2} \Rightarrow R = 25 \text{ k}\Omega$

12.29

For 15 V_{PP} output $v_z = 15/2 - 0.7$

$= 6.8$ V

For the integrator:



i.e. V_C should ramp between V_{TH} ξ V_{TL}!

$v_C(t_1) = \dfrac{1}{RC}\displaystyle\int_{t_0}^{t_1} v dt + v_c(t_0)$

$-v$ is a square wave

$\dfrac{7.5}{2} = \dfrac{1}{RC}(t_1 - t_0)(7.5 - (-7.5)) - \dfrac{7.5}{2}$

$(t_1 - t_0) = \dfrac{T}{2}$

$7.5 = \dfrac{1}{RC}\dfrac{T}{2}(15)$

$1 = \dfrac{T}{RC} \Rightarrow R = \dfrac{T}{RC} = \dfrac{1}{fC}$

$= \dfrac{1}{10^4(0.5 \times 10^{-9})}$

$\therefore R = R_{1-6} = 200 \text{ k}\Omega$

Minimum level current = 1 mA

$\dfrac{13 - 7.5}{R_z} = 1 + \dfrac{7.5}{R_1 + R_2} + \dfrac{7.5 - V_C}{R_5}$

Maximum current into the integrator when

$V_C = \dfrac{-7.5}{2}$

$\therefore \dfrac{5.5}{7.5} = 1 + \dfrac{7.5}{400} + \dfrac{11.25}{200}$

$\therefore R_7 = 5.12\text{k}\Omega \overset{\text{use}}{\Rightarrow}$

$R_7 = 5.1\ \text{k}\Omega$

Integrator output is triangular, with period $= 100\mu s$ and ± 7.5 V peaks. (i.e. $2 \times$ voltage at capacitor)

12.30

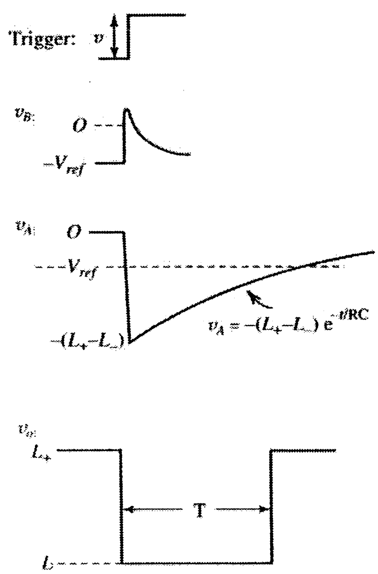
See sketches that follow:

$$V_A(t = T) = V_{ref} = -(L_+ - L_-)e^{-T/RC}$$

$$\frac{V_{ref}}{L_+ - L_-} = e^{-T/RC}$$

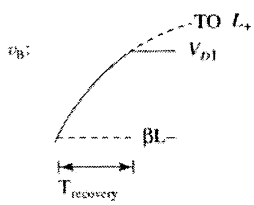
$$T = -RC\ln\left(\frac{V_{ref}}{L_+ - L_-}\right) = RC\ln\left(\frac{L_+ - L_-}{V_{ref}}\right)$$

Q.E.D.



12.31

For recovery, v_B goes from $\beta L-$ to L_+ until D_1 conducts at $V_{D1} = 0.7$ V



For recovery

$$v_B = -0.1(12) + (12 + 1.2)(1 - e^{-t/\tau})$$

$$= 12 - 13.2e^{-t/\tau}$$

At T recovery:

$$V_{D1} = 12 - 13.2e^{-Tr/\tau}$$

$$\tau = R_3C_1$$

$$Tr = -R_3C_1\ln\left(\frac{V_{D1} - 12}{13.2}\right)$$

$$= -(6171)(0.1 \times 10^{-6})\ln\left(\frac{11.3}{13.2}\right)$$

$$= 96\ \mu s$$

12.32

Choose $C_1 = 1\text{nF}$ $\qquad C_2 = 0.1\text{nF}$

$$R_1 = R_2 = 100\ \text{k}\Omega \Rightarrow \beta = \frac{1}{2}$$

$$T \cong C_1R_3\ln\left(\frac{0.7 + 13}{-13(0.5 - 1)}\right)$$

$$10^{-4} = 10^{-9}R_3\ln\left(\frac{13.7}{13(0.5)}\right)$$

$$R_3 = 134.1\ \text{k}\Omega$$

Need $R_4 >> R_1 \Rightarrow$ choose $R_4 = 470\ \text{k}\Omega$

Min trigger voltage $= (\beta L_+ - V_{D2} + V_{D1})$

$= 6.5$ V

For recovery

$$v_B = 13 - (13 - \beta L_-)e^{-t/\tau}$$

$$= 13 - 19.5e^{-t/\tau} = 0.7$$

$$\therefore t_{recovery} = -\tau\ln\left(\frac{12.3}{19.5}\right)$$

$$= -(134.1 \times 10^3)(10^{-9})(-0.4608)$$

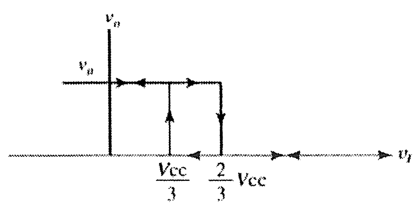
$$= 61.8\ \mu s$$

12.33

For $v_I > 2/3V_{CC}$ comp -1 = "1" and comp -2 = "0" and flip flop is reset. I.E. $v_O = 0\text{V}$.

Now v_O will not change until $v_I = 1/3V_{CC}$, when comp - 2 = "1" and comp -1 = "0" and FF is set: I.E. $V_O = V_{CC}$

For $\frac{1}{3}V_{CC} < v_I < \frac{2}{3}V_{CC}$, comp $-$ 1 = comp - 2 = "0" and no change of state will occur.



i.e. an inverting bistable circuit.

12.34

(a) C = lnF

$v_C = V_{CC}(1 - e^{-t/\tau})$

where $\tau = RC$

Pulse width of 10 μs when $v_C = V_{TH}$

$= \frac{2}{3}V_{CC}$

$\therefore \frac{2}{3} = 1 - e^{-t/RC}$

$t = T = 10\ \mu s$

$-\frac{T}{RC} = \ln\left(\frac{1}{3}\right) \Rightarrow R = \frac{-T}{C\ln(1/3)}$

$= 9.1\ k\Omega$

(b) for $T = 20\mu s$ $R = 9.1\ k\Omega$, C = lnF

$\therefore V_{TH} = 15(1 - e^{-t/RC})$

$$= 15\left(1 - e^{\frac{-20\times10^{-6}}{9.1\times10^{3}\times10^{-9}}}\right)$$

= 13.3 V

12.35

$C = 680\ pF$ $f = 50\ kHz$

$T = 20\ \mu s = T_H + T_L$

For 75% Duty $T_H = 15\ \mu s$

$T_L = 5\ \mu s$

From Eq (17.43) we have:

$T_L = CR_B \ln 2$

$\therefore R_B = \frac{5\times10^{-6}}{680\times10^{-12}\ln 2} = 10.6\ k\Omega$

From Eq (17.41)

$T_H = C(R_A + R_B)\ln(2)$

$R_A = \frac{15\times10^{-6}}{680\times10^{-12}\ln(2)} - 10.6\times10^{3}$

$= 21.2\ k\Omega$

12.36

For the rise:

$V_C = V_{CC} - (V_{CC} - V_{TL})e^{-t/C(R_A+R_B)}$

$V_{TH} = V_{CC} - (V_{CC} - V_{TL})e^{-T_H/C(R_A+R_B)}$

$\frac{V_{CC} - V_{TH}}{V_{CC} - V_{TL}} = e^{-T_H/C(R_A+R_B)}$

$T_H = C(R_A + R_B)\ln\left(\frac{V_{CC} - V_{TL}}{V_{CC} - V_{TH}}\right)$

For exponential fall:

$V_C = V_{TH}e^{-t/CR_B}$

$\therefore V_{TL} = V_{TH}e^{-T_L/CR_B}$

$T_L = CR_B\ln\left(\frac{V_{TH}}{V_{TL}}\right)$

for $V_{TH} = 2V_{TL} \Rightarrow T_L = CR_B\ln(2)$

(b) $C = $ lnF, $R_A = 7.2\ k\Omega$, $R_B = 3.6\ k\Omega$

$V_{CC} = 6\ V$ $V_{TH\ ext} = 0$

$\therefore T_H + T_L = T = \ln 2(R_A + 2R_B)C$

$T = 9.98\ \mu s \rightarrow f = 100\ kHz$

Duty cycle $= \frac{T_H}{T_H + T_L} = \frac{R_A + R_B}{R_A + 2R_B} = 0.75$

$\Rightarrow 75\%$

(c) $V_{CC} = 5V$,

$V_{TH} = \frac{2}{3}\times 5 = \frac{10}{3} = 3.33\ V$

for IV input $V^1_{TH} = 4.33\ V$

$V^1_{TL} = \frac{1}{2}V^1_{TH} = 2.17\ V$

$T^1_H = 10^{-9}(3.6 + 7.2)\times10^{3}\ln\left(\frac{5 - 2.17}{5 - 4.33}\right)$

$= 15.6\ \mu s$

$T^1_L = 10^{-9}\times 3.6\times10^{3}\ln 2 = 2.5\ \mu s$

$\therefore f = \frac{1}{(15.6 + 2.5)10^{-6}} = 55.2\ kHz$

duty cycle $= \frac{15.6}{2.5 + 15.6} = 86.2\%$

for IV input $V^{11}_{TH} = 2.33$

$V^{11}_{TL} = 1.17$

$\therefore T^{11}_H = 10^{-9}(3.6\ 7.2)10^{3}\ln\left(\frac{5 - 1.17}{5 - 2.33}\right)$

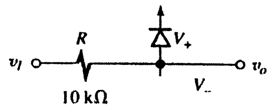
$= 3.92\ \mu s$

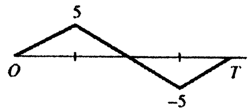
$T^{11}_L + T^1_L = 2.5\ \mu s$

$\therefore f = \frac{10^6}{(3.92 + 2.5)} = 156\ kHz$

duty cycle $= \frac{3.92}{2.5 + 3.92} = 61\%$

12.37





$v_O = A\sin\frac{2\pi}{T}t$

Slope of v_O at $t = 0$:

$\frac{\partial v_O}{\partial t} = A\frac{2\pi}{T}\cos\left(\frac{2\pi}{T}\right)t|_t = 0$

$= \frac{A2\pi}{T} =$ SLOPE AT ZERO CROSSING

Slope of Δ^- wave $= \frac{5}{T/4} = \frac{20}{T}$

$\therefore \frac{20}{T} = \frac{A2\pi}{T}$

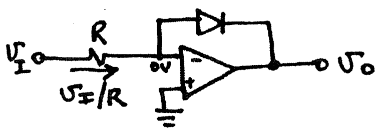
$A = 3.18$ V

∴ Clamp voltage:

$V_T = -V_- = 3.18 - 0.7$

$= 2.48 = 2.5$ V

12.38



→ $v_I > 0$

→ voltage across diode is $-v_O$

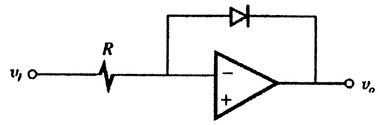
$i_D = \frac{v_I}{R} = I_s e^{-v_O/nV_T}$

$-\frac{v_O}{nV_T} = \ln\left(\frac{v_I}{RI_s}\right)$

$v_O = -nV_T\ln\left(\frac{V_I}{RI_s}\right)$, $v_I > 0$

Q.E.D.

12.39



$= -nV_T\ln\left(\frac{v_I}{RI_S}\right)$

Now.

$V_A = -nV_T\ln\frac{V_1}{RI_S}$ $\quad R = 1\ \text{k}\Omega$

$V_B = -nV_T\ln\frac{V_2}{RI_S}$ $\quad V_1, V_2 > 0$

$V_C = +nV_T\ln\frac{1}{RI_S}$

$V_D = -(V_A + V_B + V_C)$

$= nV_T\left(\ln\left[\frac{V_1}{RI_S} \times \frac{V_2}{RI_S} \times \frac{RI_S}{1}\right]\right)$

$= nV_T\ln\left(\frac{V_1V_2}{RI_S}\right)$

$i_{D4} = I_s e^{V_D/nV_T}$

$= I_s \times \frac{V_1V_2}{RI_S} = \frac{V_1V_2}{R}$

$v_O = -i_{D4}R = -\frac{V_1V_2}{R} \times R$

$\therefore v_O = -v_1v_2$ ANALOG MULTIPLIER

To check $v_1 = 0.5$, $v_2 = 2$

$I_{D1} = 0.5\ \text{mA} \rightarrow V_A = -0.7 + nV_T\ln\left(\frac{0.5}{1}\right)$

$= 0.7 + 2(0.025)\ln\left(\frac{1}{2}\right)$

$= -0.6653$ V

$I_{D2} = 2\text{ mA} \rightarrow V_B = (0.7 + 0.05\ln(2))(-1)$
$= -0.7347\text{ V}$
$I_{D3} = 1\text{ mA} \rightarrow V_C = 0.700\text{ V}$
$V_D = -(-0.6653 - 0.7347 + 0.7) = 0.7\text{ V}$
$V_D = V_{D4} = 0.7\text{ V} \Rightarrow I_{D4} = 1\text{ mA}$
$\therefore v_O = -1\text{ V}$ i.e. $2 \times 0.5 = 1$

For $v_1 = 3$, $v_2 = 2$:

$I_{D1} = 3\text{mA} \rightarrow V_A = -(0.7 + 0.05\ln 3)$
$= -0.7549\text{ V}$
$I_{D2} = 2\text{mA} \rightarrow V_A = -(0.7 + 0.05\ln 2)$
$= -0.7347\text{ V}$
$I_{D3} = 1\text{mA} \rightarrow V_C = 0.7\text{ V}$
$\therefore V_D = V_{D4} = -(V_A + V_B + V_C) = +0.7896\text{ V}$

$$\therefore \frac{I_{D4}}{1\text{ mA}} = \frac{I_S e^{V_D/0.05}}{I_S e^{0.7/0.05}}$$

$$I_{D4} = e^{\frac{0.7986 - 0.7}{0.05}} = 6\text{ mA}$$

$\therefore v_O = -6\text{ V}$ i.e. $2 \times 3 = 6$.

For squarer: $v_i = 2$ through $\frac{1}{2}\text{ k}\Omega$ resistor

$I_{D1} = 4\text{mA} \rightarrow V_A = -(0.7 + 0.05\ln 4)$
$= -0.7693$
$V_D = -(-0.7693) = 0.7693\text{ V}$

$$I_{D4} = e^{\frac{0.7693 - 0.7}{0.05}} = 3.999\text{ mA}$$

$\therefore V_D = -3.999\text{ V}$ i.e. $2^2 = 4$.

12.40

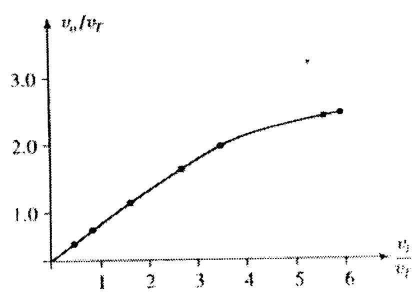
Say $V_{BE} = \tilde{V}_D @ I\ n = 1$

for $v_O = 0.25 v_T$:

$$I_R = \frac{0.25V_T}{R} = \frac{0.25V_T}{\frac{2.5V_T}{I}} = \frac{I}{10}$$

$$V_{BE1} = \tilde{V}_D + nV_T\ln\left(\frac{I + I/10}{I}\right)$$

$\cong \tilde{V}_D + V_T\ln(1.1)$



$$V_{BE2} = \tilde{V}_D + nV_T\ln\left(\frac{I - I/10}{I}\right)$$

$\cong \tilde{V}_D + V_T\ln(0.9)$
$V_I = -V_{BE2} + V_D + V_{BE1}$
$= V_T[\ln(1.1) + 0.25 - \ln(0.9)]$
$= 0.451V_T$

For $v_O = 0.5V_T$

$$I_R = \frac{0.5\text{ I}}{2.5} = 0.2\text{ I}$$

$V_I = V_T[\ln(1.2) + 0.5 - \ln(0.8)]$
$= 0.905V_T$
$V_O = V_T\ I_R = 0.4\text{ I}$
$V_I = V_T[\ln 1.4 + 1 - \ln 0.6]$
$= 1.847V_T$
$V_O = 1.5V_T\ I_R = 0.6\text{ I}$
$V_I = V_T(\ln 1.6 + 1.5 - \ln 0.4)$
$= 2.886V_T$
$V_O = 2V_T\ I_R = 0.8\text{ I}$
$V_I = V_T(\ln 1.8 + 2 - \ln 0.2) = 4.197V_T$
$V_O = 2.4V_T\ I_R = 0.96\text{ I}$
$V_I = V_T(\ln 1.96 + 2.4 - \ln 0.04) = 6.292V_T$
$V_O = 2.42V_T\ I_R = 0.968\text{ I}$
$V_I = V_T(\ln 1.968 + 2.42 - \ln 0.032)$
$= 6.519V_T$

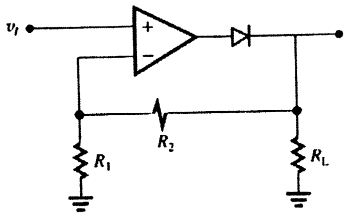
Ideal curve given by

$$v_O = 2.42V_T\sin\left(\frac{v_I}{6.6V_T} \times 90^\circ\right)$$

$$\frac{v_I}{v_T} = \frac{6.6}{90}\sin^{-1}\left(\frac{v_O}{2.42V_T}\right)$$

v_O/v_T	0.25	0.50	1.00	1.50	2.00	2.40	2.42
v_I/v_T	0.451	0.905	1.85	2.89	4.20	6.29	6.52
v_I/v_T (ideal)	0.435	0.874	1.79	2.81	4.09	6.06	6.60

12.41

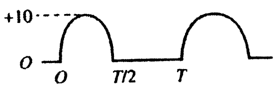


for $v_I \geq 0$

$$v_O = \left(1 + \frac{R_2}{R_1}\right) v_I$$

for a gain of 2 $R_1 = R_2 = 10\ \text{k}\Omega$

for $v_I = 10V_{pp}$ sine wave $v_O \Rightarrow$



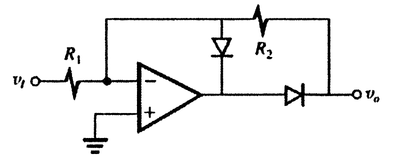
$$\text{Avg} = \frac{1}{T}\int_0^{T/2} 10\sin\frac{2\pi}{T}t\ dt$$

$$= \frac{1}{T}\frac{T}{2\pi}\cos\frac{2\pi}{T}t \times (-10)\Big|_{t=0}^{T/2}$$

$$= \frac{-10}{2\pi}(\cos\pi - \cos 0)$$

$$= 10/\pi = 3.18\ \text{V}$$

12.42



for $v_I < 0 \Rightarrow v_O = -R_2/R_1$

$R_{in} = R_1 = 100\ \text{k}\Omega \quad \therefore R_2 = 200\ \text{k}\Omega$

12.43

for high R_{in}, use $R_1 = 1\ \text{M}\Omega$

Ac gain is given by R_2/R_1

$\Rightarrow R_2 = 1\ \text{M}\Omega$

Now for 1 *VrMs* sine, peak is 1.414 V. The value of V_1 is then $\frac{1.414}{\pi} = 0.450\ \text{V}$

For 10 V out at second stage gain (dc)

$$= \frac{10}{0.450} = 22.2$$

$\therefore R_4/R_3 = 22.2$

& Choose $\frac{1}{2\pi R_4 C} = 10\ \text{Hz}$ (i.e. corner frequency)

To make C small, make $R_4 = 1\ \text{M}\Omega$

$\therefore C = 15.9\ \text{nF}$

$$R_3 = \frac{1\ \text{M}\Omega}{22.2} = 45\ \text{k}\Omega$$

12.44

At the +ve terminal $V_+ = -5\text{V}$

for $v_I > -5$ D_1 is "ON" and faces virtual short.

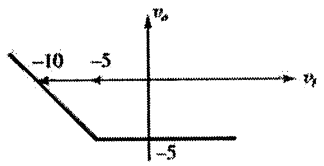
$\therefore V_- = -5$, and no current will flow in feedback R.

$\therefore v_O = -5\ \text{V}$

for $v_I < -5$ D_1 is "off" and

$$\frac{v_O}{v_I} = \frac{-5 - v_I}{R} = \frac{v_O + 5}{R}$$

$$\Rightarrow v_O = -v_I$$



12.45

for $v_I < 0$

D_2 "off"

$\frac{v_{01}}{v_I} = -1$

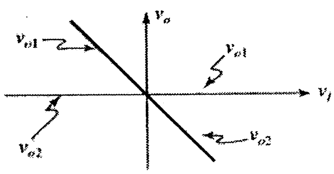
$\frac{v_{02}}{v_I} = 0$

for $v_I > 0$

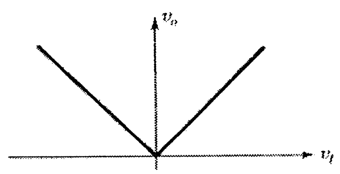
D_1 "off"

$\frac{v_{01}}{v_I} = 0$

$\frac{v_{02}}{v_I} = -1$



12.46



For $v_I < 0$ – Diode is on, and cathode is forced to $\approx$ 0 V.

$\therefore (v_O / v_I) = -1$

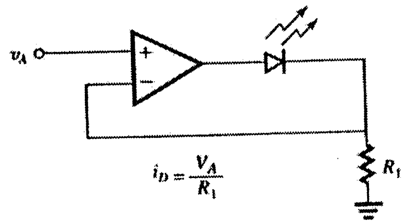
For $v_I > 0$ – Diode is off, and the cathode now follows v_I since no current flows in resistor. So v_o must follow

v_I so that no current flows in feedback resistor.

$\therefore \frac{v_O}{v_I} = +1$

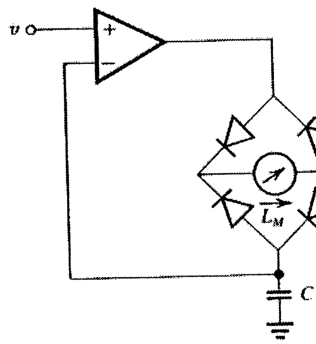
12.47

Simply place the LED in the feedback path.



$i_D = \frac{V_A}{R_1}$

12.48



$i_m = |i_c| = c\frac{dv}{dt}$ using $R = 1\ \text{k}\Omega$

$i_m = |i_R| = \frac{|v|}{R} = \frac{|v|}{1\ \text{k}\Omega} \Rightarrow i_m = |v|$ mA

Now $v = V\sin 2\pi 60t$

$\Rightarrow i_m = C \times 2\pi 60 |\cos(2\pi 60t)|$

for equivalence:

$\frac{V}{10^3}|\sin 2\pi 60t| = 2\pi 60 VC|\cos 2\pi 60t|$

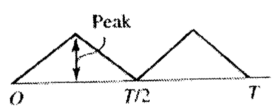
$\therefore C = \frac{1}{2\pi 60 10^3} = 2.65\ \mu\text{F}$

At 120 Hz: $i_m = 2\pi 120 VC|\cos 2\pi 60t|$

$i_{m120} = 2 i_{m60}$

At 180 Hz: $i_{m180} = 3 i_{m60}$

For Δ-wave



with R.

$i_m = 1$ mA, $R = 1$ kΩ

∴ Full wave rectified wave has average voltage $= N$.

$\therefore V_{peak} = 2$ V

with C:

$$\text{slope} = \frac{V_{peak}}{T/4} = 4V_{peak}f$$

$= 4 \times 2 \times 60 = 480$

Now: current through the capacitor will be a square wave (50% duty cycle)

Peak current $= 2.65 \times 10^{-6} \times 480$

$= 1.27$ mA.

$\therefore i_m = i_{avg} = 1.27$ mA

12.49

10 V pulses of 10 μs, and large C_{LOAD}, will cause the op amp to current limit.

Charge transfered in one pulse:

$Q = (10\text{ mA})(10\ \mu\text{S})$

$= 10^{-7}$ C

Voltage change per pulse:

$$\Delta V = Q/C = \frac{10^{-7}}{10 \times 10^{-6}} = 10\text{ mV}$$

after: 1 pulse	$V_c = 10$ mV
2 pulses	20 mV
10 pulses	100 mV
to reach 0.5 V	require 50 pulses
1.0 V	100 pulses
2.0 V	200 pulses

12.50

For $1V_{PP}$, peak detector output $V_O = 0.5$ V.

Ripple voltage = (1%) 0.5 = 5 mV

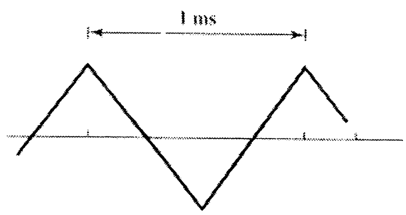
Total leakage = 10 + 1 = 11 nA

∴ total charge lost:

$\Delta Q = 11\text{ nA} \times 1\text{ ms} = 11$ PC

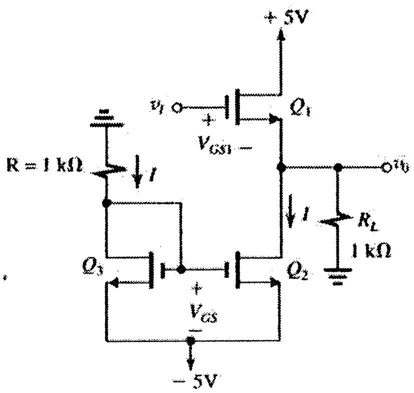
∴ Required capacitance:

$$C = \frac{Q}{\Delta V} = \frac{11 \times 10^{-12}}{5 \times 10^{-3}} = 2.2\text{ nF}$$



13.1

First we determine the bias current I as follows:



$$I = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_t)^2$$

But $V_{GS} = 5 - IR$

$= 5 - I$

Thus

$$I = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(5 - I - V_t)^2$$

$I = 10(5 - I - 1)^2$

$\Rightarrow I^2 - 8.1I + 16 = 0$

$I = 3.416$ mA and $V_{GS} = 1.584$ V

The upper limit on v_O is determined by Q_1 leaving the saturation region (and entering the triode region). This occurs when v_I exceeds V_{DI} by V_t volts,

$v_{I\max} = 5 + 1 = +6$ V

To obtain the corresponding value of v_o we must find the corresponding value of V_{GS1}, as follows:

$v_o = v_I - V_{VSI}$

$$i_L = \frac{v_o}{R_L} = \frac{v_I - V_{GSI}}{1}$$

$= v_I - V_{GS1} = 6 - V_{GS1}$

$i_1 = I + i_L$

$= 3.416 + 6 - V_{GS1}$

$= 9.416 - V_{GS1}$

But $i_1 = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS1} - V_t)^2$

Thus, $9.416 - V_{GS1} = 10(V_{GS1} - 1)^2$

$\Rightarrow V_{GS1}^2 - 1.9V_{GS1} + 0.0584 = 0$

$V_{GS1} = 1.869$ V

$v_{O\max} = 6 - 1.869$

$= +4.131$ V

The lower limit of v_o is determined either by Q_1 cutting off,

$v_o = -IR_L = -3.416 \times 1 = -3.416$ V

or by Q_2 leaving sturation,

$v_o = V_{GS} - V_t$

$= -5 + 1.584 - 1 = -4.416$ V

Thus, $v_{O\min} = -3.416$ V

The corresponding value of v_I is determined by moving that since Q_1 is on the verge of cut-off,

$V_{GS1} = V_t = 1$ V and

$v_I = -3.416 + 1 = -2.416$ V

13.2

For a load resistance of 100 Ω and v_o ranging between −5 V and +5 V, the maximum current through Q_1 is

$I + \frac{5}{0.1} = I + 50$. mA and the minimum current is $I - \frac{5}{0.1} = I - 50$. mA.

For a current ratio of 10,

$$\frac{I + 50}{I - 50} = 10$$

$\Rightarrow I = 61.1$ mA

$$R = \frac{9.3\text{ V}}{61.1\text{ mA}} = 152\ \Omega$$

The incremental voltage gain is $A_v = \frac{R_L}{R_L + r_{e1}}$

For $R_L = 100\ \Omega$;

At v_o = +5 V, I_{E1} = 61.1+50 = 111.1 mA

$$r_{e1} = \frac{25}{111.1} = 0.225\ \Omega$$

$$A_v = \frac{100}{100 + 0.225} = 0.998\ \text{V/V}$$

At v_o = 0 V, I_{E1} = 61.1 mA

$$r_{e1} = \frac{25}{61.1} = 0.409\ \Omega$$

$$A_v = \frac{100}{100.409} = 0.996\ \text{V/V}$$

At v_o = −5 V, I_{E1}=61.1−50=11.1 mA

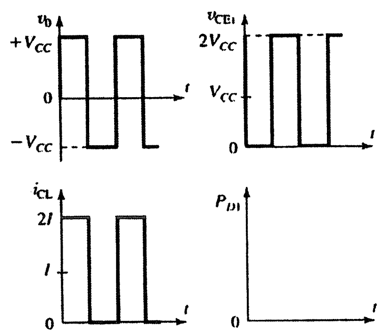
$$r_{e1} = \frac{25}{11.1} = 2.25\ \Omega$$

$$A_v = \frac{100}{102.25} = 0.978\ \text{V/V}$$

Thus the incremental gain changes by 0.998−0.978 = 0.02 or about 2% over the range of v_o.

13.3

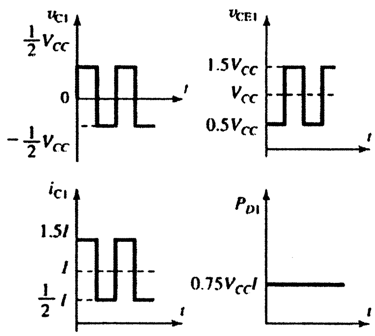
For v_o being a square wave of $\pm V_{CC}$ levels:



$P_{D1}|_{average}$ = 0 For the corresponding sine wave curve

$$P_{D1}|_{avg} = \frac{1}{2} V_{CC} I$$

For v_o a square wave of $\pm V_{CC}/Z$ levels :



$$P_{D1}|_{average} = 0.75 V_{CC} I$$

For a sine-wave output of $V_{CC}/2$ peak amplitude,

$$v_{C1} = \frac{1}{2} V_{CC} \sin\theta$$

$$i_{C1} = I + \frac{\frac{1}{2}V_{CC}}{R_L}\sin\theta = I + \frac{1}{2} I \sin\theta$$

$$v_{CE1} = V_{CC} - \frac{1}{2} V_{CC} \sin\theta$$

$$P_{D1} = \left(V_{CC} - \frac{1}{2}V_{CC}\sin\theta\right)\left(I + \frac{1}{2}I\sin\theta\right)$$

$$= V_{CC}I - \frac{1}{4}V_{CC}I\sin^2\theta$$

$$= V_{CC} - \frac{1}{4}V_{CC}I \times \frac{1}{2}(1 - \cos 2\theta)$$

$$= \frac{7}{8} V_{CC}I + \frac{1}{8} V_{CC}I \cos 2\theta$$

$$P_{D1}|_{average} = \frac{7}{8}V_{CC}I$$

13.4

In all cases, the average voltage across Q_2 is equal to V_{CC}. Thus, since Q_2 conducts a constant current I, its angle power dissipation is $V_{CC} \cdot I$.

13.5

V_{CC} = 16, 12, 10 and 8 V

I = 100 mA R_L = 100 Ω

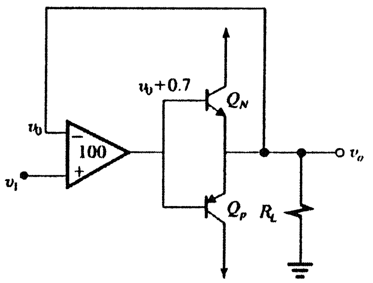
$\hat{V}_o$ = 8 V

$$\eta = \frac{1}{4}\left(\frac{\hat{V}_o}{IR_L}\right)\left(\frac{\hat{V}_o}{V_{CC}}\right)$$

$$= \frac{1}{4}\left(\frac{8}{10}\right)\left(\frac{8}{V_{CC}}\right) = \frac{1.6}{V_{CC}}$$

V_{CC}	16	12	10	8
η	10%	13.3%	16%	20%

13.6



With v_I sufficiently positive so that Q_N is conducting the situation shown obtains. Then,

$(v_I - v_o) \times 100 = v_o + 0.7$

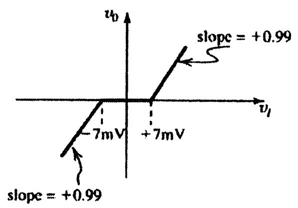
$$\Rightarrow v_o = \frac{1}{1.01}(v_I - 0.007)$$

This relationship applies for $v_I \geq 0.007$. Similarly, for v_I sufficiently negative so that Q_p conducts, the voltage at the output of the amplifier becomes $v_o - 0.7$, thus

$(v_I - v_o) \times 100 = v_o - 0.7$

$$\Rightarrow v_o = \frac{1}{1.01}(v_I + 0.007)$$

This relationship applies for $v_I \geq -0.007$. The result is the transfer characteristic



Without the feedback arrangement, the deadband becomes ±700 mV and the slope change a little (to nearly +1 V/V).

13.7

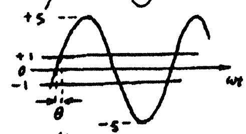
With $R_L = \infty$ and $v_I = +5$ V, v_O will be $v_I - V_{GS_1} = v_I - V_t = 4$ V (since the current is virtually zero and thus $V_{GS} \simeq V_t$). Thus the resulting peak output voltage will be $\underline{4\text{ V}}$.

$\sin\theta = \frac{1}{5}$

$\Rightarrow \theta = 11.54°$

Cross-over interval $= 4\theta$

Fraction of Cycle $= \frac{4\theta}{360°} = \underline{12.8\%}$

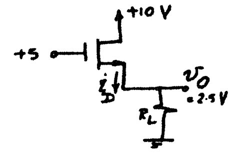


For $v_I = +5$ V and $v_O = +2.5$ V, $V_{GS} = 2.5$ V, then

$i_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_t)^2$

$= 0.1\,(2.5-1)^2 = 0.225$ mA

Thus, $R_L = \frac{2.5}{0.225} = \underline{11.1\text{ k}\Omega}$



13.8

For $V_{CC} = 10$ V and $R_L = 100\ \Omega$, the maximum sine-wave output power occurs when $\hat{V}_o = V_{CC}$

and is $P_{L\max} = \frac{1}{2}\frac{V_{CC}^2}{R_L}$

$$= \frac{1}{2} \times \frac{100}{100} = 0.5 \text{ W}$$

Correspondingly,

$$P_{S+} = P_{S-} = \frac{1}{\pi}\frac{\hat{V}_o}{R_L}V_{CC}$$

$$= \frac{1}{\pi} \times \frac{10}{100} \times 10 = 0.318 \text{ W}$$

For a total supply power of

$P_s = 2 \times 0.318 = 0.637$W

The power conversion efficency η is

$$\eta = \frac{P_L}{P_S} \times 100 = \frac{0.5}{0.637} \times 100 = 78.5\%$$

For $\hat{V}_o = 5$ V,

$$P_L = \frac{1}{2}\frac{\hat{V}_o^2}{R_L} = \frac{1}{2} \times \frac{25}{100} = \frac{1}{8} \text{ W}$$

$$P_{S+} = P_{S-} = \frac{1}{\pi}\frac{\hat{V}_o}{R_L}V_{CC}$$

$$= \frac{1}{\pi} \times \frac{5}{100} \times 10 = \frac{1}{2\pi}$$

$$P_S = \frac{1}{\pi} \text{ W} = 0.318 \text{ W}$$

$$\eta = \frac{1/8}{1/\pi} \times 100 = \frac{\pi}{8} \times 100 = 39.3\%$$

13.9

$V_{CC} = 5\text{ V}$

For maximum η,

$\hat{V}_o = V_{CC} = 5\text{ V}$

The output voltage that results in maximum device dissipation is given by Eq. (12.20),

$$\hat{V}_o = \frac{2}{\pi}V_{CC}$$

$$= \frac{2}{\pi} \times 5 = 3.18\text{ V}$$

If operation is always at full output voltage, $\eta = 78.5\%$ and thus

$$P_{\text{dissipation}} = (1-\eta)P_s$$

$$= (1-\eta)\frac{P_L}{\eta} = \frac{1-0.785}{0.785}P_L = 0.274P_L$$

$$P_{\text{dissipation/device}} = \frac{1}{2} \times 0.274P_L = 0.137P_L$$

For a rated device dissipation of 1 W, and using a factor of 2 safety margin,

$$P_{\text{dissipation/device}} = 0.5\text{ W}$$

$$= 0.137P_L$$

$$\Rightarrow P_L = 3.65\text{ W}$$

$$3.65 = \frac{1}{2} \times \frac{25}{R_L}$$

$$\Rightarrow R_L = 3.425\ \Omega \text{ (i.e. } R_L \geq 3.425\ \Omega)$$

The corresponding output power (i.e., greatest possible output power) is 3.65W.

If operation is allowed at $\hat{V}_o = \frac{1}{2}V_{CC} = 2.5\text{ V}$,

$$\eta = \frac{\pi}{4}\frac{\hat{V}_o}{V_{CC}} \text{ (Eq. 12.15)}$$

$$= \frac{\pi}{4} \times \frac{1}{2} = 0.393$$

$$P_{\text{dissipation/device}} = \frac{1}{2}\ \frac{1-\eta}{\eta}P_L = 0.772P_L$$

$$0.5 = 0.772P_L$$

$$\Rightarrow P_L = 0.647\text{ W}$$

$$= \frac{1}{2}\ \frac{2.5^2}{R_L}$$

$$\Rightarrow R_L = 4.83\ \Omega \text{ (i.e., } \geq 4.83\ \Omega)$$

13.10

$$P_L = \frac{1}{2}\ \frac{\hat{V}_o^2}{R_L}$$

$$100 = \frac{1}{2}\ \frac{\hat{V}_o^2}{16}$$

$$\hat{V}_o = 56.6\text{ V}$$

$$V_{CC} = 56.6 + 4 = 60.6 \rightarrow 61\text{ V}$$

Peak current from each supply $= \frac{\hat{V}_o}{R_L} = \frac{56.6}{16}$

$$= 3.54\text{ A}$$

$$P_{s+} = P_{s-} = \frac{1}{\pi} \times 3.54 \times 61$$

Thus, $P_s = \frac{2}{\pi} \times 3.54 \times 61$

$$= 137.4\text{ W}$$

$$\eta = \frac{P_L}{P_S} = \frac{100}{137.4} = 73\%$$

Using Eq. (12.22),

$$P_{DN\max} = P_{DP\max} = \frac{V_{CC}^2}{\pi^2 R_L} = \frac{61^2}{\pi^2 \times 16}$$

$$= 23.6\text{W}$$

13.11

$$P_L = \frac{\hat{V}_o^2}{R_L}$$

$$P_{S+} = P_{S-} = \frac{1}{2}\left(\frac{\hat{V}_o}{R_L}\right)V_{SS}$$

$$P_S = \frac{\hat{V}_o}{R_L}V_{SS}$$

$$\eta = \frac{P_L}{P_S} = \frac{\hat{V}_o^2/R_L}{\hat{V}_o V_{SS} R_L} = \frac{\hat{V}_o}{V_{SS}}$$

$\eta_{\max} = 1(100\%)$, obtained for $\hat{V}_o = V_{SS}$

$$P_{L\max} = \frac{V_{SS}^2}{R_L}$$

$$P_{\text{dissipation}} = P_S - P_L$$

$$= \frac{\hat{V}_o}{R_L}V_{SS} - \frac{\hat{V}_o^2}{R_L}$$

$$\frac{\partial P_{\text{dissipation}}}{\partial \hat{V}_o} = \frac{V_{SS}}{R_L} - \frac{2\hat{V}_o}{R_L}$$

$$= 0 \text{ for } \hat{V}_o = \frac{V_{SS}}{2}$$

Correspondingly, $\eta = \frac{V_{SS}/2}{V_{SS}} = \frac{1}{2}$ or 50%

13.12

$$A_v = \frac{R_L}{R_L + R_{out}}$$

where, $R_{out} = \frac{r_e}{2} = \frac{V_T}{2 I_Q}$.

For $A_v \geq 0.99$ with $R_L \geq 100\ \Omega$,

$$0.99 = \frac{100}{100 + R_{out}} \Rightarrow R_{out} = 1\ \Omega$$

$$\frac{V_T}{2 I_Q} = 1 \Rightarrow I_Q = \underline{\underline{12.5\ mA}}$$

$$V_{BB} = 2 V_{BE}$$

$$= 2\left[0.7 + V_T \ln \frac{12.5}{100}\right]$$

$$= \underline{\underline{1.296\ V}}$$

This table is for 13.13

v_s(V)	i_L (mA)	i_N (mA)	i_P (mA)	V_{BEN} (V)	V_{EBP} (V)	V_o (V)	V/V	R_{out} (Ω)	V/V	i_I	R_{in} (Ω)
+10.0	100	100.04	0.04	0.691	0.495	10.1	0.99	0.25	1.00	2	5050
+5.0	50	50.08	0.08	0.673	0.513	5.08	0.98	0.50	1.00	1	5080
+1.0	10	10.39	0.39	0.634	0.552	1.041	0.96	2.32	0.98	0.2	5205
+0.5	5	5.70	0.70	0.619	0.567	0.526	0.95	4.03	0.96	0.1	5260
+0.2	2	3.24	1.24	0.605	0.581	0.212	0.94	5.58	0.95	0.04	5300
+0.1	1	2.56	1.56	0.599	0.587	0.106	0.94	6.07	0.94	0.02	5300
0	0	2	2	0.593	0.593	0	-	6.25	0.94		
−0.1	−1	1.56	2.56	0.587	0.599	−0.106	0.94	6.07	0.94	−0.02	5300
−0.2	−2	1.24	3.24	0.581	0.605	−0.212	0.94	5.58	0.95	−0.04	5300
−0.5	−5	0.70	5.70	0.567	0.619	−0.526	0.95	4.03	0.96	−0.1	5260
−1.0	−10	0.39	10.39	0.552	0.634	−1.041	0.96	2.32	0.98	−0.2	5205
−5.0	−50	0.08	50.08	0.513	0.673	−5.08	0.98	0.50	1.00	−1	5080
−10.0	−100	0.04	100.04	0.495	0.691	−10.1	0.99	0.25	1.00	−2	5050

$$I_Q = \frac{V_T}{4} = \frac{25 \times 10^{-3}}{4} = 6.25 \text{ mA}$$

$$V_{BB} = 2V_{BE} = 2\left[0.7 + V_T \ln\left(\frac{6.25}{100}\right)\right]$$

$= 1.26$ V

13.13

The current i_I can be obtained as

$$i_I = \frac{i_N}{\beta_N + 1} - \frac{i_P}{\beta_P + 1} = \frac{i_L}{\beta + 1}$$

$\therefore \beta_N = \beta_P = \beta = 49$

Using values of v_I from the table one can evaluate R_{in}

$$\therefore R_{in} = \frac{v_I}{i_I}$$

Using resistance reflection rule

$R_{in} \cong \beta R_L = 49 \times 100$

$= 4900\ \Omega$

For large input signal the two values of R_{in} are somewhat same. For the small values of v_I, the calculated value in the table is larger.

13.14

$$\frac{v_o}{v_i} = \frac{R_L}{R_L + R_{out}} \text{ and}$$

$$R_{out} = \frac{V_T}{i_P + i_N} = \frac{V_T}{I_Q + I_Q} \text{ at } v_o = 0$$

a. $\epsilon = 1 - \left.\frac{v_o}{v_I}\right|_{v_o = 0}$

$$= 1 - \frac{R_L}{R_L + R_{out}} = 1 - \frac{R_L}{R_L + \frac{V_T}{2I_Q}} = \frac{V_T/2I_Q}{R_L + (V_T/2I_Q)}$$

$$\epsilon = \frac{V_T/2I_Q}{R_L + (V_T/2I_Q)} = \frac{V_T}{2R_LI_Q + V_T}$$

If $2I_QR_L >> V_T$

$$\epsilon = \frac{V_T}{2I_QR_L}$$

b. Quiescent Power Dissipation $= 2V_{CC}I_Q = P_D$

c. ϵ × Quiescent Power Dissipation =

$$\frac{V_T}{2I_QR_L} \times 2V_{CC}I_Q = V_T \times \left(\frac{V_{CC}}{R_L}\right)$$

$$\therefore \epsilon P_D = V_T\left(\frac{V_{CC}}{R_L}\right)$$

d. $\epsilon P_D = V_T\frac{V_{CC}}{R_L} = 25 \times 10^{-3} \times \frac{15}{100}$

$= 3.75$ mW

$$P_D = \frac{3.75 \times 10^{-3}}{\epsilon}$$

ϵ	P_D in mW
0.05	75
0.02	187.5
0.01	375

13.15

$I_Q \simeq I_{bias} = 0.5$ mA, neglecting the base current of Q_N. More precisely,

$$I_Q = I_{bias} - \frac{I_Q}{\beta + 1}$$

$$\Rightarrow I_Q = \frac{I_{bias}}{1 + \frac{1}{\beta + 1}} \simeq 0.98 \times 0.5 = 0.49 \text{ mA}$$

The largest positive output is obtained when all of I_{bais} flows into the base of Q_N, resulting in

$$v_o = (\beta_N + 1) I_{bias} R_L$$

$$= 51 \times 0.5 \times 100\ \Omega = 2.55 \text{ V}$$

The largest possible negative output voltage is limited by the saturation of

Q_P to $-10 + V_{ECSat} = -10$ V

To achieve a maximum positive output of 10 V without changing I_{bias}, β_N must be

$$10 = (\beta_N + 1) \times 0.5 \times 100\ \Omega$$

$$\Rightarrow \beta_N = 199$$

Alternatively, if β_N is held at 50, I_{bias} must be increased so that

$$10 = 51 \times I_{bias} \times 100\Omega$$

$$\Rightarrow I_{bias} = 1.96\text{mA}$$

for which,

$$I_Q = \frac{I_{bias}}{1 + \frac{1}{\beta + 1}} = 1.92 \text{ mA}$$

13.16

At 20°C, $I_Q = 1\text{mA} = I_S\, e^{(0.6/0.025)}$

$\Rightarrow I_S$ (at 20°C) $= 3.78 \times 10^{-11}$ mA

At 70°C, $I_S = 3.78 \times 10^{-11}(1.14)^{50}$

$= 2.64 \times 10^{-8}$ mA

At 70°C, $V_T = 25\,\frac{273 + 70}{273 + 20} = 29.3$ mV

Thus, I_Q (at 70°C) $= 2.64 \times 10^{-8}\, e^{0.6/0.0293}$

$= 20.7$ mA

Additional current $= 20.7 - 1 = 19.7$ mA

Additional power $= 2 \times 20 \times 19.7 = 788$mW

Additional temperature rise $= 10 \times 0.788 = 7.9$°C,

At 77.9°C:

$$V_T = \frac{25}{293}(273 + 77.9) = 29.9 \text{ mV}$$

$$I_Q = 3.78 \times 10^{-11} \times (1.14)^{57.9}\, e^{(0.6/0.0299)}$$

$= 37.6$ mA

etc., etc.

13.17

Since the peak positive output current is 200 mA, the base current of Q_N can be as high as

$$\frac{200}{\beta_N + 1} = \frac{200}{51} \simeq 4 \text{ mA. We select}$$

$I_{bias} = 5$ mA, thus providing the multiplier with a minimum current of 1 mA.

Under quiescent conditions ($v_o = 0$ and $i_L = 0$) the base current of Q_N can be neglected.

Selecting $I_R = 0.5$ mA leaves $I_{C1} = 4.5$ mA. To obtain a quiescent current of 2 mA in the output transistors, V_{BB} should be

$$V_{BB} = 2V_T \ln\frac{2 \times 10^{-3}}{10^{-15}} = 1.19 \text{ V}$$

Thus

$$R_1 + R_2 = \frac{V_{BB}}{I_R} = \frac{1.19}{0.5} = 2.38 \text{ k}\Omega$$

At a collector current of 4.5 mA, Q_1 has

$$V_{BEL} = V_T \ln\frac{4.5 \times 10^{-3}}{10^{-14}} = 0.671 \text{ V}$$

The value of R_1 can now be determined as

$$R_1 = \frac{0.671}{0.5} = 1.34 \text{ k}\Omega \text{ and}$$

$$R_2 = 2.58 - 1.34 = 1.04 \text{ k}\Omega$$

13.18

(a) $V_{BE} = 0.7$ V at 1 mA

At 0.5 mA,

$$V_{BE} = 0.7 + 0.025 \ln\frac{0.5}{1} = 0.683 \text{ V}$$

Thus $R_1 = \frac{0.683}{0.5} = 1.365\ k\Omega$

and $R_2 = 1.365\ k\Omega$

(b) For $I_{bias} = 2$ mA, I_C increases to nearly 1.5 mA for which

$V_{BE} = 0.7 + 0.025\ \ln\frac{1.3}{1} = 0.710$ V

Note that $I_R = \frac{0.710}{1.365} = 0.52$ mA is very nearly equal to the assumed value of 0.50 mA. Thus no further iterations are required.

$V_{BB} = 2V_{BE} = 1.420$ V

(c) For $I_{bias} = 10$ mA, assume that I_R remains constant at 0.5 mA, thus $I_{C1} = 9.5$ mA

and $V_{BE} = 0.7 + 0.025\ \ln\frac{9.5}{1} = 0.756$ V

at which

$I_R = \frac{0.755}{1.365} = 0.554$ mA

Thus,

$I_{C1} = 10 - 0.554 = 9.45$ mA

and $V_{BE} = 0.7 + 0.025\ \ln\frac{9.45}{1} = 0.756$ V

Thus $V_{BB} = 2 \times 0.756 = 1.512$ V

(d) Now for $\beta = 100$,

$I_{R1} = \frac{0.756}{1.365} = 0.554$ mA

$I_{R2} = 0.554 + \frac{9.45}{101} = 0.648$ mA

$I_C = 10 - 0.648 = 9.352$ mA

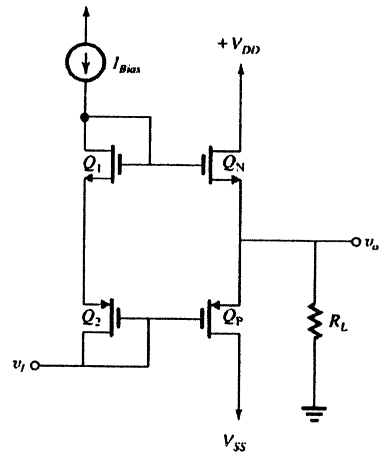
Thus, $V_{BE} = 0.7 + 0.025\ \ln\frac{9.352}{1} = 0.756$ V

$V_{BB} = 0.756 + I_{R2} R_2$

$= 0.756 + 0.648 \times 1.365$

$= 1.641$ V

13.19



a. under quiescent condition

Voltage gain $= \frac{v_O}{v_i} = \frac{R_L}{R_L + R_{out}}$

As shown in problem 11.24, for matched transistors

$R_{out} = \frac{1}{2g_m}$

Substitute for R_{out} above for $\frac{v_O}{v_i}$

$\frac{v_O}{v_i} = \frac{R_L}{R_L + \frac{1}{2g_m}}$

b. Voltage gain $= 0.98 = \frac{R_L}{R_L + \frac{1}{2g_m}}$

$0.98 = \frac{1000}{1000 + \frac{1}{2g_m}}$

$\Rightarrow g_m = 24.5$ mA/V

For Q_1, $I_{Bias} = I_D$

$\therefore 0.1 = \frac{1}{2}k_1 V_{OV}^2$

$$0.1 = \frac{1}{2} \times 20 \times V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.1 \text{ V}$$

For Q_N

$$g_m = k_n V_{OV}$$

$$24.5 = k_n \times 0.1$$

$$k_n = 245 \text{ mA/V}^2$$

$$n = \frac{k_n}{k_1} = \frac{245}{20}$$

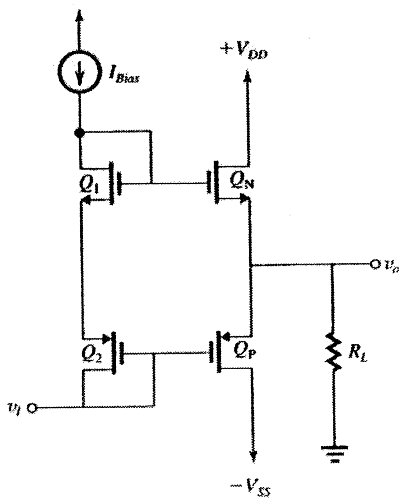
$$= 12.25$$

and $I_Q = nI_{\text{bias}}$

$$= 12.25 \times 0.1$$

$$= 1.225 \text{ mA}$$

13.20



$$I_Q = I_{\text{Bias}} \frac{(W/L)_n}{(W/L)_1}$$

$$1 = 0.1 \frac{(W/L)_n}{(W/L)_1}$$

$$\frac{(W/L)_n}{(W/L)_1} = 10$$

$$Q_1\text{: } I_{\text{Bias}} = \frac{1}{2} k_n' \left(\frac{W}{L}\right)_1 V_{ov}^2$$

$$0.1 = \frac{1}{2} \times 0.250 \times \left(\frac{W}{L}\right)_1 \times (0.2)^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_1 = 20$$

$$Q_2\text{: } 0.1 = \frac{1}{2} \times 0.100 \times \left(\frac{W}{L}\right)_2 \times (0.2)^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_2 = 50$$

$$Q_N\text{: } 1 = \frac{1}{2} \times 0.250 \times \left(\frac{W}{L}\right)_N \times (0.2)^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_N = 200$$

$$Q_P\text{: } 1 = \frac{1}{2} \times 0.100 \times \left(\frac{W}{L}\right)_P \times (0.2)^2$$

$$\left(\frac{W}{L}\right)_P = 500$$

b. From the circuit we get $-v_I + V_{GSP} + v_o = 0$

Since $v_o = 0$

$v_I = V_{GSP}$

$$V_{SGP} = |V_{ov}| + |V_t|$$

$$= 0.2 + 0.45$$

$$= -0.65 \text{ V}$$

$$\therefore v_I = V_{GSP} = -0.65 \text{ V}$$

c. Using equation 11.4

$$v_{o\max} = V_{DD} - V_{ov}|_{\text{Bias}} - V_{\text{GSN}}$$

To find V_{GSN},

$$i_{\text{DNmax}} = \frac{1}{2} k_n' \frac{W}{L} (V_{\text{GSN}} - V_t)^2$$

$$10 = \frac{1}{2} \times 0.250 \times 200\ (V_{\text{GSN}} - V_t)^2$$

$$\Rightarrow V_{\text{GSN}} - V_t = 0.63 \text{ V}$$

$$V_{\text{GSN}} = V_t + 0.63 = 0.45 + 0.63 \simeq 1.1 \text{ V}$$

$$\therefore v_{o\max} = 2.5 - 0.2 - 1.1 = 1.2 \text{ V}$$

13.21

$I_Q = 3$ mA, $|V_{ov}| = 0.15$ V

$$g_{mn} = g_{mp} = \frac{2I_D}{V_{ov}} = \frac{2 \times 3}{0.15} = 0.04 \text{ A/V}$$

$$= 40 \text{ mA/V}$$

Using equation 11.57

$$R_{\text{out}} = \frac{1}{\mu(g_{mp} + g_{mn})} = \frac{1}{5(0.04 + 0.04)}$$

$$= 2.5\ \Omega$$

13.22

$$|\text{Gain Error}| = \frac{1}{2\mu g_m R_L}$$

From equation 11.57

$$R_{\text{out}} = \frac{1}{\mu(g_{mp} + g_{mp})}$$

$$= \frac{1}{2\mu g_m} \text{ when } g_{mp} = g_{mn} = g_m$$

$$\therefore |\text{Gain Error}| = \frac{1}{2\mu g_m} \times \frac{1}{R_L}$$

$$= \frac{R_{\text{out}}}{R_L}$$

$$|\text{Gain Error}| = \frac{1}{2\mu g_m R_L}$$

$$0.05 = \frac{1}{2 \times 10 \times g_m \times 100}$$

$g_m = 0.01$ A/V $= 10$ mA/V

$$g_m = \frac{2I_Q}{V_{ov}}$$

$$V_{ov} = \frac{2I_Q}{g_m} = \frac{2 \times 1}{10}$$

$V_{ov} = 0.2$ V

13.23

a. $I_Q = \frac{1}{2} k_n' \frac{W}{L} V_{ov}^2$

$$1.5 \times 10^{-3} = \frac{1}{2} \times 100 \times 10^{-6} \left(\frac{W}{L}\right)_p (0.15)^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_p \simeq 1333.3$$

$$\left(\frac{W}{L}\right)_N = \frac{(W/L)_P}{(k_n'/k_{Q_p}')} = \frac{1333.3}{(250/100)}$$

$= 533.3$

b. $g_m = \frac{2I_Q}{V_{ov}} = \frac{2 \times 1.5}{0.15} = 20$ mA/V

$= 0.02$ A/V

$$R_{out} = \frac{1}{2\mu g_m} \quad g_{mp} = g_{mn} = g_m$$

$$2.5 = \frac{1}{2\mu \times 0.02}$$

$$\mu = \frac{1}{2.5 \times 2 \times 0.02}$$

$\mu = 10$

c. Gain Error $= -\frac{V_{ov}}{4\mu I_Q R_L}$

$$= -\frac{0.15}{4 \times 10 \times 1.5 \times 10^{-3} \times 50}$$

$= -0.05$

Gain Error $= 5\%$

d. In the quiescant state $v_o = 0$

The voltage at the output of each amplifier will be $= \mu(v_o - v_i) = -\mu v_i$

e. Q_N turn off when the voltage at its gate drops from quiescent value of -1.85 V to -2V, at which point $V_{GSN} = V_{tN}$ and an equal change of -0.15 V appear at the output of the top amplifier.

$$i_P = \frac{1}{2} k_P \left(\frac{W}{L}\right)_p (0.3)^2$$

$$= \frac{1}{2} \times 0.100 \times 1333.3 \times 0.3^2$$

$i_P = 6$ mA

$v_o = 6 \times 10^{-3} \times 50\ \Omega = 0.3$ V

So for $v_o > 0.3$ V, Q_P conducts all the current.

f. the situation at $v_o = v_{omax}$ will occur when Q_P will go from saturation to triode region and it will be approximatly 2 V.

Linear range of v_o from 2 to -2 V

13.24

$$\theta_{JA} = \frac{150 - 25}{0.2} = 625°\text{C/W} = 0.625°\text{C/mW}$$

At 70°C, Power rating

$$= \frac{150 - 70}{0.625} = 128 \text{ mW}$$

$$T_J = 50 + 0.625 \times 100 = 112.5°\text{C}$$

13.25

(a) $\theta_{JA} = \frac{T_{Jmax} - T_{AO}}{P_{DO}}$

$$= \frac{100 - 25}{2} = 37.5°\ \text{C/W}$$

(b) At $T_A = 50°$C

$$P_{Dmax} = \frac{T_{Jmax} - T_A}{\theta_{JA}}$$

$$= \frac{100 - 50}{37.5} = 1.33 \text{ W}$$

(c) $T_J = 25° + 37.5 \times 1 = 62.5°$C

13.26

$$T_C - T_A = \theta_{CA} P_D$$

$$= (\theta_{CS} + \theta_{SA}) P_D$$

$$\Rightarrow P_D = \frac{T_C - T_A}{\theta_{CS} + \theta_{SA}} = \frac{90 - 30}{0.5 + 0.1} = 100 \text{ W}$$

$$T_J - T_C = \theta_{JC} P_D$$

$$130 - 90 = \theta_{JC} \times 100$$

$$\Rightarrow \theta_{JC} = 0.4°\ \text{C/W}$$

13.27

$$\theta_{JC} = \frac{T_J - T_C}{P_D} = \frac{180° - 50°}{50} = 2.6°\ \text{C/W}$$

$$T_J - T_S = \theta_{JS} P_D$$

$$180° - T_S = (\theta_{JC} + \theta_{CS}) P_D$$

$$\Rightarrow T_S = 180 - (2.6 + 0.6) \times 30 = 84°$$

$$T_S - T_A = \theta_{SA} P_D$$

$$84 - 39 = \theta_{SA} \times 30$$

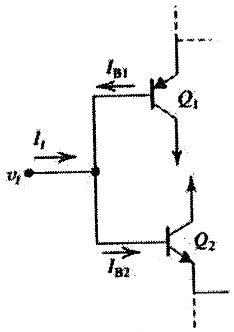
$$\Rightarrow \theta_{SA} = 1.5°\text{C/W}$$

$$\text{Required heat-sink length} = \frac{4.5°\text{C/W/Cm}}{1.5°\text{C/W}}$$

$= 3$ cm

13.28

(a) For $R_L = \infty$:



At $v_I = 0$ V,

$$I_{B1} = I_{B2} = \frac{2.87}{200}$$

$$I_I = I_{B2} - I_{B1} = 0$$

At $v_I = +10$ V,

$$I_{B1} = \frac{0.88}{200}\text{ mA} = 4.4\ \mu\text{A}$$

$$I_{B2} = \frac{4.87}{200}\text{ mA} = 24.4\ \mu\text{A}$$

$$I_I = I_{B2} - I_{B1} = 20\ \mu\text{A}$$

At $v_I = -10$ V,

$$I_{B1} = \frac{4.87}{200}\text{ mA} = 24.4\ \mu\text{A}$$

$$I_{B2} = \frac{0.88}{200}\text{ mA} = 4.4\ \mu\text{A}$$

$$I_I = I_{B2} - I_{B1} = -20\ \mu\text{A}$$

(b) For $R_L = 100\ \Omega$:

At $v_I = 0$ V, $I_I = 0$

At $v_I = +10$ V,

$$I_{B1} = \frac{0.38}{200} = 1.9\ \mu\text{A}$$

$$I_{B2} = \frac{4.87}{200} = 24.4\ \mu\text{A}$$

$$I_I = I_{B2} - I_{B1} = 22.5\ \mu\text{A}$$

At $v_I = -10$ V, $I_I = -22.5\ \mu$A

13.29

Circuit operating near $v_I = 0$ and is fed with a signal source having zero resistance.

The resistance looking as shown by the arrow X is $= R_1 \parallel r_{e1}$

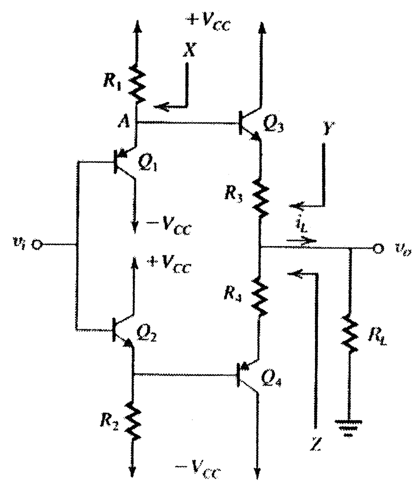
This resistance is reflected from base to the emitter of Q_3 is $= (\beta_3 + 1)/(R_1 \parallel r_{e1})$

This resistance seen as shown by arrow Y_1, from the upper half of the circuit =

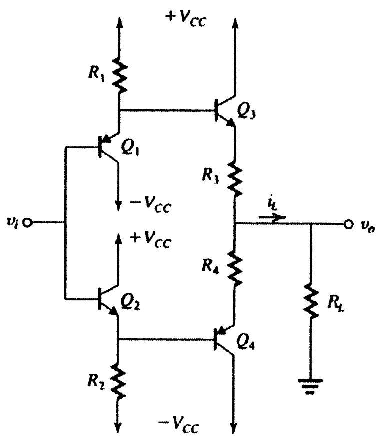
$R_3 + r_{e3} + (\beta_3 + 1)/(R_1 \parallel r_{e1})$

A similar resistance is as shown by the arrow Z and both of these resistances (seen arrow Y and arrow Z) are parallal, therefore

$$R_{out} = \frac{1}{2}[R_3 + r_{e3} + (R_1 \parallel r_{e1})/(\beta_3 + 1)]$$



13.30



At $v_I = 5$V, the voltage V_{R1} across the resistor R_1 is
$v_{R1} = V_{CC} - 0.7 - 5 = 4.3$ V
$i_{R1} = 2 \times 10$ mA
The current i_{R1} should be enough to allow for i_{B3} as much as 10 mA and only a 2 to 1 variation in i_{E1}

$$\therefore R_1 = \frac{V_{R1}}{i_{R1}} = \frac{4.3}{20\text{ m}} = 215\ \Omega = 0.215\text{ k}\Omega$$

Similarly $R_2 = 215\ \Omega = 0.215$ kΩ
Now solve for R_3 and R_4
For $v_I = 0$ and $V_{EB1} \simeq 0.7$ V

$$i_{R1} = \frac{10 - 0.7 - 0}{215} = 43.3\text{ mA}$$

$$v_{EB1} = 0.7 + 25 \times 10^{-3}\ln\left(\frac{43.3}{10}\right) = 0.7366\text{ V}$$

$\simeq 0.74$ V
In Q_3, $I_Q = 40$ mA and $I_{S3} = 3I_{S1} = 30$ mA

$$v_{BE3} = 0.7 + 25 \times 10^{-3}\ln\left(\frac{40}{30}\right) = 0.7072\text{ V}$$

$$R_3 = \frac{V_{EB1} - V_{BE3}}{I_{Q3}} = \frac{0.7366 - 0.7072}{40 \times 10^{-3}} \cong 0.74\ \Omega$$

Similarly $R_4 = 0.74\ \Omega$
Output Resistance at $v_I = 0$

$$R_{out} = \frac{1}{2}\left(R_3 + r_{e3} + \frac{r_{e1} \parallel R_1}{\beta_3 + 1}\right)$$

$$\simeq \frac{1}{2}\left(R_3 + r_{e3} + \frac{r_{e1}}{\beta_3 + 1}\right) \text{ Since } r_{e1} \parallel R_1 \simeq r_{e1}$$

This $\frac{1}{2}$ is there because of two paths to output.

$$r_{e1} \simeq r_{e3} = \frac{25\text{ mV}}{40\text{ mA}} = 0.625\ \Omega$$

$$R_{out} = \frac{1}{2}\left(0.735 + 0.625 + \frac{0.625}{50 + 1}\right)$$

$\simeq 0.69\ \Omega$
Output voltage for $v_I = 1$ V and $R_L = 2\ \Omega$
Let $v_o \simeq 1$ V

$$i_L = \frac{1\text{ V}}{2\ \Omega} = 500\text{ mA}$$

$$i_{B3} = \frac{500}{50} = 10\text{ mA}$$

$$I_{E1} \simeq \frac{10 - 0.7 - 1}{0.215\text{ k}\Omega} - 10 = 28.6\text{ mA}$$

So

$$V_{EB1} = 0.7 + 25 \times 10^{-3}\ln\left(\frac{28.6}{10}\right) = 0.726\text{ V}$$

$V_{B3} = v_I + V_{EB1} = 1 + 0.726 = 1.726$ V
Assuming $i_{E4} \simeq 0$

$$V_{BE3} = 0.7 + 0.025\ln\left(\frac{500}{30}\right)$$

$$\therefore i_{E3} = i_L = 500\text{ mA}$$

$= 0.770$ V

$$\therefore i_L = \frac{1.726 - 0.770}{0.74 + 2} = 0.349\text{ A} \simeq 0.35\text{ A}$$

This value of i_L gives
$v_o = 2\ \Omega \times 0.349$ A $= 0.698$ V
The voltage drop across the series combination of R_4 and the emitter base junction of Q_4 can be determined as follows
$V_{B4} = V_{E2} = V_I - V_{BE2} = 1 - 0.74$
$= 0.26$ V
$V_{B4} = 0.26$ V, leaves a drop across V_{BE4} and R_4 of $v_o - V_{B4}$, that is $0.698 - 0.26 = 0.438$ and this will give $i_{E4} \simeq 0$ as assumed earlier.
Do one more iteration
$i_L \simeq 0.35$ A

$$i_{B3} \cong \frac{0.35}{51} \simeq 7\text{ mA}$$

$$i_{E1} = \frac{10 - 1 - 0.73}{0.215\text{ k}\Omega} - 7 = 31.5\text{ mA}$$

$$V_{EB3} = 0.7 + 0.025\ln\left(\frac{31.5}{10}\right) = 0.729\text{ V}$$

$$V_{B3} = 1 + 0.729 = 1.729\text{ V}$$

$$V_{EB3} = 0.7 + 0.025\ln\left(\frac{31.5}{10}\right) = 0.729\text{ V}$$

$= 0.761$ V
Here $i_{E3} = i_L = 0.35$ A $= 350$ mA

$$i_L = \frac{1.729 - 0.761}{2 + 0.74} = 0.353\text{ A}$$

$v_O = 2\ \Omega \times 0.353$ A
$= 0.706$ V

13.31

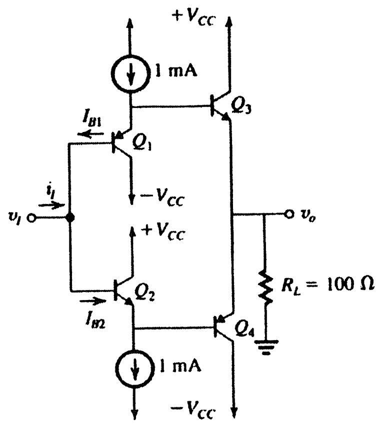
a. $v_I = 0$ and transistors have $\beta = 100$

$I_Q \cong I_{E3} = I_{E4} = I_{E1} = I_{E2} \simeq 1\text{ mA}$

More precisely $I_Q = \dfrac{\beta}{\beta + 1} \times 1 = 0.99\text{ mA}$

Input bias current in zero because $I_{B1} = I_{B2}$

output voltage = 0 V



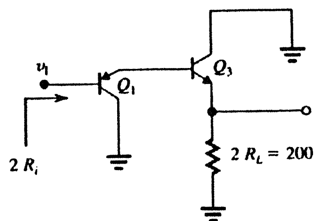
b. From the equivalent half circuit

$$2R_i = (\beta_1 + 1)[r_{e1} + (\beta_3 + 1)(r_{e3} + 2R_L)]$$

$$r_{e1} = r_{e3} = \frac{V_T}{I_E} = \frac{25}{1} = 25\ \Omega$$

$$2R_i = (100 + 1)[25 + (100 + 1)(25 + 2 \times 100)]$$

$$\Rightarrow R_i = 1.15\text{ M}\Omega$$



$$A_v = \frac{v_o}{v_i} = \frac{2R_L}{2R_L + r_{e3} + \dfrac{r_{e1}}{\beta_3 + 1}}$$

$$= \frac{200}{200 + 25 + \dfrac{25}{101}}$$

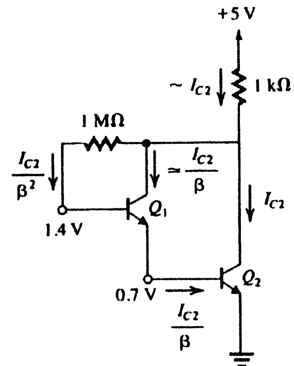
$\simeq 0.89$ V/V

$$2R_{out} = r_{e3} + \frac{r_{e1}}{\beta + 1}$$

$$= 25 + \frac{25}{101}$$

$$R_{out} = 12.6\ \Omega$$

13.32



a. D C Analysis

Current through 1 kΩ $\simeq I_{C2}$

$$5 = 1\text{k} \times I_{C2} + 1\text{M} \times \frac{I_{C2}}{\beta^2} + 1.4$$

$$I_{C2} = \frac{3.6}{1 + \dfrac{1000}{\beta^2}} \text{ in mA}$$

= 3.3 mA

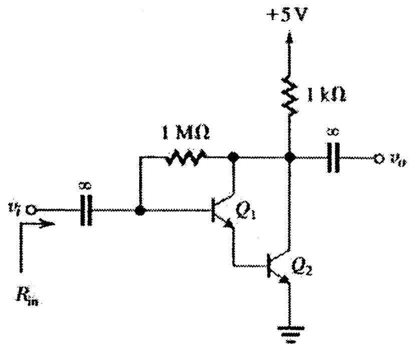
$$I_{C1} = \frac{I_{C2}}{\beta} = \frac{3.3}{100} = 0.033\text{ mA}$$

b. $i_C = g_{m2}\, v_{be2}$

$$= g_{m2} \frac{(\beta_2 + 1)r_{e2}}{r_{e1} + (\beta_2 + 1)r_{e2}} \times v_i$$

But $r_{e1} = \dfrac{V_T}{I_{E1}} = \dfrac{V_T}{\dfrac{I_{C2}}{\beta_2 + 1}} \cong r_{e2}(\beta_2 + 1)$

$$i_C = g_{m2} \frac{r_{e2}}{\dfrac{r_{e1}}{\beta_2 + 1} + r_{e2}} \cdot v_i$$



$i_C = \frac{g_{m2} r_{e2}}{r_{e2} + r_{e2}} \cdot v_i$

But $g_{m2} r_{e2} \simeq 1$

$i_C \simeq \frac{v_i}{2r_{e2}}$

For this circuit g_m equi. is

$g_m \text{ equi} \simeq \frac{i_C}{v_i} = \frac{1}{2r_{e2}} = \frac{1}{2 \times \frac{V_T}{I_{C2}}} = 66 \text{ mA/V}$

Now $v_O \simeq -i_C \times 1 \text{ k}$

$= -g_m \text{equi} \times v_i \times 1 \text{ k}$

$\frac{v_O}{v_i} = -66 \frac{\text{mA}}{\text{V}} \times 1 \text{ k} = -66 \text{ V/V}$

c. $i_i = i_{b1} + i_{1\text{M}\Omega \text{ resistor}}$

$\simeq \frac{i_C}{\beta^2} + \frac{v_i - v_o}{1 \text{ M}\Omega}$

$= \frac{1}{\beta^2} \times \frac{v_i}{2r_{e2}} + \frac{v_i - (-66\, v_i)}{1 \text{ M}\Omega}$

$i_i = \frac{v_i}{2\beta^2 r_{e2}} + \frac{67 v_i}{1 \text{ M}\Omega}$

$= v_i \left[\frac{1}{2 \times 100^2 \times \frac{25}{3.3}} + \frac{67}{1\text{M}} \right]$

$= v_i (6.6 + 67) \times 10^{-6}$

$R_{in} = \frac{v_i}{i_i} = 13.6 \text{ k}\Omega$

13.33

The quiescent current through Q_2 and Q_4 is to be 2 mA. Thus

$V_{BE2} = V_{EB4} = 0.7 + 0.025 \ln\left(\frac{2}{10}\right) = 0.660 \text{ V}$

For Q_1 and Q_3, $I_C \simeq \frac{2}{100} = 0.02$ mA, thus

$V_{BE1} = V_{EB3} = 0.7 + 0.025 \ln \frac{0.02}{1} = 0.602 \text{ V}$

$I_{B1} = \frac{20\ \mu A}{100} = 0.2\ \mu A$

$I_{bias} = 100 \times 0.2 = 20\ \mu A$

$I_{R_1, R_2} = \frac{1}{10} \times 20\ \mu A = 2\ \mu A$

$I_{C5} = 20 - 2 = 18\ \mu A$

$V_{BE5} = 0.7 + 0.025 \ln \frac{0.018}{1} = 0.600 \text{ V}$

$V_{BB} = V_{BE1} + V_{BE2} + V_{EB3} = \underline{1.864 \text{ V}}$

$R_1 + R_2 = \frac{1.864}{2\ \mu A} = 932 \text{ k}\Omega$

$R_1 = \frac{0.600}{2\ \mu A} = \underline{300 \text{ k}\Omega}$

$R_2 = 932 - 300 = \underline{632 \text{ k}\Omega}$

For $V_O = -10$ V and $R_L = 1$ kΩ:

$i_L = \frac{-10}{1} = -10 \text{ mA}$

Assume that the current through Q_2 becomes almost zero, thus

$I_{C4} = 10 \text{ mA}$

i.e. the current through Q_4 increases by a factor of 5. It follows that the current through Q_3 must increase by the same factor, thus V_{EB3} becomes

$V_{EB3} = 0.602 + 0.025 \ln 5$

$= 0.642 \text{ V}$ (an increase of 0.04V)

Let us check the current through Q_2. Since we assumed Q_1 and Q_2 to be almost cut off, all of I_{bias} now flows through the V_{BE} multiplier, an increase of 0.2 μA. Assuming that most of this increase occurs in I_{C5}, V_{BE5} becomes

$V_{BE5} = 0.7 + 0.025 \ln \frac{0.0182}{1} = 0.600 \text{ V}$

Thus the voltage across the V_{BE}-multiplier remains approximately constant and the voltage $(V_{BE1} + V_{BE2})$ decreases by the same value that V_{EB3} increases by. That is

$$V_{BE1} + V_{BE2} = 0.660 + 0.602 - 0.04$$

Since the current through each of Q_1 and Q_2 decreases by the same factor (call it m),

$$0.025 \ln m + 0.025 \ln m = -0.04 \text{ V}$$

$$\Rightarrow m = 0.45$$

Thus $I_{C2} = 0.45 \times 2 = 0.9$ mA

New iteration: $I_{C4} = 10.9$ mA (an increase by a factor $\simeq 5.5$).

$$V_{EB3} = 0.602 + 0.025 \ln 5.5 = 0.645 \text{ V}$$

$$v_I \simeq -10.645 \text{ V}$$

For $v_O = +10$ V and $R_L = 1$ kΩ:

Assume that Q_4 is now conducting a negligible current. Thus, $I_{C2} \simeq I_L = 10$ mA. i.e. the current through each of Q_1 and Q_2 increases by a factor of 5. Thus

$$V_{BE2} = 0.66 + 0.025 \ln 5 = 0.700 \text{ V}$$

$$V_{BE1} = 0.602 + 0.025 \ln 5 = 0.642 \text{ V}$$

$$I_{B1} = 5 \times 0.2 = 1 \ \mu\text{A}$$

Thus the current through the multiplier becomes 19 μA, and assuming that most of the decrease occurs in I_{C5},

$$V_{BE5} = 0.7 + 0.025 \ln \frac{0.017}{1} = 0.598 \text{ V}$$

Thus the voltage across the multiplier becomes

$$V_{BB} = 0.598 \times \frac{932}{300} = 1.858 \text{ V}$$

It follows that V_{EB3} becomes

$$V_{EB3} = 1.858 - 0.700 - 0.642 = 0.516 \text{ V}$$

i.e. V_{EB3} decreases by $0.600 - 0.516 = 0.084$ V and correspondingly I_{C3} decreases by a factor of $e^{\frac{-0.084}{0.025}} = 0.035$. ~~becoming~~ Thus I_{C4} becomes $0.035 \times 2 = 0.07$ mA, close to the zero value assumed. Thus no further iteration are required and

$$v_I \simeq 10 + 0.7 + 0.642 - 1.858 = +9.484 \text{ V}$$

13.34

Now Q_5 has $I_S = 10^{-13}$A. Thus,

$$2 \times 10^{-3} = 10^{-13} e^{V_{BE}/V_T}$$

$$V_{BE} = 0.025 \ln \frac{2 \times 10^{-3}}{10^{-13}} = 0.593 \text{ V}$$

$$R_{E1} = \frac{0.593}{150 \text{ mA}} \simeq 4\Omega$$

For a normal peak current of 100 mA, the voltage drop across R_{E1} is 400 mV and its collector current is $10^{-13} e^{400/25} = 0.89$ μA

13.35

$$2 \times 10^{-3} = 10^{-14} e^{V_{BE}/V_T}$$

$$\Rightarrow V_{BE} = 0.650 \text{ V}$$

$$R_{E1} = \frac{0.650 \text{ V}}{50 \text{ mA}} = 13\ \Omega$$

For a peak output current of 33.3 mA,

$$V_{BE} = 13 \times 33.3 = 433 \text{ mV}$$

$$I_{CS} = 10^{-14} e^{433/25} = 0.33\ \mu\text{A}$$

13.36

$$2 \times 10^{-3} = 10^{-14} e^{V_{EB5}/V_T}$$

$$V_{EB5} = 0.025 \ln(2 \times 10^{-11})$$

$$= 0.650 \text{ V}$$

$$R = \frac{0.650 \text{ V}}{150 \text{ mA}} = 4.3\ \Omega$$

For a peak output current of 100 mA,

$$V_{EB5} = 430 \text{ mV}$$

$$I_{CS} = 10^{-14} e^{430/25} = 0.3\ \mu\text{A}$$

13.37

At 125°C,

$$V_Z = 6.8 + (125 - 25) \times 2 = 7.0 \text{ V}$$

$$V_{E1} = 7.0 - (0.7 - 100 \times 0.002)$$

$$= 6.5 \text{ V}$$

$$V_{BE2} = 0.5 \text{ V}$$

$$R_2 = \frac{0.5 \text{ V}}{100\ \mu\text{A}} = 5 \text{ k}\Omega$$

$$R_1 = \frac{6.5 - 0.5}{100\ \mu\text{A}} = 60 \text{ k}\Omega$$

At 25°C, $V_Z = 6.8$ V,

$$V_{E1} = 6.8 - 0.7 = 6.1 \text{ V}$$

$$V_{B2} = 6.1 \times \frac{5}{60 + 5} = 0.469 \text{ V}$$

$$I_{C2} = 100\, e^{(469 - 700)/25} = 0.01\ \mu\text{A}$$

13.38

$V_{B1} \simeq 0$

$V_{E1} \simeq +0.7$ V

$V_{E3} \simeq +1.4$ V

$V_{C10} = 20 - 0.7 = 19.3$ V

$$I_{E3} = \frac{19.3 - 1.4}{50} = 0.358 \text{ mA}$$

$$I_{B5} = I_{E1} = \frac{0.358}{21} = 17.05\ \mu\text{A}$$

$$I_{B1} = \frac{17.05}{21} = 0.81\ \mu\text{A}$$

$$V_{B1} = 0.81\ \mu\text{A} \times 150 \text{ k}\Omega = 0.122 \text{ V} \simeq 0$$

i.e. $I_{E1} = I_{E2} \simeq 17\ \mu$A

$I_{E3} = I_{E4} \simeq 358\ \mu$A

$$I_{E5} \simeq I_{E6} = \frac{20}{21} \times 358 = 341\ \mu\text{A}$$

$$I_{R1} = I_{R2} = 358\ \mu\text{A}$$

$V_o = 0.12 + 1.4 + 25 \text{ k}\Omega \times 0.358 \text{ mA}$

$= 10.5$ V

13.39

for 8Ω load, we see that V_S=16V allows more than 1.5 W power dissipation for some input signals. Thus we use

$$V_S = 14 \text{ V}$$

For THD = 3%, $P_{Lmax} = 1.9$ W

$$1.9 = V_o^2/R_L = V_o^2/8$$

$$V_o = \sqrt{8 \times 1.9}$$

Peak-to-Peak output sinusoid

$$= 2\sqrt{2}\sqrt{8 \times 1.9} = 11 \text{ V}$$

13.40

For $i_L = 1\text{A}$, $i_{C5} \simeq 1\text{A}$ and $i_{B5} = \frac{1\text{A}}{50} = 20\text{ mA}$

$i_{E3} = 0.9 \times 20 = 18\text{ mA}$,

↑ 10%

For $i_L = 20\text{ mA}$,

$i_{C5} = 2\text{ mA} \quad I_{B5} = \frac{2}{50} = 0.04\text{ mA}$

$$i_{C3} = \frac{50}{51} \times 18 = 17.65\text{ mA}$$

$$i_{R3} = i_{C3} - i_{B5} = 17.61\text{ mA}$$

Thus, $R_3 = \frac{0.7}{17.61} = 39.8 \simeq 40\ \Omega$

Similarly, $R_4 = 40\ \Omega$

Since $i_{B5} \le 20\text{ mA}$, $i_{C3} \le \frac{0.7\text{ V}}{40\ \Omega} + 20\text{ mA}$

i.e. $i_{C3} \le 37.5\text{ mA}$

$$i_{B3} \le \frac{37.5}{50} = 0.75\text{ mA}$$

Allowing for a factor of safety of 2, we select R_1 so that the current through it is 1.5 mA. Now, for $V_o = 11\text{ V}$, $V_{E1} = 11.7\text{ V}$,

$$R_1 = \frac{15 - 11.7}{1.5} = 2.2\text{ k}\Omega$$

Similarly,
$R_2 = 2.2\text{ k}\Omega$

13.41

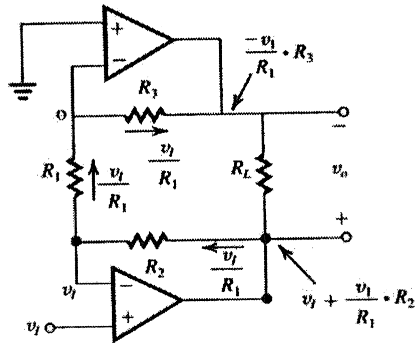
$$\frac{v_o}{v_I} = 2K = 2\left(1 + \frac{R_2}{R_1}\right) = 10$$

$$\Rightarrow \frac{R_2}{R_1} = 4 \qquad R_2 = 40\text{ k}\Omega$$

Also, $K = \frac{R_4}{R_3} = 5$

$$\Rightarrow R_4 = 50\text{ k}\Omega$$

13.42



As shown on the diagram

$$v_O = \left(v_I + \frac{V_I}{R_1} \cdot R_2\right) - \left(\frac{v_I}{R_1} \cdot R_3\right)$$

$$= v_I\left(1 + \frac{R_2}{R_1} + \frac{R_3}{R_1}\right) = v_I\left(1 + \frac{R_2 + R_3}{R_1}\right)$$

The largest sine wave output is obtained when the output voltage of one op amp is +13 V and the output voltage of the other op amp is –13 V, which results in A 26 V peak output

For $\frac{v_O}{v_I} = 10 = 1 + \frac{R_2 + R_3}{R_1}$ choose

$R_I = 1\text{ k}\Omega$ and $(R_2 + R_3) = 9\text{ k}\Omega$

To keep the output complementary

$\frac{R_3}{R_1} = 1 + \frac{R_2}{R_1}$ here $R_1 = 1\text{ k}\Omega$

$$\Rightarrow R_3 = 1 + R_2$$

So $R_2 = 4\text{ k}\Omega$, $R_3 = 5\text{ k}\Omega$

13.43

For

$$I_{QN} = I_{QP} = 10 \text{ mA} = \mu C_{OX}\frac{W}{L}(V_{GS} - V_t)^2$$

$$10 = 100(V_{GS} - 2)^2$$

$$\Rightarrow V_{GS} = 2.32 \text{ V}$$

$$V_R = 2|V_{GS}| = 4.63 \text{ V}$$

For $I_R = 10$ mA,

$$R = \frac{4.63 \text{ V}}{10 \text{ mA}} = 463 \ \Omega$$

$$V_{BB} = 4.63 + 4 \times 0.7 = 7.43 \text{ V}$$

$$I_{R2} = I_{R4} = \frac{100}{2} = 50 \ \mu\text{A}$$

$$R_2 = R_4 = \frac{700 \text{ mV}}{50 \ \mu\text{A}} = 14 \text{ k}\Omega$$

Now, since V_{GS} changes by $2 \times -3 \text{ mV}/°\text{C} = -6 \text{ mV}/°\text{C}$ while V_{BE1}, V_{BE2}, V_{BE3} and V_{BE4} remain constant, V_{BB} changes by -6 mV/°C. But the voltage across the Q_5 multiplier remains constant. Thus the voltage across the Q_6 multiplier should by made to change by -6 mV/°C which can be achieved by making

$$1 + \frac{R_3}{R_4} = 3$$

$$\Rightarrow R_3 = 2R_4 = 28 \text{ k}\Omega$$

The voltage across the Q_5 multiplier is $V_{BB} - 3V_{BE6} = 7.43 - 2.1 = 5.33$ V

$$\text{Thus, } 5.33 = \left(1 + \frac{R_1}{R_2}\right) \times 0.7$$

$$\Rightarrow \frac{R_1}{R_2} = 6.61$$

But $R_2 = 14$ kΩ, thus

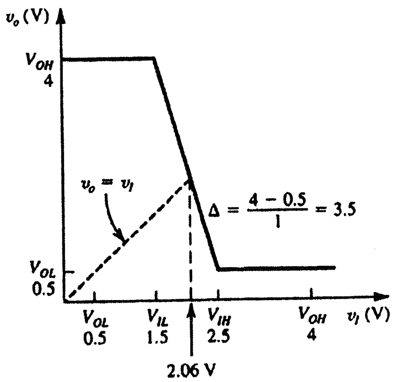
$R_1 = 92.6$ kΩ

14.1

$NM_H = V_{OH} - V_{IH} = 3.3 - 1.7 = 1.6$ V

$NM_L = V_{IL} - V_{OL} = 1.3 - 0 = 1.3$ V

14.2



(a) $NM_H = V_{OH} - V_{IH} = 4 - 2.5 = 1.5$ V

$NM_L = V_{IL} - V_{OL} = 1.5 - 0.5 = 1$ V

(b) In the transition region

$$V_O = 4 - 3.5(V_I - 1.5)$$

$$= 9.25 - 3.5V_I$$

If

$$V_O = V_I \Rightarrow 4.5V_O = 9.25$$

$$V_O = V_I = 2.06 \text{ V}$$

(c) Slope = −3.5 V/V

14.3

$NM_H = V_{OH} - V_{IH} = 0.8V_{DD} - 0.6V_{DD} = 0.2V_{DD}$

$NM_L = V_{IL} - V_{OL} = (0.4 - 0.1)V_{DD} = 0.3V_{DD}$

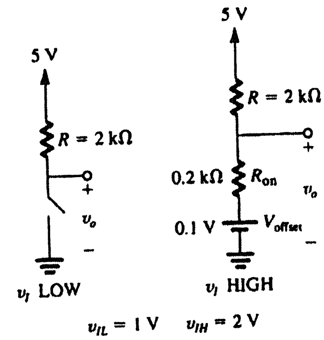
width of transition region

$= V_{IH} - V_{IL} = 0.2V_{DD}$ for a minimum NM of

$1 \text{ V} \Rightarrow 0.2V_{DD} = 1$

$V_{DD} = 5$ V

14.4



(a) $V_{OL} = \frac{5 - 0.1}{2.2} = 0.2 + 0.1 = 0.545$ V

$V_{OH} = 5$ V

$NM_H = V_{OH} - V_{IH} = 3$ V

$NM_L = V_{IL} - V_{OL} = 0.455$ V

(b) $V_{OH} = 5 - N(0.2 \times 10^{-3})R = 5 - 0.4N$

$NM_H = 5 - 0.4N - 2 = 3 - 0.4N = 0.455 \quad \therefore N = 6$

(c) (i) $P_{D_{v_o,LOW}} = (5 - 0.1)^2/2.2\text{ k}\Omega = 10.9$ mW

(ii) $P_{D_{v_o,HIGH}} = 5 \times (0.2 \times 6) = 6$ mW

14.5

Ideal 3V logic implies:

$V_{OH} = V_{DD} = \underline{3.0V}$; $V_{OL} = \underline{0.0V}$;

$V_{th} = V_{DD}/2 = 3.0/2 = \underline{1.5V}$;

$V_{IL} = V_{DD}/2 = \underline{1.5V}$; $V_{IH} = V_{DD}/2 = \underline{1.5V}$

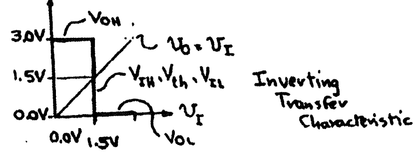
$NM_H = V_{OH} - V_{IH} = 3.0 - 1.5 = \underline{1.5V}$

$NM_L = V_{IL} - V_{OL} = 1.5 - 0.0 = \underline{1.5V}$

The gain in the transition region is:

$(V_{OH} - V_{OL})/(V_{IH} - V_{IL}) =$

$(3.0 - 0.0)/(1.5 - 1.5) = 3/0 = \underline{\infty V/V}$



14.6

Nearly ideal 3.3V logic, assumed ideal:

→ $V_{OH} = \underline{3.3V}$, $V_{OL} = \underline{0.0V}$, $V_{th} = 0.4(3.3) = \underline{1.32V}$

Now, at V_{th}, $v_O = v_I$, so to reach $v_O = 1.32V$

the required input is $1.32/(-50) = -26.4mV$

Thus, $V_{IL} = 1.32 - 26.4 \times 10^{-3} = \underline{1.294\ V}$

Likewise, $V_{IH} = 1.32 + (3.3 - 1.32)/50 = \underline{1.360V}$

Best possible noise margins are:

$NM_H = V_{OH} - V_{IH} = 3.30 - 1.360 = \underline{1.940\ V}$

$NM_L = V_{IL} - V_{OL} = 1.294 - 0.0 = \underline{1.294V}$

For noise margins only 7/10 of these, and V_{OH}, V_{OL} still ideal:

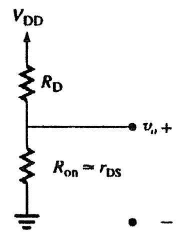
$V_{IH} = 3.3 - 0.7(1.940) = \underline{1.942\ V}$, and

$V_{IL} = 0.0 + 0.7(1.294) = \underline{0.906V}$

Correspondingly, the large-signal voltage gain is:

$G = (3.3 - 0.0)/(0.906 - 1.942) = \underline{-3.18\ V/V}$

14.7



Equivalent circuit for output-low state

The output high level for the simple inverter circuit shown in Fig 13.2 of the Text is

$V_{OH} = V_{DD} \Rightarrow V_{DD} = 2\ V$.

When the output is low, the current drawn from the supply can be calculated as:

$$I = \frac{V_{DD}}{R_D + R_{on}} = 20\ \mu A$$

Therefore: $R_D + r_{DS} = \dfrac{2}{20 \times 10^{-6}} = 100\ k\Omega$

Also:

$$V_{OL} = 0.1\ V = \frac{r_{DS}}{R_D + r_{DS}} \times V_{DD}$$

$$\Rightarrow r_{DS} = 100\ k\Omega \times \frac{0.1}{2} = 5\ k\Omega$$

Hence: $R_D = 100\ K - 5\ K = 95\ k\Omega$

$$r_{DS} = \frac{1}{\mu_n C_{ox} \frac{W}{L}(V_{GS} - V_t)} = \frac{1}{100 \times 10^{-6} \times \frac{W}{L}(2 - 0.5)}$$

$= 5\ k\Omega$

$$\frac{W}{L} = \frac{10}{1.5 \times 5} = 1.3$$

when the output is low:

$P_D = V_{DD} I_{DD} = 2 \times 20\ \mu A = 40\ \mu W$

when the output is high, the transistor is off:

$P_D = 0\ W$

14.8

$V_{OH} = V_{DD} = 2.5\ V$

The power drawn from the supply during the low-output state is:

$P_{DD} = V_{DD} \Rightarrow 125\ \mu w = 2.5 \times I_{DD}$

$\Rightarrow I_{DD} = 50\ \mu A$

In this stage:

$$I_{DD} = \frac{V_{DD} - V_{OL}}{R_D} \Rightarrow 50\ \mu A = \frac{2.5 - 0.1}{R_D}$$

$\Rightarrow R_D = 48\ k\Omega$

In order to determine $\frac{W}{L}$, we note that

$$k_n R_D = 1 / V_X \text{ or } k'_n \frac{W}{L} R_D = \frac{1}{V_X}$$

Therefore, we need to first calculae V_X.

$$V_{OL} - \frac{V_{DD}}{1 + \frac{V_{DD} - V_t}{V_X}} \text{ or equivalently:}$$

$$0.1\ V = \frac{2.5}{1 + \frac{2.5 - 0.5}{V_X}} \Rightarrow V_X = \frac{2}{24} = 0.083\ V$$

Hence, $k'_n \frac{W}{L} R_D = \frac{1}{V_x}$ gives:

$$100 \times 10^{-6} \times \frac{W}{L} \times 48 \times 10^3 = \frac{1}{0.083} \Rightarrow \frac{W}{L} = 2.5$$

$V_{IL} = V_t + V_X = 0.5 + 0.083 = 0.583V$

$V_M = V_t + \sqrt{2(V_{DD} - V_t)V_x + V_x^2} - V_x$

$= 0.5 + \sqrt{2(2.5 - 0.5)0.083 + 0.083^2} - 0.083$

$V_M = 1V$

$V_{IH} = V_t + 1.63\sqrt{V_{DD}V_x} - V_x$

$= 0.5 + 1.63\sqrt{2.5 \times 0.083} - 0.083 = 1.16V$

$NM_H = V_{OH} - V_{IH} = 2.5 - 1.16 = 1.34V$

$NM_L = V_{IL} - V_{OL} = 0.583 - 0.1 = 0.483V$

14.9

$V_{t2} = V_{t0} + r[\sqrt{V_{OH} + 2\varphi_F} - \sqrt{2\varphi_F}]$,

$V_{OH} - V_{DD} - V_{t2}$

Interation 1: $V_{t2} = 0.5V$

$V_{OH} = 1.8 - 0.5 = 1.3V$

Interation 2:

$V_{t2} = 0.5 + 0.3[\sqrt{1.3 + 0.8} - \sqrt{0.8}] = 0.67V$

$V_{OH} = 1.8 - 0.67 = 1.13V$

Interation 3:

$V_{t2} = 0.5 + 0.3[\sqrt{1.13 + 0.8} - \sqrt{0.8}] = 0.65V$

$V_{OH} = 1.8 - 0.65 = 1.15V$

Interation 4:

$V_{t2} = 0.5 + 0.3[\sqrt{1.15 + 0.8} - \sqrt{0.8}] = 0.65V$

$V_{OH} = 1.8 - 0.65 = 1.15V$

$\therefore V_{t2} = 0.65V$ and $V_{OH} = 1.15V$

$\Delta V_{OH} = 1.3 - 1.15 = 0.15V$ V_{OH} is reduced by 0.15V due to the body effect on Q_2

14.10

$V_{IH} \simeq V_M + \frac{V_M}{K_r} = 0.63 + \frac{0.63}{5} = 0.756V$

The value calculated the long way in Example 13.2 is: $V_{IH} = 0.75V$ and is very close to the above approximation.

14.11

Given: $V_{OL} \simeq 0.05V$

$V_{OH} = V_{DD} - V_t = 2.5 - 0.5 = 2V$

$V_{IL} = V_t = 0.5V$

$$V_{OL} \simeq \frac{(V_{DD} - V_t)^2}{2k_r^2(V_{DD} - 2V_t)} = \frac{(2.5 - 0.5)^2}{2k_r^2(2.5 - 2 \times 0.5)}$$

$$= \frac{4}{3k_r^2} \simeq 0.05V \Rightarrow K_r = 5.2$$

V_M

$$= \frac{V_{DD} + (K_r - 1)V_t}{(K_r + 1)} = \frac{2.5 + (5.2 - 1) \times 0.5}{5.2 + 1}$$

$= 0.74V$

$V_{IH} \cong V_M + \frac{V_M}{K_r} = 0.74V + \frac{0.74V}{5.2} = 0.88V$

$NM_H = V_{OH} - V_{IH} = 2 - 0.88 = 1.12V$

$NM_L = V_{IL} - V_{OL} = 0.5 - 0.05 = 0.45V$

To obtain $\frac{W}{L}$.

$$K_r = \sqrt{\frac{W/L_1}{W/L_2}} \Rightarrow K_r = \sqrt{\frac{W/L_1}{\frac{1}{W/L_1}}} = \left(\frac{W}{L}\right)_1$$

$$\Rightarrow \left(\frac{W}{L}\right)_1 = 5.2\left(\frac{W}{L}\right)_2 = \frac{1}{5.2} = 0.19$$

$$I_{DD} = i_{D2} = \frac{1}{2}K_{n1}(V_{DD} - V_{OL} - V_t)^2$$

$$= \frac{1}{2} \times 100 \times 10^{-6} \times 0.19 \times (2.5 - 0.05 - 0.5)^2$$

$= 36.1\ \mu A$

$P_D = V_{DD}I_{DD} = 2.5 \times 36.1\mu = 90.2\ \mu W$

14.12

$E_{dissipated/cycle} = CV_{DD}^2 = 10 \times 10^{-15} \times 2.5^2$

$E_{dissipated/cycle} = 62.5FJ$

$P_{dyn} = fcV_{DD}^2 = 1 \times 10^9 \times 10 \times 10^{-15} \times 2.5^2$

$= 62.5\mu W$

This is the power consumption for one inverter. For a chip with 1 million inverters, the power consumption is:

$P_{dyn(chip)} = 62.5 \times 10^{-6} \times 10^6 = 62.5W$

To determine the average current drawn from the supply, we note that

$$P_{dyn} = I_{DDavg}V_{DD} \Rightarrow I_{DD-avg} = \frac{62.5}{2.5} = 25A$$

14.13

$P_{dynamic} = fCV_{DD}^2 = 100 \times 10^6 \times 10 \times 10^{-12} \times 25 =$ 25 mW

$P = V_{DD} I_{avg} = 5I_{avg} = 25$ mW

$I_{avg} = 5$ mA

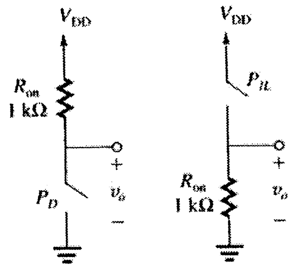
14.14

(a) $V_{OL} = 0$

$V_{OH} = 5$

$NM_L = V_{IL} - V_{OL} = 2.5 - 0 = 2.5V$

$NM_H = V_{OH} - V_{IH} = 5 - 2.5 = 2.5V$



v_I LOW v_I HIGH

(b)

$$V_o(t) = 0 - (0 - 5)e^{-t/R_{on}C} = 5e^{-t/R_{on}C}$$

For $t_{PHL} \Rightarrow V_o(t) = 5e^{-t/R_{on}C} = \frac{1}{2}(5) = 2.5$

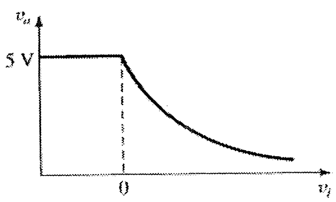
$$t_{PHL} = -(10^3)(10^{-12})\ln\frac{2.5}{5} = 0.69\text{ns}$$

For $t_{PHL} V_o(t) = 5e^{-t_H/R_{on}C} = 4.5V$

$t_1 = 0.1$ ns $V_o(t) = 5e^{-t_L/R_{on}C} = 0.5V$

$t_2 = 2.3$ ns

$\therefore t_{THL} = t_2 - t_1 = 2.2$ ns



(c)

$$V_o(t) = 5 - (5 - 0)e^{-t/R_{on}C} = 5 - 5e^{-t/R_{on}C}$$

$$V_O = 5 - 5e^{-t_{PLH}/R_{on}C} = 2.5$$

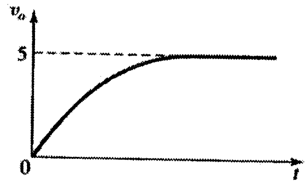
$t_{PLH} = 0.69$ ns

For t_{PLH},

$$V_o(t) = 5 - 5e^{-t_1/R_{on}C} = 0.5 \Rightarrow t_1 = 0.10 \text{ ns}$$

$$V_o(t) = 5 - 5e^{-t_2/R_{on}C} = 4.5 \Rightarrow t_2 = 2.3 \text{ ns}$$

$t_{PLH} = 2.3 - 0.1 = 2.2$ns



14.15

(a) Generally, $t_p = (t_{PHL} + t_{PLH})/2$, but due to current ratio, $t_{PHL} = 0.5t_{PLH}$.

Thus $1.5t_{PHL} = 2(1.2\text{ns})$, whence

$t_{PLH} = 2.4/1.5 = 1.6$ ns, and $t_{PHL} = 0.8$ ns

Check:

$t_p = (1.6 + 0.8)/2 = 1.2$ns

(b) Generally, $t_p = CV/I = kC$

Originally, $1.2n = kC$ (1)

Then, $1.7(1.2)n = k(C + 1p)$ (2)

Dividing $\left(\frac{2}{1}\right)$: $1.7 = (c + 1p)/C$

Thus, $1.7C = C + 1$, $0.7C = 1$,

$C = 1.43$pF

(the combined load and output capacitances)

(c) With the load inverter removed:

$0.6(1.2n) = k(1.43 - C_{in})$ (3)

Dividing $\left(\frac{3}{1}\right)$ $0.6 = (1.43p - C_{in})/1.43$

Thus,

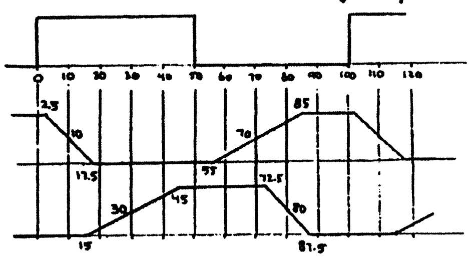
$C_{in} = 1.43(1 - 0.6) = 0.57$pF;

$C_{out} = 1.43 - 0.57 = 0.86$pF

14.16

The results depend on whether the gates are inverting or non-inverting.

For inverting gates, the timing diagram is:



Note: For simplicity, 0% to 100% (rather than 10% to 90%) both in the diagram above and calculation to follow:

For inverting gates (as shown, above):

a) For a rising input, time to 90% change of output of second gate is 10 + 20 + 30/2 = 45ns

b) For a falling input, time to 90% change of output of 2nd gate is 20 + 10 + 15/2 = 37.5ns

For non-inverting gates:

a) Time to 90% rise is 10 + 10 + 15/2 = 27.5ns

b) Time to 90% fall is 20 + 20 + 30/2 = 55ns

The propagation delay for these gates is

$t_P = (t_{PHL} + t_{PLH})/2 = (10 + 20)/2 = 15ns$

14.17

Note that this question ignores the possibility of dynamic power dissipation: Average propagation delay is $t_p = (50 + 70)/2 = 60$ ns

Average power loss at 50% duty cycle

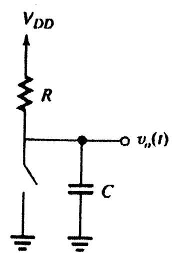
$= (1 + 0.5)/2 = 0.75$ mW

Delay-Power product is

$DP = 60 \times 10^{-9} \times 0.75 \times 10^{-3}$ or

$DP = 45 \times 10^{-12}$ J $= 45$ pJ

14.18



$v_O(t)$ begins at V_{OL} and rises toward V_{OH} (in this case $V_{OH} = V_{DD}$) according to

$v_o(t) = v_\infty - (v_\infty - v_{ot})e^{-t/CR}$

$= V_{OH} - (V_{OH} - V_{OL})e^{-t/CR}$

$= V_{OH} - (V_{OH} - V_{OL})e^{-t/\tau_1}$, $\tau_1 = CR$

Q.E.D.

$v_O(t)$ reaches $\frac{1}{2}(V_{OH} + V_{OL})$ at $t = t_{PLH}$,

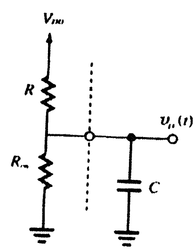
$\frac{1}{2}(V_{OH} + V_{OL}) = V_{OH} - (V_{OH} - V_{OL})e^{-t_{PLH}/\tau_1}$

$\Rightarrow t_{PLH} = \tau_1 \ln 2 = 0.69CR$ Q.E.D.

(b)

$v_O(t) = v_\infty - (v_\infty - v_{ot})e^{-t/\tau_2}$,

$\tau_2 = C(R // R_{on})$



14.19

$$V_M = \frac{r(V_{DD} - |v_{tp}|)}{1 + r} \text{ where}$$

$$r = \sqrt{\frac{kp}{k_n}} = \sqrt{\frac{\mu_p w_p}{\mu_n w_n}}$$

a) For $w_p = 3.5w_n$ or matched case:

$\frac{w_p}{w_n} = \frac{\mu_n}{\mu_p}$ we have $r = 1$ and:

$$V_M = \frac{1(2.5 - 0.5) + 0.5}{1 + 1} = \frac{V_{DD}}{2} = 1.25V,$$

$$A = (w_p + w_n)L = 4.5w_nL$$

b) For $w_p = w_n$: $r = \sqrt{\frac{1}{3.5} \times 1} = 0.53$

$$V_M = \frac{0.53(2.5 - 0.5) + 0.5}{1 + 0.53} = 1.02V$$

The shift in NM_L is approximately equal to the shift in V_M, that is:

$\Delta V_M = 1.25 = 1.02 = 0.23V$, hence NM_L is reduced by 0.23V.

$A = (w_p + w_n)L = (w_n + w_n)L = 2w_nL$, therefore the area is reduced by

$(4.5 - 2)w_nL = 2.5w_nL = 2.5 \times 1.5 \times 0.25 \times 0.25$

$= 0.23\mu m^2$ or by $\frac{25}{4.5} = 0.56, 56\%$

c) For $w_p = 2w_n$: $r = \sqrt{\frac{1 \times 2}{3.5}} = 0.76,$

$$V_M = \frac{0.76(2.5 - 0.5) + 0.5}{1 + 0.76} = 1.15V$$

The shift in V_M is $1.25 - 1.15 = 0.1V$, hence, the NM_L is approximately reduced by 0.1V or comparing to NM_L in P13.26 above, it is reduced by 9.4%.

$A = (w_p + w_n)L = (2w_n + w_n)L = 3w_nL$,

therefore the area is reduced by $1.5w_nL$ or

$1.5 \times 1.5 \times 0.25 \times 0.25 = 0.14\mu m^2$ or by

$\frac{1.5}{4.5} = 0.33$ or 33%.

14.20

In the low-output state, V_{in} is high and V_{on} is low and therefore NMos operatres in triode region:

$$I_D = \mu_n C_{ox}\left(\frac{W}{L}\right)_n\left((V_{GS} - V_{tn})V_{DS} - \frac{1}{2}V_{DS}^2\right)$$

$$= k_n'\left(\frac{W}{L}\right)_n\left((V_{DD} - 0.2V_{DD}) \times 0.1V_{DD} - \frac{1}{2}(0.1V_{DD})^2\right)$$

$$I_D = k_n'\left(\frac{W}{L}\right)_n(0.08V^2_{DD} - 0.005V^2_{DD})$$

$$= 0.075k_n'\left(\frac{W}{L}\right)_n V^2_{DD}$$

For $I = 0.5mA$,

$V_{DD} = 2.5V$, $k_n' = 115\mu A/V^2$, we'll have:

$$0.5 \times 10^{-3} = 0.075 \times 115 \times 10^{-6}\left(\frac{W}{L}\right)_n \times 2.5^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_n = 9.3$$

14.21

For $v_I = 1.5V$, the NMOS operates in triode mode while the PMOS is cut off.

$r_{DSn} = [k_n(v_I - V_t)]^{-1} = [100 \times 10^{-6}(1.5 - 0.5)]^{-1}$

$= 10k\Omega$

Thus,

$v_a = 100 \times 10^{-3} \times 10^4/(10^4 + 10^5) = 9.09mV$

For $v_I = -1.5V$, the PMOS operates with

$r_{DSp} = [k_p(|v_I|) - V_t] = [(100 \times 10^{-6})(1.5 - 0.5)]^{-1}$

$= 10^5\Omega$

Thus

$v_a = 100 \times 10^{-3} \times 10^5/(10^5 + 10^5) = 50mV$

14.22

Since at M, both Q_N and Q_P operate in saturation, their currents are given

Substituting $V_I = V_O = V_M$ and equating the two currents results in:

$$i_{DN} = i_{DP} \Rightarrow \frac{1}{2}k_n'\left(\frac{W}{L}\right)_n(V_I - V_{tn})^2$$

$$= \frac{1}{2}k_p'\left(\frac{W}{L}\right)_p(V_{DD} - V_M - |V_{tp}|)^2$$

$\frac{K_p}{k_n} = \frac{(V_M - V_{tn})^2}{(V_{DD} - V_M - |V_{tp}|)^2}$. Considering

$r = \sqrt{\frac{K_p}{R_n}}$, we have: $r = \frac{V_M - V_{tn}}{V_{DD} - |V_{tp}| - V_M}$

Now, for

$V_M = 0.6V_{DD} = 0.6 \times 1.8 = 1.08V$

$$r = \frac{1.08 - 0.5}{1.8 - 0.5 - 1.08} = 2.64$$

$$r = \sqrt{\frac{\mu_p w_p}{\mu_n w_n}} = r\frac{w_p}{w_n} = \frac{(2.64)^2}{V_4} = 27.9$$

14.23

The peak current happens when $V_I = V_M$ and since Q_p and Q_N are matched

$$V_M = \frac{V_{DD}}{2} = \frac{1.8}{2} = 0.9V$$

Noting that both transistors are in saturation region, Find the current.

$$I = \frac{1}{2}k_n'\left(\frac{W}{L}\right)_n(V_M - V_{tn})^2$$

$$= \frac{1}{2}K_n'\left(\frac{W}{L}\right)_n\left(\frac{V_{DD}}{2} - V_{tn}\right)^2$$

For $k_n' = 300\mu A/V^2$,

$\left(\frac{W}{L}\right)_n = 1.5$ $V_{DD} = 1.8V$ $V_{tn} = 0.5V$:

$$I_{peak} = \frac{1}{2} \times 300 \times 10^{-6} \times 1.5 \times \left(\frac{1.8}{2} - 0.5\right)^2$$

$$= 36\mu A$$

14.24

since $V_{tn} = |V_{tp}|$, then $\alpha_n = \alpha_p$, then $\alpha_n = \alpha_p$. From above, we have $\alpha_n = \alpha_p = 2.32$.

$$t_p = \frac{1}{2}(t_{PLH} + t_{PHL})$$

$$= \frac{1}{2}\left(\frac{\alpha_n C}{k'_n\left(\frac{W}{L}\right)_n V_{DD}} + \frac{\alpha_p C}{k'_p\left(\frac{W}{L}\right)_n V_{DD}}\right)$$

Since QN and QP are matched, then $k_p = k_n$ and

$$t_p = \frac{\alpha_n C}{k_n V_{DD}} = \frac{2.32 \times 20 \times 10^{-15}}{K_n \times 1.2V}$$

$$t_p \le 20ps \Rightarrow \frac{38.7 \times 10^{-15}}{k_n} \le 2.0 \times 10^{-12} \Rightarrow k_n \approx 1.9m$$

Now if we substitude in

$$k_n = (k_n')\left(\frac{W}{L}\right) = 430 \times 10^{-6} \times \left(\frac{W}{L}\right)_n = 1.9m$$

$$\Rightarrow \left(\frac{W}{L}\right)_n = 4.4$$

Since Q_P and Q_N are matched and $k_n = k_p$, then

$$\left(\frac{W}{L}\right)_p = \left(\frac{W}{L}\right)_n \times \frac{k_n}{k_p} 4.4 \times 4 = 17.6$$

14.25

Using the equivalent resistance approach, we first find R_N

$$R_N = \frac{12.5}{\left(\frac{W}{L}\right)_n} = \frac{12.5}{1.5} = 8.33k\Omega$$

to determine t_{PHL}.

$$t_{PHL} = 0.69R_NC = 0.69 \times 8.33 \times 10^3 \times 10 \times 10^{-15}$$

$= 57.5$ ps.

to determine R_p:

$$R_p = \frac{30}{\left(\frac{W}{L}\right)_p} = \frac{30}{3} = 10k\Omega$$

to determine t_{PLH}:

$$t_{PLH} = 0.69R_pC = 0.69 \times 10 \times 10^3 \times 10 \times 10^{-15}$$

$= 69$ ps

$$t_p = \frac{1}{2}(57.5 + 69) = 63.2 \text{ ps}$$

Note that while the value obtained for t_{PHL} is higher than that found using the average currents method, the value for t_{PLH} is about the same.

14.26

$t_{PHL} = 0.69R_NC$, $t_{PLH} = 0.69R_pC$

Since $t_{PHL} = t_{PLH}$, then $R_N = R_p = R$

For $t_p \le 40ps$, we have to have:

$$\frac{1}{2}(t_{PHL} + t_{PLH}) \le 40 \text{ ps or}$$

$$\frac{1}{2}(0.69 \times 2R \times C) \le 40ps$$

$$\therefore R \le \frac{40 \times 10^{-12} \times 2}{0.69 \times 2 \times 10^{-15} \times 10} \Rightarrow R \le 5.8k\Omega$$

To determine the transistor widths in 0.18 μm technology,

$L_n = L_p = .18$ μm

$$R_N = \frac{12.5}{\left(\frac{W}{L}\right)_n} k\Omega \text{ or}$$

$$\frac{12.5}{\left(\frac{W}{L}\right)_n}K \le 5.8k\Omega \Rightarrow \left(\frac{W}{L}\right)_n \ge 2.2$$

$$\Rightarrow w_n \ge 0.4\mu m$$

$$R_p = \frac{30}{\left(\frac{W}{L}\right)_p} k\Omega \text{ or}$$

$$\frac{30}{\left(\frac{W}{L}\right)_p}K \le 5.8k\Omega \Rightarrow \left(\frac{W}{L}\right)_p \ge 5.2$$

$$\Rightarrow w_p \ge .94\mu m$$

14.27

$$\alpha_n = 2/\left[\frac{7}{4} - \frac{3V_{tn}}{V_{DD}} + \left(\frac{V_{tn}}{V_{DD}}\right)^2\right]$$

$$= 2/\left[\frac{7}{4} - \frac{3 \times 0.7}{3.3} + \left(\frac{0.7}{3.3}\right)^2\right] = 1.73$$

$$t_{PHL} = \frac{\alpha_n C}{k_n'\left(\frac{W}{L}\right)_n V_{DD}} = \frac{1.73 \times (2fF \times 0.75 + 1fF)}{180 \times 10^{-6} \times \frac{0.75}{0.5} \times 3.3}$$

$= 4.85$ ps

Since, $V_{tn} = |V_{tp}|$, then $\alpha_n = \alpha_p = 1.73$. We also have $\left(\frac{W}{L}\right)_n = \left(\frac{W}{L}\right)_p$, hence:

$$t_{PLH} = t_{PHL} \times \frac{k_n'}{k_p'} = 4.85 \times 3 = 14.55\text{ps}$$

$$t_p = \frac{1}{2}(t_{PHL} + t_{PLH}) = \frac{1}{2}(4.85 + 14.55) = 9.7 \text{ ps}$$

If both devices are matched, then $k_p' = k_n'$.

$t_{PLH} = t_{PHL}$ and

$$t_p = \frac{1}{2}(t_{PLH} + t_{PHL}) = t_{PHL} = 4.85 \text{ ps}$$

14.28

In order to determine the propagation delay, we first need to calculate the total value for C, using

$$C = 2C_{gd1} + 2C_{gd2} + C_{db1} + C_{db2} + C_{g3} + C_{g4} + C_w$$

where $C_{gd1} = 0.4w_n = 0.4 \times 0.75 = 0.3\text{fF}$

Since transistors are matched

$$k_p'\left(\frac{W}{L}\right)_p = k_n'\left(\frac{W}{L}\right)_n \Rightarrow w_p = \frac{180}{45} \times 0.75 = 3\mu\text{m}$$

$C_{gd2} = 0.4 \times w_p = 0.4 \times 3 = 1.2\text{fF}$

$C_{db1} = 1.0 \times w_n = 1 \times 0.75 = 0.75\text{fF}$

$C_{db2} = 1.0 \times w_p = 1.0 \times 3 = 3.0\text{fF}$

$C_{g3} = (WL)_3 C_{ox} + C_{gdov3} + C_{gsov3}$

$= (0.75 \times 0.5) \times 3.7 + 0.4 \times 0.75 + 0.4 \times 0.75$

$= 1.99\text{fF}$

$C_{g4} = 3 \times 0.5 \times 3.7 + 2 \times 0.4 \times 3 = 7.95\text{fF}$

$C = 2 \times 0.3 + 2 \times 1.2 + 0.75 + 3 + 1.99 + 7.95 + 2$

$= 18.7\text{fF}$

to determine t_{PHL}:

$$\alpha_n = 2/\left[\frac{7}{4} - \frac{3 \times 0.7}{3.3} + \left(\frac{0.7}{3.3}\right)^2\right] = 1.74$$

then, t_{PHL}

$$= \frac{1.74 \times 18.7 \times 10^{-15}}{180 \times 10^{-6} \times \frac{0.75}{0.5} \times 3.3} = 36.5\text{ps}$$

Since $|V_{tp}| = V_{tn}$ and transistors are matched,

$t_{PHL} = t_{PLH} = t_p \Rightarrow t_p = 36.5$ ps.

Considering that t_{PHL} and t_{PLH} both are proportional to C, then for an increase of 50% in t_p, C also has to be increased by 50%. Hence,

$\Delta C = 18.7 \times 0.5 = 9.4$ fF

14.29

$$\frac{t_{pnew}}{t_{pold}} = \frac{C_{int} + C_{ext}/s}{C_{int} + C_{ext}} \Rightarrow \frac{30}{60} = \frac{10 + 20/s}{10 + 20}$$

$$\Rightarrow 15 = 10 + \frac{20}{5} \Rightarrow s = 4$$

Note that $S = \frac{R_{eq}}{R_{eq}}$ and hence R_{eq} has to be reduced by a factor of 4 or equivalently $\left(\frac{W}{L}\right)_n$ and $\left(\frac{W}{L}\right)_p$ have to be increased by a factor of 4.

14.30

Dynamic power is $P_D = fCV^2_{DD}$; Static Power is P_s.

Now, $9.0 = P_s + 120 \times 10^6 C^2 5$ and

$4.7 = P_s + 50 \times 10^6 C^2 5$

Subtracting, $4.3 = 70 \times 10^6 C(25)$

Whence

$C = 4.3/(25 \times 70 \times 10^6) = 2457\text{pF}$

and $P_s = 9.0 - 120 \times 10^6 (25) 2457 \times 10^{-12}$

$= 9.0 - 7.37 = 1.63\text{W}$

For 70% of the gates active, total gates

$= 0.7 \times 10^6$

Capacitance per gate is

$2457 \times 10^{-12}/(0.7 \times 10^6) = 3.5\text{fF}$

14.31

$C = 2C_{gd1} + C_{gd2} + C_{db1} + C_{db2} + C_{g3} + C_{g4} + C_n$

$W = W_n = W_p = 0.75\mu m$

$C_{gd1} = \frac{.4f}{\mu m} \cdot W_n = \frac{.4f}{\mu m} \times .75\mu m = 0.3fF$

$C_{gd2} = \frac{0.4f}{\mu m} \cdot W_p = \frac{.4f}{\mu m} \times .75\ \mu m = 0.3fF$

$C_{db1} = C_{db2} = \frac{1f}{\mu m} \cdot W = \frac{1f}{\mu m} \times 0.75\mu m$

$= 0.75fF$

$C_{g3} = C_{g4} = (WL)C_{ox} + C_{gdov} + C_{gsov}$

$= (0.75\ \mu m \times 0.5\ \mu m)\frac{3.7f}{\mu m^2} + 2 \times \frac{0.4f}{\mu m} \times 0.75\ \mu m$

$C_{g3} = C_{g4} = 1.99fF$

$C = 2 \times 0.3f + 2 \times 0.3f + 2 \times 0.75f$

$+ 2 \times 1.99f + 2f = 8.7\ fF$

$$\alpha_n = 2/\left[\frac{7}{4} - \frac{3 \times 0.7}{3.3} + \left(\frac{0.7}{3.3}\right)^2\right] = 1.74$$

$$t_{PHL} = \frac{1.74 \times 8.7 \times 10^{-15}}{180 \times 10^{-6} \times \frac{0.75}{0.5} \times 3.3} = 17\ ps$$

$$\alpha_p = 2\left[\frac{7}{4} - \frac{3 \times 0.7}{3.3} + \left(\frac{0.7}{3.3}\right)^2\right] = 1.74$$

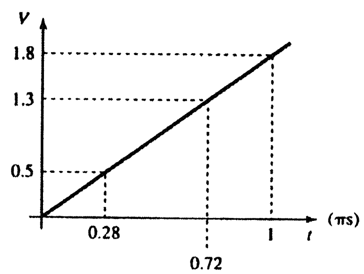
$$t_{PLH} = \frac{1.74 \times 8.7 \times 10^{-15}}{45 \times 10^{-6} \times \frac{0.75}{0.5} \times 3.3} = 68ps$$

$$t_P = \frac{1}{2}(t_{PHL} + t_{PLH}) = \frac{1}{2}(17p + 68p) = 42.5ps$$

$P_D = fCV_{DD}^2 = 250 \times 10^6 \times 8.7 \times 10^{-15} \times (3.3)^2$

$= 23.7\mu W$

14.32



$$I_{peak} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_n\left(\frac{V_{DD}}{2} - V_{tn}\right)^2$$

$$I_{peak} = \frac{1}{2} \times 450\frac{\mu A}{V^2}\left(\frac{1.8}{2} - 0.5\right)^2 = 36\ \mu A$$

The time when the input reaches V_t is:

$$\frac{0.5}{1.8} \times 1^{ns} = 0.28ns$$

The time when the input reaches $V_{DD} - V_t$ is

$$\frac{1.8 - 0.5}{1.8} \times 1^{ns} = 0.72^{ns}$$

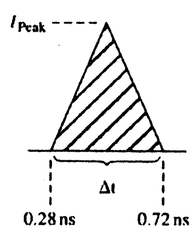
So the base of the triangle is

$\Delta t = 0.72 - 0.28 = 0.44ns$ wide

$E = \frac{1}{2}I_{peak} \times V_{DD} \times \Delta t = \frac{1}{2} \times 36\ \mu A \times 1.8 \times 0.44\ ns$

$= 14.3fJ$

$P = f \times E = 100 \times 10^6 \times 14.3 \times 10^{-15} = 1.43\ \mu W$



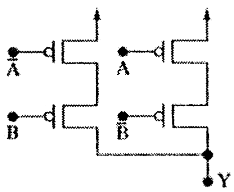
14.33

$Y = A\bar{B} + \bar{A}B \rightarrow \bar{Y} = \overline{A\bar{B} + \bar{A}B} = \overline{A\bar{B}} \cdot \overline{\bar{A}B}$

or $Y = (\bar{A} + B)(A + \bar{B}) = AB + \bar{A}\bar{B}$

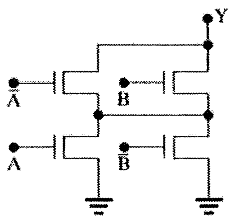
PUN for $Y = A\bar{B} + \bar{A}B$:

u1



PDN dual to u1

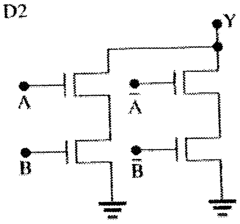
D1



*Note, however that D1 can be redrawn as shown, then its colums (series links) converted to rows (parallal links of a PUN:

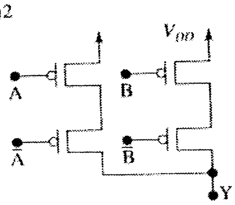
PDN for $Y = AB + \bar{A}\bar{B}$:

D2



PUN dual to D2:

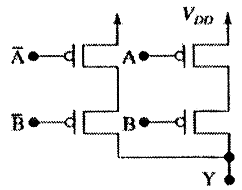
u2



The two circuits required are U_1 with D1 and U_2 with D_2.

14.34

$Y = AB + \bar{A}\bar{B}$. Directly, the PUN is as follows: u1

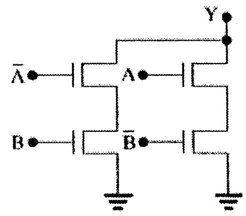


Now,

$\bar{Y} = \overline{AB + \bar{A}\bar{B}} = \overline{AB} \, . \, \overline{\bar{A}\bar{B}} = (\bar{A} + \bar{B})/(A + B)$

or $\bar{Y} = \bar{A}B + A\bar{B}$

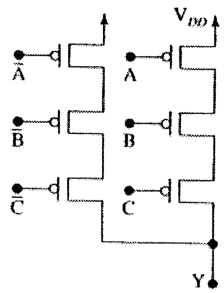
Directly, the PDN is:



14.35

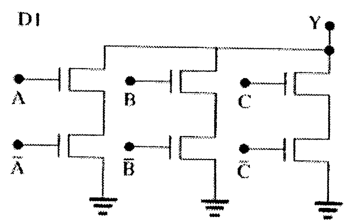
$Y = ABC + \bar{A}\bar{B}\bar{C}$. Directly, the PUN is as shown below:

u1



The corresponding dual PDN is shown above below.

D1



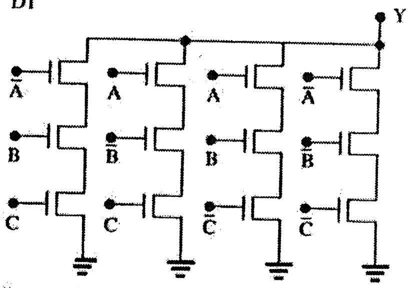
14.36

a) Even-parity circuit:

$\overline{Y} = \overline{A}BC + A\overline{B}C + AB\overline{C} + \overline{A}\overline{B}\overline{C}$,

b) PDN directly is:

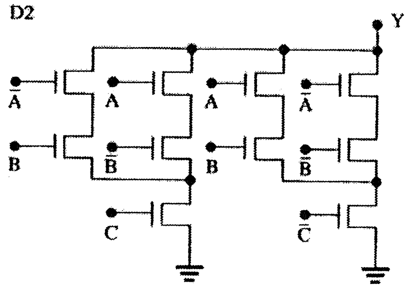
D1



It uses 12 transistors.

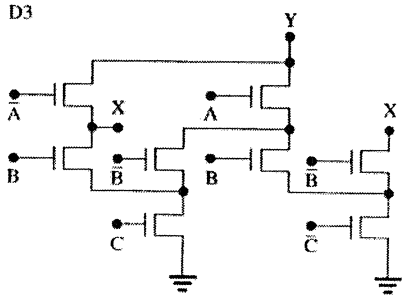
(c) PDN reduced to 10 transistors:

D2



PDN reduced to 8 transistors: (X and X are joined)

D3



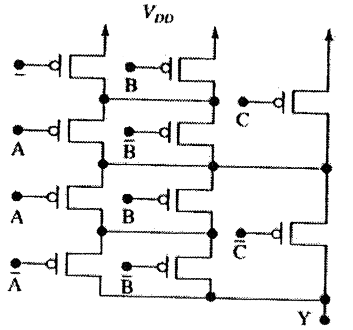
[Think circuit is not "planar", but has one "cross-over" (x-x); it has no convenient dual]

PUN as the dual of D_2:

[Think of the structure of the dual of D_1 when constructing this]

The complete circuit, using u_2 and D_2 has 20 transistors.

u2



14.37

Sum, $S = A\overline{B}\overline{C} + \overline{A}B\overline{C} + \overline{A}\overline{B}C + ABC$

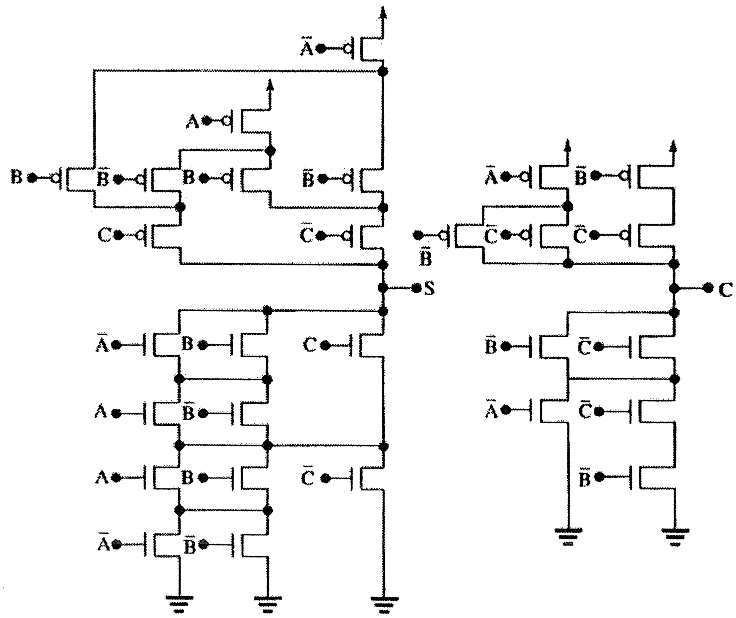
Carry $C_o = AB\overline{C} + A\overline{B}C + \overline{A}BC + ABC$

$= AB + AC + BC = A(B + C) + BC$

Create the PUN, directly, simplifying that for S as in P13.50 above as

$S = \overline{A}(B\overline{C} + \overline{B}C) + A(\overline{B}\overline{C} + BC)$

This figure is for 14.37



14.38

For matched-inverter equivalence of the circuit

$p_A = p : p_B = p_C = p_D = 2p$

and

$n_A = n_B = 2n : n_C = n_D = 2(2n) = 4n.$

14.39 blank

14.40

Ignore the capacitances of the transistors themselves; For the matched NAND,

$t_{PLH} = t_{PHL} = t_P$. For the "uncompensated NAND, $t_{PLH} = t_P$, $t_{PHL} = t_P/4$. Thus, t_{PLH} are the same, but t_{PHL} is 4 times greater with no matching.

14.41

For design a), there are $2(6)+2 = 14$ transistors;

All 7 NMOS use $(W/L)_n = n$

1 PMOS uses $(W/L)_p = p$

6 PMOS use $(W/L)_p = 6p$

Total Area $= 7(1.2)0.8 + 1(3.6)0.8 + 6(6)(3.6)0.8 = 113.3\ \mu m^2$

For design b), there are $2(3)2 + 1(2)2 = 16$ transistors;

6 NMOS use $(W/L)_n = n$

6 PMOS use $(W/L)_p = 3p$

2 PMOS use $(W/L)_p = p$

2 NMOS use $(W/L)_n = 2n$

Total equivalent devices is $6n + 18p + 2p + 2n = 10n + 20p$

Total equivalent area is $[10 + 3(20)]n = 70n$, and

Total Area $= 70(1.2)0.8 = 67.2\ \mu m^2$, or 59% of a)

14.42

Corresponding to a matched inverter characterized by n and p where $k_p = k_n = k$, the two-input NOR uses transistors n and 2p where $k_p = 2k_n$

a) For A grounded, V_{thB} occurs near $V_{DD}/2$, with Q_{PB} and Q_{NB} in saturation and Q_{PA} in trade. Let $V_{th} = v$ and the voltage across Q_{PA} be x.

Thus $i_D = k_p[(5-1)x - x^2/2]$

and $i_D = \frac{1}{2}k_p(5 - x - v - 1)^2$

and $i_D = \frac{1}{2}k_n(v-1)^2$

For $k_p = 2k_n$

$i_p = 2k_n(4x - x^2/2) = k_n(8x - x^2)$ (1)

and $i_D = k_n(4x - x - v)V^2$ (2)

and $i_D = \frac{1}{2}k_n(v-1)^2$ (3)

From 2) 3): $\pm(v-1)(0.707) = 4 - x - v$

Thus,

$1.707v = 4.707 - x$ or
$x = 4.707v - 1.707v$

$0.293v = 3.293 - x$
$x = 3.293 - 0.293v$

Now $x \approx 0$, in which case

$v = 4.707/1.707 = 2.38$ or

$v = 3.293/0.293 = 11.2$ (clearly too large)

Thus

$x = 4.707 - 1.707v$ (4)

Now, from 1), 3): $(v-1)^2 = 2(8x - x^2)$

with 4)

$v^2 - 2v + 1 = 16(4.707 - 1.707v) - 2$

$(4.707 - 1.707v)^2$ or

$v^2 - 2v + 1 = 75.32 - 27.32v - 44.31 + 32.13v$

$-5.83v^2$

or

$6.83v^2 + v(-2 + 27.32 - 32.13)$

$+ (1 - 75.32 + 44.31) = 0$

or

$6.833v^2 - 6.81v - 30.01 = 0$

whence

$v = (-6.81 \pm (6.81^2 - 4(6.83)30.01)^{\frac{1}{2}}/2(6.83)$

$= (6.81 \pm 29.43)/13.66 = 2.65\text{V}$

Check: [>2.5V probably OK since one PMOS is full on]

Thus $V_{th} = 2.65\text{V}$

b) For A and B joined, the PMOS can be approximated as a single device with twice the length for which the width is twice that in a matched inverter. Thus, for the equivalent PMOS device $(W/L)_{peq} = P$ and $k_p = k$. For each of the two NMOS $(W/L)_n = n$ and $k_n = k$.

Thus at $v_{th} = v$ with all devices in saturation:

$i_D = 2k/2(v-1)^2 = (k/2)(5 - v - 1)^2$

$2(v-1)^2 = (4-v)^2$, and

$\pm\sqrt{2}(v-1) = (4-v)$

Thus, $1.414v - 1.414 = 4 - v$,

$2.414v = 5.414$,

whence $V_{th} = v = 2.24$ V

See this is reduced from the single-input value (of 2.65V)!

Note that this fact can be used to control the relative threshold of multiple gates connected to a single fanout node in order to guarantee operation sequence for slowly changing signals.

14.43

a) $t_p \alpha \frac{\alpha C}{k' V_{DD}}$, and k' is sealed by S, and C and V_{DD} are scaled by $\frac{1}{S}$ thus t_p is scaled by

$$\frac{\frac{1}{S}}{S \times \frac{1}{S}} = \frac{1}{S}$$

$S = 4 \Rightarrow t_p$ is scaled by $\frac{1}{4}$ (t_p decreases)

The maximum operating speed is $\frac{1}{2t_p}$ and therefore is scaled by 4.

$P_{dyn} = f_{max} C V^2_{DD}$ and thus is scaled by

$S \times \frac{1}{S} \times \frac{1}{S^2} = \frac{1}{S^2} = \frac{1}{16}$ (P_{dyn} decreases) power density $= \frac{P_{dyn}}{\text{area}}$ and thus is scaled by $\frac{\frac{1}{S^2}}{\frac{1}{S^2}} = 1$

i.e., remains unchanged.

PDP is scaled by $\frac{1}{S^3}$ (power is scaled by $\frac{1}{S^2}$ and delay by $\frac{1}{S}$ and thus it is scaled by $\frac{1}{64}$ (PDP decreases)

b) If V_{DD} and V_{tn} only scaled by $\frac{1}{2}$ while $S = 4$ we have:

$t_p = \frac{\alpha C}{k' V_{DD}}$ and $\alpha = \frac{2}{\frac{7}{4} - \frac{3V_{tn}}{V_{DD}} + \left(\frac{V_{tn}}{V_{DD}}\right)^2}$ so

α remains unchanged and t_p is scaled by

$$\frac{\frac{1}{S}}{S \times \frac{1}{2}} = \frac{\frac{1}{4}}{4 \times \frac{1}{2}} = \frac{1}{8}$$

The maximum operating speed is $\frac{1}{2t_p}$ and therefore is scaled by 8.

$P_{dyn} = f_{max} C V^2_{DD}$ and thus is scaled by

$$8 \times \frac{1}{5} \times \frac{1}{2^2} = 8 \times \frac{1}{4} \times \frac{1}{2} = 1$$

Power density $= \frac{P_{dyn}}{\text{area}}$ is thus scaled by

$$\frac{1}{\frac{1}{S^2}} = \frac{1}{\frac{1}{16}} = 16$$

PDP is scaled by $1 \times \frac{1}{8} = \frac{1}{8}$

14.44

$V_{DSsat} = \left(\frac{L}{\mu_n}\right) v_{sat} \Rightarrow \mu_n = \frac{L}{V_{DDsat}} v_{sat}$

$v_{sat} \simeq 10^7 \text{cm/s} = 10^5 \text{m/s}$, $L = 0.18\ \mu\text{m}$ and $V_{DSsat} = 0.6$ V

$$\mu_n = \frac{0.18 \times 10^{-6} \text{m}}{0.6\text{V}} \times 10^5 \text{m/s} = 0.03 \frac{\text{m}^2}{\text{V} \cdot \text{S}}$$

$$= 300 \frac{\text{cm}^2}{\text{V} \cdot \text{S}}$$

For PMOS we have $V_{DSsat} = 1$ V, thus

$$\mu_p = \frac{0.18 \times 10^{-6}\ \text{m}}{1\ \text{V}} \times 10^5 \text{m/s} = 0.018 \frac{\text{m}^2}{\text{V} \cdot \text{S}}$$

$$= 180 \frac{\text{cm}^2}{\text{V} \cdot \text{S}}$$

From equation (13.92) of the Text, we have:

$E_{cr} = \frac{V_{DSsat}}{L}$

For NMOS: $E_{cr} = \frac{0.6\ \text{V}}{0.18\ \mu\text{m}} = 3.33 \times 10^6 \frac{\text{V}}{\text{m}}$;

$= 3.33 \times 10^{-4} \frac{\text{V}}{\text{cm}}$

For PMOS: $E_{cr} = \frac{1\ \text{V}}{0.18\ \mu\text{m}} = 5.56 \times 10^6 \frac{\text{V}}{\text{m}}$

$= 5.56 \times 10^4 \frac{\text{V}}{\text{cm}}$

14.45

assuming $g_{sat} = 10^7$cm/s, then:

$$V_{Dsat_n} = \frac{L}{\mu_n} \times g_{sat} = \frac{0.13 \times 10^{-6}}{325 \times 10^{-4}} \times 10^7 \times 10^{-2}$$

$= 0.4$ V

$$V_{Dsat_p} = \frac{L}{\mu_p} \times g_{sat} = \frac{0.13 \times 10^{-6}}{200 \times 10^{-4}} \times 10^7 \times 10^{-2}$$

$= 0.65$ V

14.46

$t_{PHL} = \frac{CV_{DD}}{2I_{av}}$

Since based on the assumption in this problem, Q_N turns on immediately (V_I rises instantaneously to V_{DD}) and it operates in the velocity-saturation region then $I_{av} = I_{Dsat}$ thus, $t_{PHL} = \frac{CV_{DD}}{2I_{Dsat}}$

b) From equations (13.68) and (13.70) of the Text we have:

$t_{PHL} = 0.69 R_N C$ and

$$R_N = \frac{12.5\ \text{k}\Omega}{(W/L)_n} \Rightarrow t_{PHL} = 0.69C\frac{12.5 \times 10^3}{(W/L)_n}$$

c) If the formula in (a) and (b) yield the same result we have:

$\frac{CV_{DD}}{2I_{Dsat}} = 0.69C\frac{12.5 \times 10^3}{(W/L)_n}$ and from equation (13.94) of the Text we have:

$$I_{Dsat} = \mu_n C_{ox} \left(\frac{W}{L}\right)_n V_{DSsat} \left(V_{GS} - V_t - \frac{1}{2} V_{DSsat}\right)$$

where in this case $V_{GS} = V_{DD}$ thus:

$$\frac{V_{DD}}{2 \times 0.69 \times 12.5 \times 10^3} = \frac{\mu_n C_{ox}\left(\frac{W}{L}\right)_n V_{DSsat}\left(V_{DD} - V_t - \frac{1}{2}V_{DSsat}\right)}{\left(\frac{W}{L}\right)_n}$$

$$\Rightarrow \frac{1.2}{17250} = 325 \times 10^{-6}\left(1.2 - 0.4 - \frac{V_{DSsat}}{2}\right)$$

$$\frac{1.2}{17250} = 325 \times 10^{-6} V_{DSsat}\left(1.2 - 0.4 - \frac{V_{DSsat}}{2}\right)$$

$$1.2 = 5.61 V_{DSsat}\left(0.8 - \frac{V_{DSsat}}{2}\right)$$

$$\Rightarrow 2.805 V^2_{DSsat} - 4.488 V_{DSsat} + 1.2 = 0$$

$V_{DSsat} = 1.261$ V or $V_{DSsat} = 0.339$ V

The answer $V_{DSsat} = 1.261$ V is not acceptable as it is above V_{DD}. Thus, $V_{DSsat} = 0.339$ V

14.47

$$I_{DSsatn} = \mu_n C_{ox}\left(\frac{W}{L}\right)_n V_{DSsatn}\left(V_{GS} - V_{tn} - \frac{1}{2}V_{DSsatn}\right)$$

and

$$I_{DSsatp} = \mu_n C_{ox}\left(\frac{W}{L}\right)_p |V_{DSsatp}||V_{GS}| - |V_{tp}| - \left(-\frac{1}{2}|V_{DSsatp}|\right)$$

Since $|V_{GS}| = V_{DD}$(i.e., for NMOS $V_{GS} = V_{DD}$ and for PMOS $|V_{GS}| = V_{DD}$ and $I_{DSsatn} = I_{DSsatp}$ we have:

$$\mu_n C_{ox}\left(\frac{W}{L}\right)_n V_{DSsatn}\left(V_{DD} - V_{tn} - \frac{1}{2}V_{DSsatn}\right)$$

$$= \mu_p C_{ox}\left(\frac{W}{L}\right)_p |V_{DSsatp}|\left(V_{DD} - |V_{tp}| - \frac{1}{2}|V_{DSsatp}|\right)$$

$L_n = L_p \Rightarrow$ Thus,

$$\frac{w_p}{w_n} = \frac{\mu_n}{\mu_p}\frac{V_{DSsatn}}{|V_{DSsatp}|}\frac{V_{DD} - V_{tn} - \frac{1}{2}V_{DSsatn}}{V_{DD} - |V_{tp}|\left|\frac{1}{2}V_{DSsatp}\right|}$$

b) $$\frac{w_p}{w_n} = \frac{\mu_n}{\mu_p}\frac{V_{DSsatn}}{|V_{DSsatp}|}\frac{V_{DD} - V_{tn} - \frac{1}{2}V_{DSsatn}}{V_{DD} - |V_{tp}|\frac{1}{2}V_{DSsatp}}$$

$$= 4 \times \frac{0.34}{0.6}\,\frac{1.2 - 0.4 - \frac{0.34}{2}}{1.2 - 0.4 - \frac{0.6}{2}} = 2.86$$

14.48

a) $R = 27\ \text{m}\Omega/\square \times \frac{10\ \text{mm}}{0.5\ \mu\text{m}} = 540\ \Omega$

b) $C = 0.1\ \text{fF}/\mu\text{m} \times 10\text{mm}$

$= 0.1\ \text{fF}/\mu\text{m} \times 10000\ \mu\text{m} = 1000\ \text{fF} = 1\ \text{pF}$

c) $t_{delay} = 0.69RC = 372.6$ ps

15.1

Here $V_{DD}/4 = 5/4 = 1.25V$

Now, for v_O rising, the NMOS is cutoff, and the PMOS is in triode mode with:

$i_{D_p} = k_p[(v_{SG} - V_t)v_{SD} - v_{SD}^2/2]$, and here

$i_D = k_p[(5-0.8)(5-1.25) - (5-1.25)^2/2]$
$= k_p(15.75 - 7.03) = \underline{8.72k_p}$

Now, for v_O falling, the net current extracted from the load is $i_{D_n} - i_{D_p}$ which should be i_{D_p}

Thus $i_{D_n} = 2i_{D_p} = 2(8.72)k_p$, for triode operation where $i_{D_n} = k_n[(5-0.8)1.25 - 1.25^2/2]$

Overall, $i_{D_n} = 2(8.72)k_p = k_n(5.25 - 0.78) = 4.47k_n$
Thus $k_p = (4.47/(2(8.72)))k_n = \underline{0.256k_n}$

Check using Eq 10.39, where $r = k_n/k_p = 3.91$:
$V_{OL} = (V_{DD} - V_t)[1 - (1 - 1/r)^{1/2}]$
$= (5 - 0.8)[1 - (1 - 1/3.91)^{1/2}] = \underline{0.577V}$ ✓

From Eq 13.35, $V_{IL} = V_t + (V_{DD} - V_t)/(r(r+1))^{1/2}$
$= 0.8 + 4.2/(3.91 \times 4.91)^{1/2} = \underline{1.76V}$

From Eq 13.38, $V_{IH} = V_t + (2/\sqrt{3r})(V_{DD} - V_t)$
$= 0.8 + (2/\sqrt{3(3.91)})4.2 = \underline{3.25V}$

From Eq 13.36, $V_M = V_t + (V_{DD} - V_t)/(r+1)^{1/2}$
$= 0.8 + 4.2/(4.91)^{1/2} = \underline{2.70V}$

Now, $NM_H = V_{OH} - V_{IH} = 5.00 - 3.25 = \underline{1.75V}$

and $NM_L = V_{IL} - V_{OL} = 1.76 - 0.58 = \underline{1.18V}$

15.2

$$\alpha_p = 2/\left[\frac{7}{4} - 3\left(\frac{0.4}{1.2}\right) + \left(\frac{0.4}{1.2}\right)^2\right] = 2.3$$

$$t_{PLH} = \frac{\alpha_p C}{k_p V_{DD}} = \frac{2.3(10f)}{\left(\frac{430\ \mu}{4}\right) \cdot \left(\frac{W}{L}\right)_p \cdot 1.2}$$

$$r = \frac{\mu_n C_{ox}\left(\frac{W}{L}\right)_n}{\mu_p C_{ox}\left(\frac{W}{L}\right)_p} = 4$$

$$\frac{430\ \mu(1)}{\frac{430\ \mu}{4}\left(\frac{W}{L}\right)_p} = 4 \Rightarrow \left(\frac{W}{L}\right)_p = \frac{4}{4} = 1$$

t_{PLH} = 0.18 nsec or 180 psec

using eq. (14.17) and (14.18)

$$\alpha_n = 2/\left[1 + \frac{3}{4}\left(1 - \frac{1}{r}\right) - \left(3 - \frac{1}{r}\right)\left(\frac{V_+}{V_{DD}}\right)^2\right] = 2.6$$

$$t_{PHL} \cong \frac{\alpha_n C}{k_n V_{DD}} = \frac{2.6(10f)}{(430\ \mu)(1)(1.2)} = 50 \text{ psec}$$

$$t_P = \frac{1}{2}(t_{PHL} + t_{PLH}) = \frac{1}{2}(180p + 50p) = 115 \text{ psec}$$

15.3

$NM_L = V_t - (V_{DD} - V_t)[1 - (1 - \frac{1}{r})^{1/2} - (r(r+1))^{-1/2}]$

Now, $\partial NM_L/\partial r = -(V_{DD} - V_t)[-\frac{1}{2}(1 - \frac{1}{r})^{-1/2}(-\frac{1}{r^2}) - (-\frac{1}{2}(r(r+1))^{-3/2}(2r+1))$

Maximum occurs where:
$-\frac{1}{2}(1 - 1/r)^{-1/2}(1/r^2) = -\frac{1}{2}(r(r+1))^{-3/2}(2r+1)$

Square both sides: $(1 - 1/r)^{-1}/r^4 = r^{-3}(r+1)^{-3}(2r+1)^2$

or $\frac{1}{1 - 1/r} = \frac{r}{(r+1)^3}(2r+1)^2 = \frac{r}{r-1}$

15.4 blank

15.5

$NM_H = (V_{DD} - V_t)(1 - 2/\sqrt{3r})$

This is zero, when $1 - 2/\sqrt{3r} = 0$, or $\sqrt{3r} = 2$, or $3r = 4$, or $r = \underline{1.33}$

For $r = 1$, $NM_H = 4.2(1 - 2/\sqrt{3}) = \underline{-0.65}V$

For $r = 2$, $NM_H = 4.2(1 - 2/\sqrt{6}) = \underline{0.77}V$

For $r = 4$, $NM_H = 4.2(1 - 2/\sqrt{12}) = \underline{1.78}V$

For $r = 8$, $NM_H = 4.2(1 - 2/\sqrt{24}) = \underline{2.48}V$

For $r = 16$, $NM_H = 4.2(1 - 2/\sqrt{48}) = \underline{2.99}V$

But, what about NM_L? (For $r = 16$, it is 0.92V)

15.6

noise margins are equal when

$$V_t - (V_{DD}-V_t)[1-(1-\tfrac{1}{r})^{1/2} - 1/(r(r+1))^{1/2}] = (V_{DD}-V_t)[1-2/(3r)^{1/2}]$$

or $V_t/(V_{DD}-V_t) = 2 - 2/(3r)^{1/2} - (1-1/r)^{1/2} - 1/(r(r+1))^{1/2}$ — (1)

Here $V_t/(V_{DD}-V_t) = 0.8/(5.0-0.8) = \underline{0.1904}$

Try various values of r to solve (1):

For $r=2$, $f(r) = 2 - 2/6^{1/2} - (1-1/2)^{1/2} - 1/(2(3))^{1/2}$
$= 2 - 0.816 - 0.707 - 0.408 = \underline{0.069}$

For $r=3$, $f(r) = 2 - 2/9^{1/2} - (1-1/3)^{1/2} - 1/(3(4))^{1/2}$
$= 2 - 0.667 - 0.816 - 0.289 = \underline{0.228}$

Try $r=2.8$, $f(r) = 2 - 2/(3(2.8))^{1/2} - (1-1/2.8)^{1/2} - 1/(2.8(3.8))^{1/2}$
$= 2 - 0.690 - 0.802 - 0.307 = \underline{0.201}$

Try $r=2.7$, $f(r) = 2 - 2/(3(2.7))^{1/2} - (1-1/2.7)^{1/2} - 1/(2.7(3.7))^{1/2}$
$= 2 - 0.703 - 0.793 - 0.316 = \underline{0.188}$

Conclude $r \simeq \underline{2.72}$, for which the margins are:

$NM = NM_H = NM_L = (V_{DD}-V_t)(1-2/(3r)^{1/2})$
$= 4.2(1-2/(3(2.72))^{1/2}) = \underline{1.26V}$

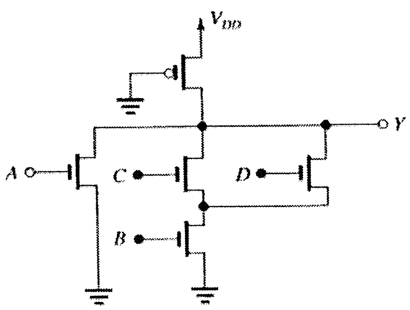
15.7 blank

15.8

$Y = \overline{A + B(C+D)}$,

whence $\overline{Y} = A + B(C+D)$

Thue the PDN can be formed directly as shown :



Now, $t_{PLH}/t_{PHL} = (k_n/k_p)(1-0.46/r)$

$= r(1-0.46/r) = 2.72 - 0.46 = 2.26$

Now,

$t_{PLH} = 1.7(1\times10^{-12})/(25\times10^{-6}(1.33/0.8)(5))$

$= 8.24\text{ns}$

and $t_{PHL} = 8.24/2.26 = 3.65\text{ns}$

and $t_p = (8.24+3.65)/2 = 5.95\text{ns}$

Now, dynamic power is approximately $fC\,V^2_{DD}$ since the output swing is not quite V_{DD}.

For equal static and dynamic power

$f \times 1\times10^{-12}\times5^2 = 1.82\times10^{-3}$

whence

$f = 1.82\times10^{-3}/(25\times10^{-12}) = 72.8\text{MHz}$

for which the period is

$1/(72.8\times10^6) = 13.7\text{ ns}$

Now, for transition times in the same proportion as propagation delays

$t_{TLH}/t_{TTHL} = 8.24/3.65 = 2.26$

Now, for full output swing, there must be time for 2 full transitions in each cycle :

Thus

$t_{THL} = 13.7/(1+2.26) = 4.19\text{ns}$ and

$t_{TLH} = 4.19(2.26) = 9.47\text{ns}$

Since these values are of the same order as the propagation delays. Full swing operation is likely not possible at 72.8MHz.

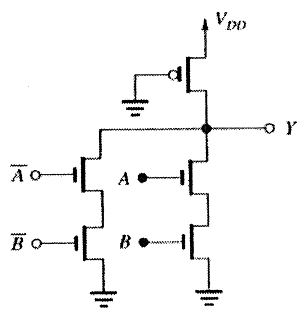
15.9

For an Exclusiue OR, $Y = A\overline{B} + \overline{A}B$, and

$\overline{Y} = \overline{A\overline{B} + \overline{A}B} = \overline{A\overline{B}}\cdot\overline{\overline{A}B} = (\overline{A}+B)(A+\overline{B})$

or $\overline{Y} = \overline{A}\overline{B} + AB$

The PDN results directly :



15.10

For a pseudo-NMOS NOR gate, independant of the number of inputs, the worst-case value of V_{OL} occurs for one input high (and a single NMOS conducting

From Eq. 10.39, $V_{OL} = (V_{DD} - V_t)[1 - (1 - 1/r)^{1/2}]$, for which $0.2 = (5.0 - 0.8)[1 - (1 - 1/r)^{1/2}]$, and $(1 - 1/r)^{1/2} = 1 - 0.2/4.2 = 0.952$

Thus $1 - 1/r = 0.907$

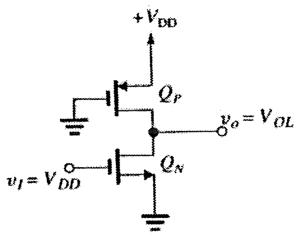
$1/r = 0.093$, and $r = \underline{10.76}$

Thus $k_n/k_p = 10.76 = 75(1.8/1.2)/(25(W/L)_p)$

Thus $(W/L)_p = (75/25)(1.8/1.2)/10.76 = \underline{0.418}$

Thus for $W_p = 1.8\mu m$, $L_p = 1.8/0.418 = \underline{4.31\mu m}$ and $(W/L)_p = \underline{(1.8/4.31)}$

15.11



$V_{Dsat} = 0.6\text{V}$

$V_{DD} = 1.2\text{V}$

$V_t = 0.4\text{V}$

$\mu_n C_{ox} = 4\mu_p C_{ox} = 430\mu\text{A}/\text{V}^2$

$L = 0.13\mu\text{m}$

In the case of Q_p,

$V_{SG} - V_t = 1.2\text{V} - 0.4\text{V} = 0.8\text{V}$, which is $> V_{Dsat}$

For reliable logic levels and noise margins, $V_{SD} > V_{Dsat}$ so that Q_p is operating in the velocity saturation region.

Ignoring channel-length modulation,

$$I_{Dsat} = \mu_p C_{ox}\left(\frac{W}{L}\right)_p |V_{Dsat}| \cdot \left[V_{SG} - |V_t| - \frac{1}{2}|V_{Dsat}|\right]$$

$$= \frac{1}{4}(430\mu\text{A}/\text{V}^2)\left(\frac{W}{L}\right)_p (0.6\text{V}) \bullet$$

$$\left[1.2\text{V} - 0.4\text{V} - \frac{1}{2}(0.6\text{V})\right]$$

$$I_{Dsat} = 32.25\left(\frac{W}{L}\right)_p \mu\text{A}$$

For Q_N,

$V_{GS} - V_t = 1.2\text{V} - 0.4\text{V} = 0.8\text{V}$

and $V_{DSat} = 0.6\text{V}$

$V_{GS} - V_t > V_{DSat}$, but $V_{DS} < V_{DSat}$

This defines the triode region.

$$i_{DN} = \mu_n C_{ox}\left(\frac{W}{L}\right)_n V_{DS}\left[(V_{GS} - V_t) - \frac{1}{2}V_{DS}\right]$$

$$i_{DN} = (430\mu\text{A}/\text{V}^2)\left(\frac{W}{L}\right)_n\left[(0.8\text{V})V_{DS} - \frac{1}{2}V^2_{DS}\right]$$

The unknowns in this problem are $\left(\frac{W}{L}\right)_n$, $\left(\frac{W}{L}\right)_p$ and V_{DS}. One possibility would be to match the source and sink currents for charging the output capacitance.

Without further information, let us assume as in Exercise 13.26

that $\left(\frac{W}{L}\right)_n = \left(\frac{W}{L}\right)_p = 1.5$

In this case,

$I_{DSat} = 32.25(1.5)\mu\text{A} = 48.4\mu\text{A}$

In the static state,

$i_{DN} = i_{DP}$ so that

$$(430\mu\text{A}/\text{V}^2)(1.5)\left[(0.8\text{V})v_{DS} - \frac{1}{2}v^2_{DS}\right]$$

$$= 48.4\mu\text{A}$$

$$0.8v_{DS} - \frac{1}{2}v^2_{DS} = \frac{48.4\mu\text{A}}{430\mu\text{A}(1.5)} = 0.075$$

$$v^2_{DS} - 1.6v_{DS} + 0.15 = 0$$

$$v_{DS} = \frac{1.6 \pm \sqrt{(1.6)^2 - 4(1)(0.15)}}{2} = 0.8 \pm 0.7\text{ V}$$

$$V_{OL} = v_{DS} = 0.1\text{V}$$

15.12

(a) $V_{OH} = V_{DD} - V_t$

and $V_t = V_{t0} + \gamma(\sqrt{V_{OH} + 2\phi_f} - \sqrt{2\phi_f})$

so, $V_t = V_{t0} + \gamma(\sqrt{V_{DD} - V_t + 2\phi_f} - \sqrt{2\phi_f})$

Substituting values, we have

$V_t = 0.5\text{V} + 0.3\text{V}^{1/2} \times$

$(\sqrt{1.8\text{V} - V_t + 0.85\text{V}} - \sqrt{0.85\text{V}})$

$V_t - 0.22\text{V} = (0.3\text{V}^{1/2})(\sqrt{2.65\text{V} - V_t})$

Squaring both sides,

$V_t^2 - 0.44V_t + 0.048 = 0.09(2.65 - V_t)$

or $V_t^2 - 0.35V_t - 0.191 = 0$

Solving this qundratic, we obtain

$V_t = 0.646\text{V}$

So that,

$V_{OH} = V_{DD} - V_t = 1.8\text{V} - 0.646\text{V} = 1.15\text{V}$

(b) $i_{DP} = \frac{1}{2}\mu pC_{ox}\left(\frac{W}{L}\right)_p (V_{DD} - V_{OH} - V_{to})^2$

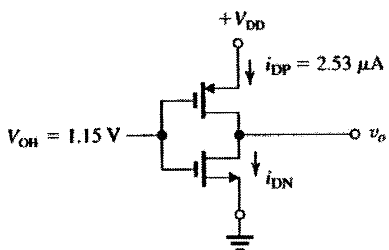
$= \frac{1}{2}75\mu(\text{A}/\text{V}^2)\left(\frac{0.54}{0.18}\right)(1.8\text{V} - 1.15\text{V} - 0.5\text{V})^2$

$i_{DP} = 2.53\mu\text{A}$

$P_D = V_{DD}i_{DP} = 1.8\text{V}(2.53\mu\text{A}) = 4.6\mu W$

To find the inverter's output voltage, we note that

$i_{DN} = i_{DP} = 2.53\mu\text{A}$



Since $V_{DS} < V_{GS} - V_t$ (triode region), we can to find v_o:

$i_{DN} = k_n\left[(v_t - V_t)v_o - \frac{1}{2}v_o^2\right]$

where $V_t = V_{to}$

$2.53\mu\text{A} = 300\mu\text{A}/\text{V}^2\left(\frac{0.54}{0.18}\right) \times$

$\left[(1.15\text{V} - 0.5\text{V})v_o - \frac{1}{2}v_o^2\right]$

$\frac{2.53\mu\text{A}}{300\mu\text{A}/\text{V}^2(1.5)} = 0.65\text{ V } v_o - \frac{1}{2}v_o^2$

or,

$v_o^2 - 1.3v_o + 0.0112 = 0$

solving for v_o, we get

$v_o = 0.01\text{V}$

(c) To find t_{PLH}, we can follow the procedure of

$i_{D(0)} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_n (V_{DD} - V_{to})^2$

$= \frac{1}{2}300\mu(\text{A}/\text{V}^2)1.5(1.8 - 0.5)^2\text{V}^2 = 380.3\mu\text{A}$

$V_t(at\ v_o = 0.9\text{V}) = V_{to} + \gamma(\sqrt{v_O + 2\phi_f} - \sqrt{2\phi_f})$

$= 0.5\text{ V} + 0.3\text{V}^{1/2}(\sqrt{0.9\text{V} + 0.85\text{V}} - \sqrt{0.85\text{V}})$

$V_t = 0.62\text{ V}$

$i_D(t_{PLH}) = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_n (V_{DD} - V_o - V_t)^2$

$= \frac{1}{2}(300\mu\text{A}/\text{V}^2)(1.5)(1.8\text{V} - 0.9\text{V} - 0.62\text{V})^2$

$= 17.6\mu\text{A}$

$i_D|_{av} = \frac{380.3\mu\text{A} + 17.6\mu\text{A}}{2} = 199\mu\text{A}$

$t_{PLH} = \frac{C\left(\frac{V_{DD}}{2}\right)}{i_{D|av}} = \frac{10(10^{-15})F\left(\frac{1.8\text{ V}}{2}\right)}{199(10^{-6})\text{A}}$

$= 0.045\text{ ns}$

(d) For V_I going LOW

$i_{D(o)} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_n (V_{DD} - V_{to})^2$

$= \frac{1}{2}(300\mu\text{A}/\text{V}^2)(1.5)(1.8\text{V} - 0.5\text{V})^2 = 380.3\mu\text{A}$

At $t = t_{PHL}$,

$i_D(t_{PHL}) = k_n\left[(V_{DD} - V_{to})v_o - \frac{1}{2}v_o^2\right]$

$= 300\mu\text{A}/\text{V}^2\left(\frac{1}{2}\right) \times$

$\left[(1.8\text{V} - 0.5\text{V})(0.9\text{V}) - \frac{1}{2}(0.9\text{V})^2\right]$

$i_D(t_{PHL}) = 114.8\mu\text{A}$

$i_D|_{av} = \frac{1}{2}(380.3\mu\text{A} + 114.8\ \mu\text{A}) = 247.6\mu\text{A}$

so that,

$t_{PHL} = \frac{C\left(\frac{V_{DD}}{2}\right)}{i_D|_{av}} = \frac{10(10^{-15})F\left(\frac{1.8\text{V}}{2}\right)}{247.6(10^{-6})\text{A}}$

$= 0.036\text{ns}$

(e) t_p

$\simeq \frac{1}{2}(t_{PLH} + t_{PHL}) = \frac{1}{2}(0.045\text{ ns} + 0.036\text{ ns})$

$= 0.04\text{ ns}$

15.13

For a) see directly that $X = 1.\bar{A} = \bar{A}$

and $Y = X \cdot \bar{B} = \bar{A} \cdot \bar{B}$

For b) see directly that $Y = \bar{A} \cdot \bar{B}$

For each circuit node Y nominally satisfies both conditions. However in a) with A high and B low, Y is not pulled down completely to ground, but remains at V_{tp}, due to the PMOS threshold. Circuit b) does not have this problem, but node X is floating for A,B both high. However, X is not an output node. The body effect makes this worse! Notice that b) is exactly a complemantary CMOS NOR gate for which $Y = \bar{A} \cdot \bar{B} = \overline{A + B}$

For V_{DD} replaced by an inverter driven by C,

$Y = \bar{C}(\bar{A} \cdot \bar{B}) = \bar{A} \cdot \bar{B} \cdot \bar{C} = \overline{A + B + C}$,

a 3-input NOR for both a) and b).

Practically speaking, however, there is a problem because as noted above, the series PMOS do not operate well with a low input. In fact Y is pulled down only to one thueshold drop below ground, when C is high.

15.14

For the switch gate and input both at $V_{DD} = 3.3\text{V}$, the switch output is

$V_{OH} = V_{DD} - V_t$

where $V_t = V_{to} + \gamma[\sqrt{V_{OH} + 2\phi_F} - \sqrt{2\phi_F}]$

Substituting for V_{OH}, we get :

$V_t = V_{to} + \gamma[\sqrt{V_{DD} - V_t + 2\phi_F} - \sqrt{2\phi_F}]$

$= 0.8\text{V} + 0.5\text{V}^{1/2}[\sqrt{3.3\text{V} - V_t + 0.6\text{V}} - \sqrt{0.6\text{V}}]$

So that,

$V_t = 0.413\text{ V} + 0.5\text{ V}^{1/2}\sqrt{3.9\text{ V} - V_t}$

$V_t - 0.413\text{ V} = 0.5\text{V}^{1/2}\sqrt{3.9\text{ V} - V_t}$

Squaring both sides, we get

$V_t^2 - 0.826V_t + 0.171 = 0.975 - 0.25V_t$

or, $V_t^2 - 0.576V_t - 0.804 = 0$

Solving this quadratic, we find that

$V_t = 1.23\text{V}$

$V_{OH} = V_{DD} - V_t = 3.3\text{V} - 1.23\text{V} = 2.07\text{V}$

with the input Low and the gate switch HIGH, $V_{OL} \to 0\text{V}$

If $V_{OH} = 2.07\text{V}$, the PMOS transistor of the inverter is in the saturation region. Since the inverter transistors are matched,

$\left(\frac{W}{L}\right)_p = \frac{k_n'}{k_p'}\left(\frac{W}{L}\right)_n$ so that

$i_{DP} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_p (V_{DD} - V_{OH} - V_{to})^2$

$i_{DP} = \frac{1}{2}(25\mu\text{A}/\text{V}^2)\left(\frac{1.2}{0.8}\right)(3) \times$

$(3.3\text{V} - 2.07\text{V} - 0.8\text{V})^2 = 10.4\mu\text{A}$

For t_{PLH}, at $t = 0$,

$i_D(0) = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_n (V_{DD} - V_{to})^2$

$= \frac{1}{2}(75\mu\text{A}/\text{V}^2)\left(\frac{1.2}{0.8}\right)(3.3\text{V} - 0.8\text{V})^2 = 352\mu\text{A}$

At $v_O = \frac{V_{DD}}{2}$,

$V_t = V_{to} + \gamma\left[\sqrt{\frac{V_{DD}}{2} + 2\phi_F} - \sqrt{2\phi_F}\right]$

$= 0.8\text{ V} + 0.5\text{ V}^{1/2}\left[\sqrt{\frac{3.3\text{ V}}{2} + 0.6\text{ V}} - \sqrt{0.6\text{ V}}\right]$

$= 1.16\text{ V}$

$i_D(t_{PLH}) = \frac{1}{2}\mu_n C_{OX}\left(\frac{W}{L}\right)_n (V_{DD} - v_o - v_t)^2$

$= \frac{1}{2}(75\mu\text{A}/\text{V}^2)\left(\frac{1.2}{0.8}\right)\left(3.3\text{V} - \frac{3.3\text{V}}{2} - 1.16\text{V}\right)^2$

$= 13.5\mu\text{A}$

$i_D|_{av} = \frac{(352\mu\text{A} + 13.5\mu\text{A})}{2} = 183\mu\text{A}$

$t_{PLH} = \frac{C\left(\frac{V_{DD}}{2}\right)}{i_D|_{av}} = \frac{100(10^{-15})F(1.65V)}{183\mu\text{A}}$

$= 0.9\text{ ns}$

For t_{PHL}, $V_t = V_{to}$ and

$i_{D(o)} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_n (V_{DD} - V_{to})^2$

$= \frac{1}{2}(75\mu\text{A}/\text{V}^2)\left(\frac{1.2}{0.8}\right)(3.3\text{V} - 0.8\text{V})^2 = 352\mu\text{A}$

$i_D(t_{PHL}) = \mu_n C_{ox}\left(\frac{W}{L}\right)_n \left[(V_{DD} - V_{to})v_o - \frac{1}{2}v_o^2\right]$

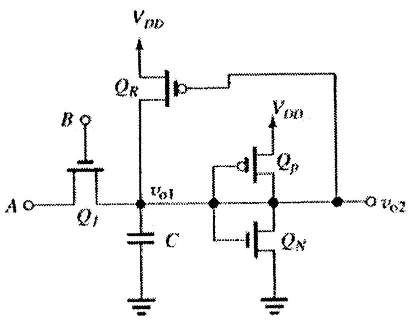
$= 75\mu\text{A}/\text{V}^2\left(\frac{1.2}{0.8}\right)$

$\left[(3.3\text{ V} - 0.8\text{ V})\left(\frac{3.3\text{ V}}{2}\right) - \frac{1}{2}\left(\frac{3.3\text{ V}}{2}\right)^2\right] = 311\mu\text{A}$

$i_D|_{av} = \frac{1}{2}(352\mu\text{A} + 311\mu\text{A}) = 332\mu\text{A}$

$t_{PHL} = \frac{C\left(\frac{V_{DD}}{2}\right)}{i_D|_{av}} = \frac{100(10^{-15})F(1.65\text{ V})}{332\mu\text{A}} = 0.5\text{ ns}$

15.15



For the inverter, with

$v_{o2} = V_{DD} - |V_{to}| = 3.3\text{ V} - 0.8\text{ V} = 2.5\text{ V}$

Q_N is in the saturation region, so that

$$i_{DN} = \frac{1}{2}k_n'\left(\frac{W}{L}\right)_n (v_{o1} - v_{to})^2$$

$$= \frac{1}{2}(75\mu A/V^2)\left(\frac{1.2}{0.8}\right)(v_{o1} - 0.8\text{ V})^2$$

$$= 56.25(v_{o1} - 0.8\text{ V})^2 \mu A$$

Q_p is operating in the triode region so

$$i_{DP} = k_P'\left(\frac{W}{L}\right)_p [(V_{DD} - v_{o1} - V_{to})(v_{DS}) - \frac{1}{2}(v_{DS})^2]$$

$$= (25\mu A/V^2)\left(\frac{3.6}{0.8}\right)\times \left[(3.3V - v_{o1} - 0.8\text{ V})(0.8\text{ V}) - \frac{1}{2}(0.8\text{ V})^2\right]$$

$$= 112.5[1.68 - 0.8v_{o1}]\mu A$$

Since $i_{DP} = i_{DN}$, we set these equal :

$$56.25(v_{o1} - 0.8V)^2 = 189 - 90v_{o1}$$

$$56.25(v^2_{o1} - 1.6v_{o1} + 0.64) = 189 - 90v_{O1}$$

Simplifying, we get

$$v_{o1}^2 - 2.72 = 0$$

$$v_{o1} = \sqrt{2.72} = 1.65\text{ V}$$

for Q_1,

$$v_t = v_{to} + \gamma[\sqrt{v_{o1} + 2\phi_f} - \sqrt{2\phi_f}]$$

$$V_t = 0.8V + 0.5V^{1/2}[\sqrt{1.65V + 0.6V} - \sqrt{0.6V}]$$

$$= 1.16\text{ V}$$

Capacitor charging current before Q_R turns on is due to the current supplied by Q_t.

$$\text{At } v_{o1}: i_D = \frac{1}{2}k_n'\left(\frac{W}{L}\right)_n (V_{DD} - v_{O1} - v_t)^2$$

$$= \frac{1}{2}(75\mu A/V^2)\left(\frac{1.2}{0.8}\right)(3.3V - 1.65V - 1.16V)^2$$

$$= 13.5\mu A$$

At $v_{O1} = 0V$,

$$i_D = \frac{1}{2}k_n'\left(\frac{W}{L}\right)_n (V_{DD} - v_{to})^2$$

$$= \frac{1}{2}(75\mu A/V^2)\left(\frac{1.2}{0.8}\right)(3.3V - 0.8V)^2 = 351.6\mu A$$

$$i_D|_{av} = \frac{1}{2}(13.5\mu A + 351.6\mu A) = 182.6\mu A$$

$$t_{PLH} = \frac{Cv_{O1}}{i_D|_{av}} = \frac{20(10^{-15})F(1.65V)}{182.6(10^{-6})A} = 0.18\text{ ns}$$

(b) For the inventer,

$$V_{IH} = \frac{1}{8}(5V_{DD} - 2V_t)$$

$$= \frac{1}{8}[5(3.3\text{ V}) - 2(0.8\text{ V})] = 1.86\text{ V}$$

For this value,

$$i_{D1} = k_n'\left(\frac{W}{L}\right)_n\left[(V_{DD} - v_{to})(v_{DS1}) - \frac{1}{2}(v_{DS1})^2\right]$$

$$i_{D1} = (75\mu A/V^2)\left(\frac{1.2}{0.8}\right)\times \left[(3.3\text{ V} - 0.8\text{ V})(1.86\text{ V}) - \frac{1}{2}(1.86\text{ V})^2\right]$$

$$i_{D1} = 328.5\mu A$$

The current in

$$Q_R = k_P'\left(\frac{W}{L}\right)_R (V_{DD} - v_{to})^2 = \frac{i_{D1}}{2}$$

So,

$$(25\mu A/V^2)\left(\frac{W}{L}\right)_R (3.3\text{ V} - 0.8\text{ V})^2 = \frac{328.5\mu A}{2}$$

$$\left(\frac{W}{L}\right)_R = \frac{328.5\mu A}{2} = \frac{1}{(25\mu A/V^2)(2.5\text{ V})^2} = 1.05$$

OR,

$$\left(\frac{W}{L}\right)_R = \frac{W}{0.8\mu m} \Rightarrow W = 0.84\mu m, \text{ and}$$

$$\left(\frac{W}{L}\right)_R = \frac{0.84\mu m}{0.8\mu m}$$

Initially, at $v_{O1} = V_{DD}, i_{DR} = 0$, since

$$V_{DSR} = o \text{ ,and } i_{D1} = \frac{1}{2}k_n'\left(\frac{W}{L}\right)_n (V_{DD} - v_{to})^2$$

$$= \frac{1}{2}(75\mu A/V^2)\left(\frac{1.2}{0.8}\right)(3.3V - 0.8V)^2 = 352\mu A$$

At $V_{O1} = V_{IH} = 1.86\text{ V}$,

$$i_{DR} = k_P'\left(\frac{W}{L}\right)_R [(V_{SG} - v_{to})\times (V_{DD} - V_{IH}) - \frac{1}{2}(V_{DD} - V_{IH})^2]$$

$= (25\mu A/V^2)(1.05)[(3.3V - 0.8V)(3.3V - 1.86V)$

$-\frac{1}{2}(3.3V - 1.86V)^2]$

$= 67.3\mu A$

$$i_{D1} = k_n'\left(\frac{W}{L}\right)_n[(V_{GS} - v_{to})(V_{DS}) - \frac{1}{2}(V_{DS})^2]$$

$$= (75\mu A/V^2)\left(\frac{1.2}{0.8}\right)[(3.3V - 0.8V)(1.86V)$$

$$-\frac{1}{2}(1.86V)^2] = 328.5\mu A$$

$$i_C|_{av} = \frac{1}{2}(328.5 + 352 - 67.3 - 0)\mu A = 306.6\mu A$$

$t_{PHL} \simeq$

$$\frac{C\Delta v_{O1}}{i_C|_{av}} = \frac{20(10^{-15})F(3.3V - 1.86V)}{306.6\mu A} = 94 \text{ ps}$$

15.16

(a) When the input goes HIGH, Q is ON and V_{OH} will approach $+V_{DD}$

(b) when the input goes Low and Q is ON, $V_{OL} \to |V_{tp}|$

(c) $i_D(o) = \frac{1}{2}k_P(V_{DD} - |V_{to}|)^2$

$$i_D(o) = \frac{1}{2}(225\mu A/V^2)(1.8\text{ V} - 0.5\text{ V})^2 = 190\mu A$$

when $v_o = \frac{V_{DD}}{2} = 0.9\text{ V}$,

$$i_D(t_{PLH}) = k_P[(V_{DD} - |V_{to}|)\left(\frac{V_{DD}}{2}\right)$$

$$-\frac{1}{2}\left(\frac{V_{DD}}{2}\right)^2]$$

$$i_D(t_{PLH}) = 225\mu A/V^2[(1.8\text{ V} - 0.5\text{ V})(0.9\text{ V})$$

$$-\frac{1}{2}(0.9\text{ V})^2]$$

$= 172\mu A$

$$i_D|_{av} = \frac{1}{2}(190\mu A + 172\mu A) = 181\mu A$$

$$t_{PLH} = \frac{C\left(\frac{V_{DD}}{2}\right)}{i_D|_{av}} = \frac{C(0.9V)}{181\mu A} = 5000C$$

15.17

$V_{OH} = V_{DD} = 1.8\text{ V}$

$V_{OL} = 0\text{ V}$

(b) $i_{DN}(o) = \frac{1}{2}k_n'\left(\frac{W}{L}\right)_n(V_{DD} - V_{to})^2$

$$= \frac{1}{2}(300\mu A/V^2)(1.5)(1.8V - 0.5V)^2 = 380.3\mu A$$

$$i_{DP}(o) = \frac{1}{2}k_P'\left(\frac{W}{L}\right)_p(V_{DD} - V_{to})^2$$

$$= \frac{1}{2}(75\mu A/V^2)(1.5)(1.8 - 0.5)^2V^2$$

$= 95.1\mu A$

$i_{DN}(t_{PLH})$ can be found by finding V_t

when $v_O = \frac{V_{DD}}{2}$:

$$V_t = V_{to} + \gamma(\sqrt{v_O + 2\phi_f} - \sqrt{2\phi_f})$$

$$V_t = 0.5\text{ V} + 0.3\text{ V}^{1/2}(\sqrt{0.9\text{ V} + 0.85\text{ V}} - \sqrt{0.85\text{ V}})$$

$V_t = 0.62 \text{ V}$

$i_{DN}(t_{PLH}) = \frac{1}{2}k_n'\left(\frac{W}{L}\right)_n\left(V_{DD} - \frac{V_{DD}}{2} - V_t\right)^2$

$= \frac{1}{2}(300\mu A/V^2)(1.5)(1.8 - 0.9 - 0.62)^2$

$= 17.64\mu A$

$i_{DP}(t_{PLH}) = k_p'\left(\frac{W}{L}\right)_p[(V_{DD} - V_{to})\left(\frac{V_{DD}}{2}\right)$

$-\frac{1}{2}\left(\frac{V_{DD}}{2}\right)^2]$

$= (75\mu A/V^2)(1.5)[(1.8 \text{ V} - 0.5 \text{ V})(0.9 \text{ V})$

$-\frac{1}{2}(0.9 \text{ V})^2]$

$= 86.1\mu A$

$i_D|_{av} \simeq$

$\frac{1}{2}[i_{DN}(0) + i_{DP}(0) + i_{DN}(t_{PLH}) + i_{DP}(t_{PLH})]$

$i_D|_{av} \simeq$

$\frac{1}{2}[380.3 + 95.1 + 17.6 + 86.1]\mu A = 290\mu A$

$t_{PLH} = \frac{C\frac{V_{DD}}{2}}{i_D|_{av}} = \frac{15(10^{-15})F\left(\frac{1.8V}{2}\right)}{290\mu A} = 0.047 \text{ ns}$

(c) For the situation in Fig. 14.12(b),

$i_{DN}(o) = \frac{1}{2}k_n'\left(\frac{W}{L}\right)_n(V_{DD} - V_{to})^2$

$= \frac{1}{2}(300\mu A/V^2)(1.5)(1.8 \text{ V} - 0.5 \text{ V})^2 = 380.3\mu A$

$i_{DP}(o) = \frac{1}{2}k_p'\left(\frac{W}{L}\right)_p(V_{DD} - |V_{to}|)^2$

$= \frac{1}{2}(75\mu A/V^2)(1.5)(1.8V - 0.5V)^2 = 95.1\mu A$

At $V_o = \frac{V_{DD}}{2} = 0.9 \text{ V}$,

$i_{DN}(t_{PHL}) = k_n'\left(\frac{W}{L}\right)_n\left[(V_{DD} - V_{to})v_o - \frac{1}{2}v_o^2\right]$

$= (300\mu A/V^2)(1.5)[(1.8V - 0.5V)(0.9V)$

$-\frac{1}{2}(0.9 \text{ V})^2]$

$= 344.3\mu A$

To estimate $i_{DP}(t_{PHL})$, we find $|V_{tp}|$ at

$v_o = \frac{V_{DD}}{2}$:

$|V_{tp}|(at\ V_o = 0.9 \text{ V}) = |V_{to}| + \gamma \times$

$[\sqrt{V_{DD} - v_o + 2\phi_f} - \sqrt{2\phi_f}]$

$= 0.5 \text{ V} + 0.3 \text{ V}^{1/2}[\sqrt{1.8 \text{ V} - 0.9 \text{ V} + 0.85 \text{ V}}$

$-\sqrt{0.85 \text{ V}}]$

$= 0.62 \text{ V}$

$i_{DP}(t_{PHL}) = \frac{1}{2}k_p'\left(\frac{W}{L}\right)_p(V_{DD} - v_o - V_t)^2$

$= \frac{1}{2}(75\mu A/V^2)(1.5)(1.8 \text{ V} - 0.9 \text{ V} - 0.62 \text{ V})^2$

$= 4.41\mu A$

$i_D|_{av} \simeq$

$\frac{1}{2}[i_{DN}(0) + i_{DP}(0) + i_{DN}(t_{PHL}) + i_{DP}(t_{PHL})]$

$= \frac{1}{2}[380.3 + 95.1 + 344.3 + 4.41]\mu A = 412\mu A$

$t_{PHL} = \frac{C\left(\frac{V_{DD}}{2}\right)}{i_D|_{av}} = \frac{15(10^{-15})F(0.9 \text{ V})}{412\mu A}$

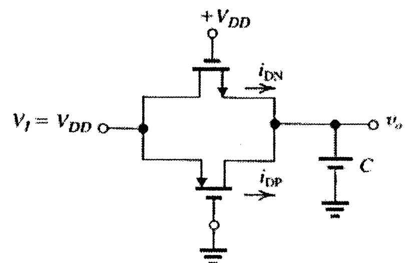
$= 0.033 \text{ ns}$

Q_P will turn off when $v_o = |V_{tp}| = 0.5 \text{ V}$

(d) $t_p \simeq \frac{1}{2}(t_{PLH} + t_{PHL}) = \frac{1}{2}(0.047 + 0.033) \text{ ns}$

$= 0.04 \text{ ns}$

15.18



With V_I going to V_{DD} and $v_o(o) = 0$ V,

$R_{Neq} = \frac{V_{DD} - v_o}{\frac{1}{2}k_n'\left(\frac{W}{L}\right)_n(V_{DD} - V_{tn} - v_o)^2}$

$= \frac{1.8 \text{ V} - 0 \text{ V}}{\frac{1}{2}(300\mu A/V^2)(1.5)(1.8 \text{ V} - 0.5 \text{ V} - 0v)^2}$

$= 4.7 \text{ k}\Omega$

$R_{Peq} = \frac{V_{DD} - 0}{\frac{1}{2}k_p'\left(\frac{W}{L}\right)_p(V_{DD} - |V_{tp}|)^2}$

$= \frac{1.8 \text{ V} - 0}{\frac{1}{2}(75\mu A/V^2)(1.5)(1.8 \text{ V} - 0.5)^2}$

$R_{Peq} = 18.9 \text{ k}\Omega$

$R_{TG}(V_o = 0) = R_{Neq} \| R_{Peq} = 4.7 \text{ k}\Omega \| 18.9 \text{ k}\Omega$

$= 3.76 \text{ k}\Omega$

when $V_o = 0.9$ V Q_N is still considered in the saturation region.

So, R_{Neq} (V_o = 0.9 V)

$$= \frac{V_{DD} - v_O}{\frac{1}{2}k_n'(W/L)(V_{DD} - V_{tn} - v_o)^2}$$

Where $V_{tn} = V_{to} + \gamma(\sqrt{V_o + 2\phi_f} - \sqrt{2\phi_f})$

$= 0.5 + 0.3\ V^{1/2}(\sqrt{0.9\ V + 0.85\ V} - \sqrt{0.85\ V})$

$= 0.62\ V$

$$R_{Neq} = \frac{1.8\ V - 0.9\ V}{\frac{1}{2}(300\mu A/V^2)(1.5)(1.8\ V - 0.62\ V - 0.9\ V)^2}$$

$= 51\ k\Omega$

$$R_{Peq} = \frac{1}{k_P'\left(\frac{W}{L}\right)_P\left[V_{DD} - |V_{tp}| - \frac{1}{2}(V_{DD} - V_O)\right]}$$

$$R_{Peq} = \frac{1}{(75\mu A/V^2)(1.5)\left[1.8\ V - 0.5\ V - \frac{1}{2}(1.8\ V - 0.9\ V)\right]}$$

$= 10\ k\Omega$

$R_{TG}(V_o = 0.9V) = R_{Neq} \| R_{Peq} = 51\ k\Omega \| 10\ k\Omega$

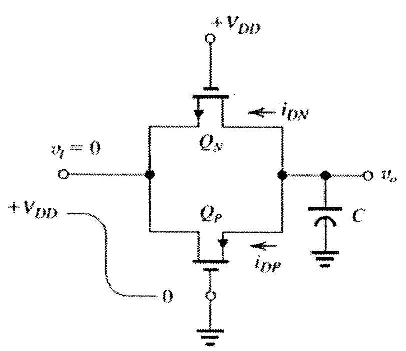
$= 8.36\ k\Omega$

$R_{TG}|_{av} = \frac{1}{2}(3.76\ k\Omega + 8.36\ k\Omega) = 6.06\ k\Omega$

$t_{PLH} = 0.69R_{TG}C = 0.69(6.06\ k\Omega)(1.5)(10^{-15})F$

$= 62.7\ ps$

15.19



C is charged so that $V_O = V_{DD}$

When v_I goes Low to 0V, Q_N is initially in the saturation region with

$$i_{DN} = \frac{1}{2}k_n(V_{DD} - V_{to})^2$$

until $V_{DSN} = V_{GS} - V_{tn} = V_{DD} - V_{tn}$

$$R_{Neq} = \frac{v_o - 0}{\frac{1}{2}k_n(V_{DD} - V_{to})^2} = \frac{2v_o}{k_n(V_{DD} - V_{to})^2}$$

for $v_o \ge V_{DD} - V_{tn}$

When Q_N enters the triode region,

$$i_{DN} = k_n\left[(V_{DD} - V_{to})v_o - \frac{1}{2}v_o^2\right]$$

for $v_O \le V_{DD} - V_{to}$

$$\text{Then, } R_{Neq} = \frac{v_o}{k_n\left[(V_{DD} - V_{to})v_o - \frac{1}{2}v_o^2\right]}$$

$$= \frac{1}{k_n\left[(V_{DD} - V_{to}) - \frac{1}{2}v_o\right]}$$

For Q_P initially,

$i_{DP} = \frac{1}{2}k_P(v_o - |V_{tp}|)^2$ so that

$$R_{Peq} = \frac{2v_o}{k_P(v_o - |V_{tp}|)^2}$$

where

$|V_{tp}| = |V_{to}| + \gamma(\sqrt{V_{DD} - v_o + 2\phi_f} - \sqrt{2\phi_f})$

until $v_o = |V_{tp}|$

For $v_o = V_{DD}$

$$R_{Neq}(v_o = V_{DD}) = \frac{2V_{DD}}{k_n'\left(\frac{W}{L}\right)_n(V_{DD} - V_{to})^2}$$

$$= \frac{2(1.8V)}{(300\mu A/V^2)(1.5)(1.8\ V - 0.5\ V)^2} = 4.7\ k\Omega$$

$$R_{Neq}(v_o = V_{DD}) = \frac{2V_{DD}}{k_p'(W/L)_p(V_{DD} - |V_{to}|)^2}$$

$$= \frac{2(1.8V)}{(75\mu A/V^2)(1.5)(1.8\ V - 0.5\ V)^2} = 18.9\ k\Omega$$

$R_{TG}(v_o = V_{DD}) = R_{Neq} \| R_{Peq} = 4.7\ k\Omega \| 18.9\ k\Omega$

$= 3.76\ k\Omega$

At $v_o = \frac{V_{DD}}{2} = 0.9\ V$

$V_{DD} - V_{tn} = 1.8\ V - 0.5 = 1.3\ V$

since $V_{DS} = v_o - 0 = 0.90$

Q_N is in the triode region.

$$R_{Neq} = \frac{1}{k_n'\left(\frac{W}{L}\right)_n\left[(V_{DD} - V_{to}) - \frac{1}{2}v_o\right]}$$

$$= \frac{1}{(300\mu A/V^2)(1.5)\left(1.8V - 0.5\ V - \frac{0.9\ V}{2}\right)}$$

$R_{Neq}(0.9\ V) = 2.6\ k\Omega$

At $v_o = 0.9\ V$,

$|V_{tp}| = |V_{to}| + \gamma(\sqrt{V_{DD} - v_o + 2\phi_f} - \sqrt{2\phi_f})$

$= 0.5\ V + 0.3\ V^{1/2}(\sqrt{1.8\ V - 0.9\ V + 0.85\ V}$

$- \sqrt{0.85\ V})$

$= 0.62\ V$

$$R_{Peq} = \frac{2v_o}{k_p'\left(\frac{W}{L}\right)_p (v_o - |V_{tp}|)^2}$$

$$= \frac{2(0.9\ V)}{(75\mu A/V^2)(1.5)(0.9V - 0.62V)^2}$$

$= 204\ k\Omega$

$R_{TG}(v_o = 0.9V) = 2.6\ k\Omega \parallel 204\ k\Omega = 2.57\ k\Omega$

$R_{TG}|_{av} = \frac{1}{2}(3.76\ k\Omega + 2.57\ k\Omega) = 3.17\ k\Omega$

$t_{PHL} = 0.69R_{TG}C = 0.69(3.17\ k\Omega)(15)(10^{-15})F$

$= 32.8$ ps

(This is close to the answer of Problem 14.19)

15.20

$$R_{TG} = \frac{12.5}{\left(\frac{W}{L}\right)_n}\ k\Omega = \frac{12.5}{1.5}\ k\Omega = 8.3\ k\Omega$$

$t_P \simeq t_{PLH} \simeq t_{PLH} \simeq 0.69R_{TG}C = 0.69(8.3\ k\Omega) \times (10)(10^{-15})F = 57.3$ps

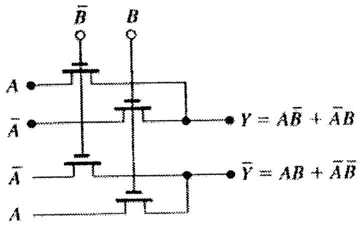
15.21

$$t_p = 0.69CR_{TG} \cdot \frac{n(n+1)}{2}$$

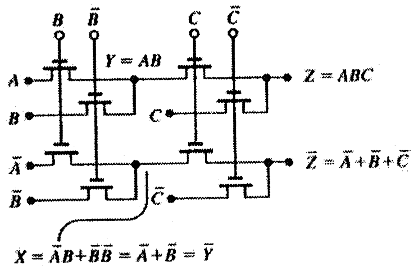
$$= 0.69(10)(10^{-15})F(10\ k\Omega)\frac{16(16+1)}{2} = 9.38\ \text{ns}$$

15.22

Need a CPL circuit for $Y = A\bar{B} + \bar{A}B$ and $\bar{Y} = AB + \bar{A}\bar{B}$ (See Exercise 14.8b



15.23

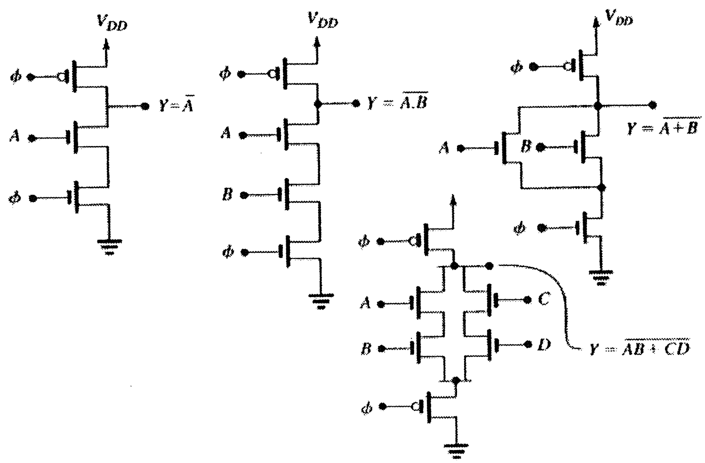


Require a CPL for Z = ABC and

$\bar{Z} = \overline{ABC} = \bar{A} + \bar{B} + \bar{C}$

Extend Fig 14.18 to 3 variables by dealing in pairs, creating Y = AB, then Z = YC with $\bar{Y} = \bar{A} + \bar{B}$, then $\bar{Z} = \bar{Y} + \bar{C}$

15.24



15.25

At $v_Y = 0.3V$, $i_{Dp} = \frac{1}{2}(75/3)(2.4/0.8)(3.0-0.8)^2 = \underline{181.5\mu A}$

At $v_Y = 2.7V$, $i_{Dp} = (75/3)(2.4/0.8)[(3.0-0.8)0.3 - 0.3^2/2]$
$= \underline{46.1\mu A}$

Thus $i_{D_{av}} = (181.5 + 46.1)/2 = \underline{114\mu A}$

and $t_{PLH} = t_r = 15\times10^{-15}(2.7-0.3)/(114\times10^{-6}) = \underline{316\,ps}$

15.26

For a 0.5V change, $t = C\Delta V/I_L = 30(10^{-15})0.5/10^{-12} = \underline{15ms}$

Since the precharge interval is much shorter than the evaluate, the period of the minimum clocking frequency can be as great as 15ms, for which $f_{min} = 1/(15\times10^{-3}) = \underline{67Hz}$

15.27

a) $C_1 = 5fF$:

Now, for v_{C1} rising to $V_{DD} - V_t = 5-1 = 4V$, and assuming Q_1 continues to conduct, v_Y will fall by an amount $(C_1/C_L)(\Delta v_{C1}) = 5/30(4) = \underline{0.67V}$ to $5.0 - 0.67 = 4.33V$. Since this exceeds 4.0, the assumption that Q_1 continues to conduct is verified. Thus v_Y drops by $\underline{0.67V}$

Note that if the body effect is included, it will likely to be impossible to raise v_{C1} to 4V. Thus 0.67V is the largest possible change.

b) $C_1 = 10fF$:

In view of the previous analysis, assume that ultimately $v_Y = v_{C1} = v$. Now, the charge change in each capacitor is the same:

$Q = CV \rightarrow 10(v-0) = 30(5-v)$

and $10v = 150 - 30v$, $40v = 150$, and $v = \underline{3.75V}$

Thus v_Y drops by $5 - 3.75 = \underline{1.25V}$ to 3.75V

15.28

(a) Since Q_1 and Q_{e1} are in series, W remains the same, but the effective length doubles. So,

$$\left(\frac{W}{L}\right)_{eq1} = \left(\frac{W}{2L}\right) = \frac{1}{2}\left(\frac{W}{L}\right)_n$$

Similarly,

$$\left(\frac{W}{L}\right)_{eq1} = \left(\frac{W}{2L}\right) = \frac{1}{2}\left(\frac{W}{L}\right)_n$$

(b)

$$i_{D1}\,(v_{y1} = V_{DD}) = \frac{1}{2}k_n'\left(\frac{W}{L}\right)_{eq1}(V_{DD} - 0.2V_{DD})^2$$

$$= \frac{1}{2}k_n'\left(\frac{W}{L}\right)_{eq1}(0.64\,V_{DD}^2)$$

$$= 0.32k_n'\left(\frac{W}{L}\right)_{eq1}V_{DD}^2$$

$$= 0.16k_n'\left(\frac{W}{L}\right)_n V_{DD}^2 = 0.16k_n V_{DD}^2$$

At $v_{y1} = V_t$:

$$i_{D1}\,(v_{y1} = V_t) = k_n'\left(\frac{W}{L}\right)_{eq1} \times$$

$$\left[(V_{DD} - 0.2V_{DD})(0.2V_{DD}) - \frac{1}{2}(0.2V_{DD})^2\right]$$

$$= k_n'\left(\frac{W}{L}\right)_{eq1}[0.16V_{DD}^2 - 0.02V_{DD}^2]$$

$$= k_n'\left(\frac{W}{L}\right)_{eq1} V_{DD}^2 = 0.07k_n'\left(\frac{W}{L}\right)_n V_{DD}^2$$

$$= 0.07k_n V_{DD}^2$$

$i_{D1}|_{av}$

$$= \frac{1}{2}\left[0.16k_n'\left(\frac{W}{L}\right)_n V_{DD}^2 + 0.07k_n'\left(\frac{W}{L}\right)_n V_{DD}^2\right]$$

$$= 0.115k_n'\left(\frac{W}{L}\right)_n V_{DD}^2 = 0.115k_n V_{DD}^2$$

(c) $\Delta t = \dfrac{C_{L1}(V_{DD} - V_t)}{i_{D1}|_{av}} = \dfrac{C_{L1}(0.8V_{DD})}{0.115k_n V_{DD}^2}$

$$= \frac{6.96C_{L1}}{k_n V_{DD}}$$

(d) Q_{eq2} will conduct during the time that v_{y1} drops from V_{DD} to V_t The transition half point is

when $v_{y1}|_{av} = \dfrac{V_{DD} - 0.2V_{DD}}{2} + 0.2V_{DD}$

$$v_{y1}|_{av} = 0.6V_{DD}$$

$$i_{D2}|_{av} = \frac{1}{2}k_n'\left(\frac{W}{L}\right)_{eq2}(0.6V_{DD} - 0.2V_{DD})^2$$

$$= 0.08k_n'\left(\frac{W}{L}\right)_{eq2} V_{DD}^2 = 0.04k_n V_{DD}^2$$

(e) $\Delta v_{y2} = -\dfrac{i_{D2}|_{av}\Delta t}{C_{L2}}$

Since $C_{L1} = C_{L2}$

$$\Delta v_{y2} = -\frac{0.04k_n V_{DD}^2(6.96C_{L1})}{C_{L1}k_n V_{DD}} = -0.278V_{DD}$$

So that v_{y2} is $V_{DD} - 0.278\ V_{DD} = 0.72\ V_{DD}$

15.29

The precharge time can be approximated as the rise time of the output voltage. In Example 14.3, $t_r \simeq 0.19$ ns. Assuming that the evaluation time is relatively short, the total cycle time can be estimated as being slighty longer than $t_r + t_{PHL}$. With $t_{PHL} \simeq 0.25$ ns, the maximum clocking frequency is $f < \dfrac{1}{T} \simeq \dfrac{1}{(t_r + t_{PHL})}$

$$= \frac{1}{(0.19 + 0.25)(10^{-9})\text{s}} = 2.27\text{ GHz}$$

15.30

(a) $V_{OH} = 0 - 0.75 = -0.75$ V

$V_{OL} = 0 - 0.75 - IR = -(0.75 + IR)$

(b)

$V_{th} = -(IR/2 + 0.75) = -(0.75 + IR/2)$

(c) For $i = 0.99$ I,

$v_{BE} = 750 + 25\ln(0.99) = 750$ mV

$i = 0.01$ I,

$v_{BE} = 750 + 25\ln(0.01) = 635$ mV

For

$0.99I$ in Q_R,

$$v_I = -\left(0.75 + \frac{IR}{2}\right) - (0.750 - 0.635)$$

$$= -(0.875 + IR/2)$$

(d) For $0.01I$ in Q_R,

$v_I = -(0.75 + IR/2) + 0.115$

$= -(0.635 + IR/2)$

(e) $V_{IH} = -(0.635 + IR/2)$

$V_{IL} = -(0.875 + IR/2)$

(f) $NM_H = -0.75 - [-(0.635 + IR/2)]$

$= IR/2 - 0.115$

$NM_L = -(0.875 + IR/2) - [-(0.75 + IR)]$

$= IR/2 - 0.115$

(g) $V_{IH} - V_{IL}$

$= -(0.635 + IR/2) - [-(0.875 + IR/2)]$

That is; $IR/2 - 0.115 = 0.230$

and $IR = 2(0.345) = 0.690$ V

(h) $V_{OH} = -0.75$ V;

$V_{OL} = -0.75 - 0.69 = -1.44$ V;

$V_{IL} = -(0.875 + 0.345) = -1.22$ V;

$V_{IH} = -(0.635 + 0.345) = -0.98$ V;

$V_R = -(0.750 + 0.345) = -1.095$ V.

15.31

See that once started the process continues; that is we have,an oscillation. In each cycle, each gate output rises and falls.

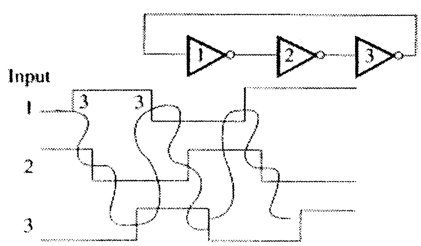
Thus the period is 3(3+7) = 30 ns

Frequency is 1/30 = 33.3 MHz.

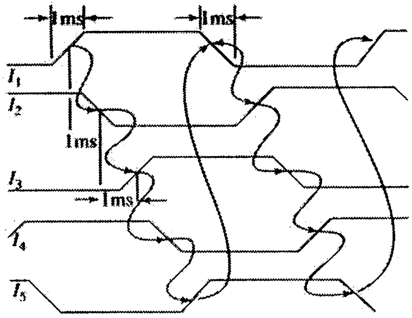
Any output is high for 3+7+3 = 13 ns

and low for 7+3+7 = 17 ns

Check: 30 ns.



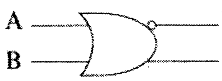
15.32



10 transitions per cycle, each of 1ns duration:
Period = 10 ns
Frequency = 100 MHz

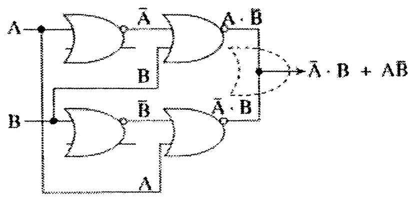
15.33

Tying two outputs together as yields a WIRED- OR operation. The most direct implementation is ORing the outputs of two AND gates. The AND function can be obtained using Demorgan's theorem:



$\overline{A+B} = \overline{A} \cdot \overline{B}$

Using NOR gates as inverters, $\overline{A}B$ and $A\overline{B}$ are obtained:



when $v_I = V_{IL} = -1.435$ V,

$I_E = 3.97$ mA, $v_{OR} = V_{OL} = -1.77$ V

when $v_I = V_R = -1.32$ V, $I_E = 4.00$ mA,

$v_{OR} = -1.31$ V

when $v_I = V_{IH} = -1.205$ V,

$I_E = 4.12$ mA, $v_{OR} = V_{OH} = -0.88$ V

At point x Transistor A's emitter current is 1% of I_E or

$I_{E_A} = (0.01)(3.97 \text{ mA}) = 39.7\ \mu\text{A}$

So that $r_{eA} = \dfrac{V_T}{I_{E_A}} = \dfrac{25 \text{ mV}}{39.7\ \mu\text{A}} = 630\ \Omega$

Transistor B's emitter current is 99% of I_E or

$I_{EB} = (0.99)(3.97 \text{ mA}) = 3.93$ mA

So that $r_{eB} = \dfrac{V_T}{I_{EB}} = \dfrac{25 \text{ mV}}{3.93 \text{ mA}} = 6.4\ \Omega$

$$I_{E2} = \frac{v_{OR} - V_{EE}}{R_T} = \frac{-1.77 \text{ V} - (-2 \text{ V})}{50\ \Omega}$$

$= 4.6$ mA

So that $r_{e2} = \dfrac{V_T}{I_{E2}} = \dfrac{25 \text{ mV}}{4.6 \text{ mA}} = 5.4\ \Omega$

$R_{in2} = (\beta + 1)(r_{e2} + R_T)$

$= (101)(5.4\ \Omega + 50\ \Omega) = 5.6$ kΩ

Solving for the incremental gain,

$$v_{OR} = \frac{V_{be2} R_T}{r_{e2} + R_T}$$

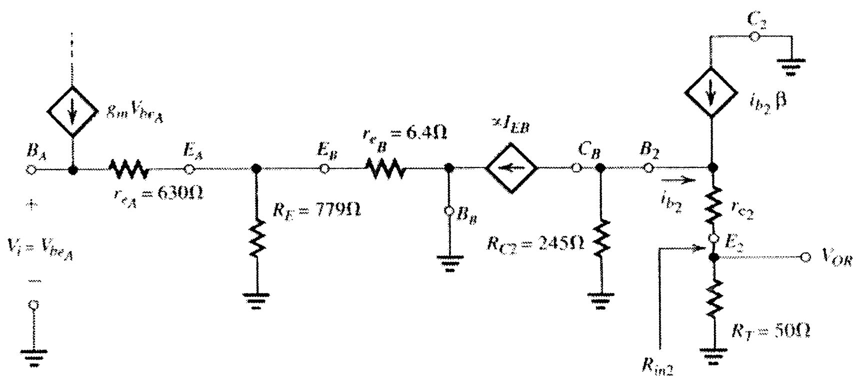
$V_{be2} = -\alpha i_{eB}(R_{c2} \| R_{in2})$

Since $R_E >> r_{eB}$,

$i_{eB} \approx \dfrac{-v_i}{r_{eA} + r_{eB}}$ so that

$$\frac{v_{OR}}{v_i} = \frac{R_T}{r_{e2} + R_T} \cdot \frac{\alpha(R_{C2} \| R_{in2})}{r_{eA} + r_{eB}}$$

15.34



$$\frac{v_{OR}}{v_i} = \frac{50\ \Omega}{5.4\ \Omega + 50\ \Omega}\cdot\left(\frac{100}{101}\right)\cdot\frac{(245\ \Omega \parallel 5.6\ k\Omega)}{630\ \Omega + 6.4\ \Omega}$$

$= 0.33$ V/V

At point m,

$$I_{EA} = I_{EB} = \frac{I_E}{2} = \frac{4\ mA}{2} = 2\ mA$$

$$r_{eA} = r_{eB} = \frac{V_T}{I_{EA}} = \frac{25mV}{2mA} = 12.5\ \Omega$$

$$I_{E2} = \frac{v_{OR} - V_{EE}}{R_T} = \frac{-1.31 - (-2V)}{50\ \Omega} = 13.8\ mA$$

$$r_{e2} = \frac{V_T}{I_{E2}} = \frac{25\ mV}{13.8\ mA} = 1.81\ \Omega$$

$$R_{in2} = (\beta + 1)(R_T + r_{e2}) = (101)(50 + 1.81)$$

$= 5.23$ kΩ

$$\text{Gain} = \frac{V_{OR}}{V_i} = \frac{50\ \Omega}{50\ \Omega + 1.81\ \Omega}\cdot\left(\frac{100}{101}\right)$$

$$\cdot\frac{(245\Omega \parallel 5.23k\Omega)}{12.5\Omega + 12.5\Omega} = 8.95\ V/V$$

At point Y,

$$I_{EA} = 0.99(4.12\ mA) = 4.08\ mA$$

$$r_{eA} = \frac{V_T}{I_{EA}} = \frac{25\ mV}{4.08\ mA} = 6.13\ \Omega$$

$$I_{EB} = (0.01)(4.12\ mA) = 41.2\ \mu A$$

$$r_{eB} = \frac{V_T}{I_{EB}} = \frac{25\ mV}{41.2\ \mu A} = 607\ \Omega$$

$$I_{E2} = \frac{v_{OR} - V_{EE}}{R_T} = \frac{-0.88\ V - (-2\ V)}{50\ \Omega}$$

$= 22.4$ mA

$$r_{e2} = \frac{25\ mA}{22.4\ mA} = 1.1\ \Omega$$

$$R_{in2} = (101)(50\Omega + 1.1\Omega) = 5.16\ k\Omega$$

$$\text{Gain} = \frac{v_{OR}}{v_i} = \frac{50\ \Omega}{50\ \Omega + 1.1\ \Omega}\cdot\left(\frac{100}{101}\right)$$

$$\cdot\frac{(245\ \Omega \parallel 5.16\ k\Omega)}{607\ \Omega + 6.13\ \Omega} = 0.37\ V/V$$

15.35

Assume I_E is constant at 4 mA.

(a) Currents are: 3.6 mA and 0.4 mA

$\therefore$ Emitter-Base voltage difference $= V_T \ln\frac{3.6}{0.4}$

or 25 ln 9 = 54.9 mV.

Thus $V_{IL} = -1.32 - .055 = -1.375$ V

$V_{IH} = -1.32 + .055 = -1.265$ V

(b) Currents are: $4(0.999) = 3.996$ mA and

$.001 \times 4 = 0.004$ mA

$\therefore$ Emitter-Base voltage difference =

$$V_T \ln\left(\frac{3.996}{.004}\right)$$

15.36

$NM_H = 0.325$ V, of which 50% is 162 mV, for $\beta = 100$, and $V_{BE2} = 0.83$ V, $I_{E2} = 22.4$ mA.

Approximately:

$$-2 + \frac{50}{50 + \frac{245}{\beta + 1}}\cdot(2 - 0.83) = -0.88 - 0.162$$

$$\text{or } \frac{50(1.17)}{50 + \frac{245}{\beta + 1}} = 0.958$$

$$50 + \frac{245}{\beta + 1} = 61.06$$

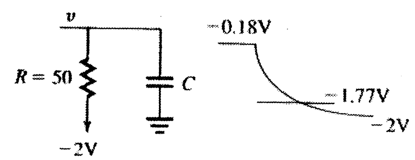
$$\text{Hence } \beta = \frac{245}{11.06} - 1 = 21.2$$

Check: For $V_o = -.88 - .162 = -1.042$ V

$$I_{E2} = \frac{2 - 1.042}{50} = 19.2\ mA$$

and $V_T \ln\left(\frac{22.4}{19.2}\right) = 3.85$ mV – OK, Since small, can ignore.

15.37



$$v = -0.88 + (.88 - 2)(1 - e^{-t/RC})$$

or $v = -2 + 1.12\, e^{-t/50\,C}$

After 1ns, $v = -1.77$ V

i.e., $-1.77 = -2 + 1.12\, e^{-1/50\,C}$

or $e^{-1/50C} = \frac{2-1.77}{1.12}$ and

$-1/50\,C = -1.583$

Thus

$$C = \frac{10^{-9}}{50(1.583)} = 12.6 \times 10^{-12}\text{ F} = 12.6\text{ pF}$$

15.38

$v = 2/3 \times 30\text{ cm/ns} \times 20\text{ cm/ns}$

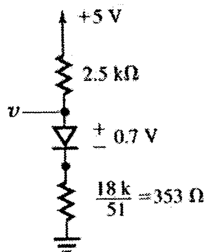
$$\text{Rate} = \frac{\text{Rise Time}}{\text{Return Time}} = 5/1 = \frac{3.5}{2L/20}$$

$$L = \frac{3.5\text{ ns} \times 20\text{ cm/ns}}{5 \times 2} = 7\text{ cm}$$

15.39

$V_{OL} \approx 0.7$ V; $V_{OH} = +5.0$ V

More precisely, for $V_{OL} = v$



$$v = \frac{0.353}{2.5 + .353}(5 - 0.7) + 0.7 = 1.23\text{ V}$$

i.e., $V_{OL} = 1.23$ V

Logically: A is high if one of A *or* B *and* one of C *or* D are high.

That is A = (A + B) · (C + D)

15.40

For $V_1 = V_o = V_{DD/2} = 5/2 = 2.5$ V.

$i_{DN} = (1/2)\,k_n'\,(W/L)_n\,(V_{GS} - V_t)^2$

$= (1/2)(100)10^{-6}(2/1)(2.5 - 0.7 - 1)^2$

$= 64\ \mu$A

Now, the collector current of

$Q_2 = \beta i_B = \beta i_{DN}$

$= 100(64 \times 10^{-6}) = 6.4$ mA

Corresponding, the totem-pole current is

$i_{EQ2} = (6400 + 64)10^{-6} = 6.46$ mA

Now, for $i_{EQ1} = i_{EQ2}$, $i_{DP} = i_{DN} = 64\ \mu$A

Thus

$64 = 1/2(100/2.5)(W/L)_p(5 - 2.5 - 0.7 - 1)^2$

where $(W/L)_p = 2.5(2/1) = (5\mu\text{m}/1\mu\text{m})$

15.41

At the threshold V_{th}, $v_o = v_I = V_{th} = v$, and the two MOS operate in saturation with equal currents. Thus

$1/2\,(100)\,(2/1)\,(5 - y - 0.7 - 1)^2$

Thus, $(3.3 - v)^2 = 2.5(v - 1.7)^2$

and $(3.3 - v) = \pm\sqrt{2.5}\,(v - 1.7)$.

Usefully, $(3.3 - v) = (1.58v - 2.69)$,

hence $2.58\,v = 5.99$, and $v = V_{th} = 2.32$ V

For this value,

$i_{DN} = 1/2\,(100)\,(2/1)\,(2.32 - 0.7 - 1)$

$= 38.4\ \mu$A

and the totem-pole current is $(\beta + 1)i_{DN}$

or $101(38.4)10^{-6} = 3.88$ mA

15.42

The problem as stated is very general, and correspondingly, its solution can be long and complex.

the specifications of

matched MOS having

$(W/L)_p = 2.5\,(W/L)_n$.

<u>For R_2:</u> With $v_{Ds} = V_{t/3} = 1/3 = 0.333$ V

$i_{DN} = 100(10^{-6})(2/1) \times$

$[(5 - 0.7 - 1)0.33 - 0.33^2/2] = 209\ \mu$A

Now, if 50% of this is lost in R_2,

$R_2 = 0.7/(0.50 \times 209) = 6.70$ kΩ

Now if 20% is lost in R_2,

$R_2 = 0.7/(0.20 \times 209) = 16.7$ kΩ

<u>For R_1:</u> $i_{Dp} = (100/2.5)10^{-6}(2.5(2/1)) \times$

$[(5 - 0 - 1)0.33 - 0.33^2/2] = 256\ \mu$A

Now, if 50% if this is lost in R_1,

$R_1 = (5 - 0.333)/(0.5 \times 256) = 36.5$ kΩ

Now, if 20% is lost in R_1

$R_1 = 2.5(36.5) = 91.1$ kΩ

<u>In comparison:</u>

For the 50% case,

$R_1/R_2 = 36.5/6.70 = 5.45$

For the 20% case,

$R_1/R_2 = 91.1/16.7 = 5.45$

(why should their equality be obvious?)

Thus, in general $R_1/R_2 = 5.45$

15.43

For t_{PLH}

At $V_O = 0$ V,

$i_{Dp} = \frac{1}{2}(100/2.5)(2/1)(5.0-1)^2 = 640\mu A$

At

$V_O = 2.5$ V,

$i_{DP} = (100/2.5)(2/1)[(5-1)2.5 - 2.5^2/2]$

$= 550\mu A$

Thus $i_{DPav} = (640+550)/2 = 595\mu A$

and $i_{Oav} = (100+1)595 = 60.1$ mA

Thus

$t_{PLH} = CV/I = 2\times10^{-12}\times 2.5/(60.1\times10^{-3})$

$= 83.2$ ps

For t_{PHL}:

At

$v_O = 5.0$ V,

$i_{DN} = \frac{1}{2}(100)(2/1)(5-0.7-1)^2 = 1.09$ mA

At

$v_O = 2.5$ V, $i_{DN} = 100(2/1)$

$[(5-0.7-1)(2.5-0.7)-(2.5-0.7)^2/2]$

$= 864\ \mu A$

Thus

$i_{DNav} = (1089+864)/2 = 977\mu A$

and $i_{Dav} = 101(977\times10^{-6}) = 98.6$ mA

Thus

$t_{PHL} = CV/I = 2\times10^{-12}(2.5)/(98.6\times10^{-3})$

$= 50.7$ ps

Thus $t_p = (83.2+50.7)/2 = 67.0$ ps

Note that this solution embodies two assumptions

1) Internal capacitances can be neglected.

2) Transitions are from ideal 0 V and 5 V output-signal level.

If outputs of $(5-0.7) = 4.3$ V and $(0+0.7) = 0.7$V apply, t_p becomes about $67\times(2.5-0.7)/2.5 = 48$ ps

15.44

$R_1 = R_2 = 5\ k\Omega$ robs the base of some of its drive current, namely $0.7/5\times10^3 = 140\mu A$. Using results from the solution of P 14.46 above:

For t_{PLH}

: $i_{Bav} = 595 - 140 = 455\mu A$ and

$i_{Bav} = 101(455\times10^{-6}) = 46.0$ mA

Thus

$t_{PHL} = 2\times10^{-12}\times2.5/4.6\times10^{-3} = 108.7$ ps

For t_{PLH}:

$i_{Dav} = 977 - 140 = 837\mu A$

and $i_{Bav} = 101(837\times10^{-6}) = 84.5$ mA

Thus

$t_{PHL} = 2\times10^{-12}\times2.5/84.5\times10^{-3} = 59.2$ ps

Thus $t_p = (59.2+108.7)/2 = 84$ ps

15.45

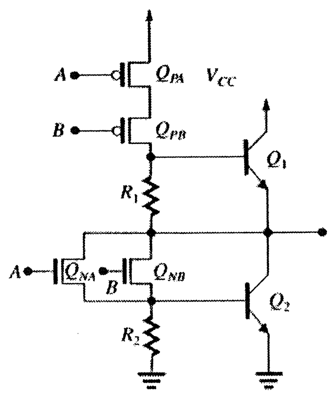
For the BiCMOS NAND of Fig 14.39 to have a dynamic response some what line that of the inverter of Fig. 14.37e:

$(W/L)_{PA} = (W/L)_{PB} = (W/L)_P$

and $(W/L)_{NA} = (W/L)_{NB} = 2(W/L)_N$

15.46

A BiCMOS 2-input NOR is as shown:



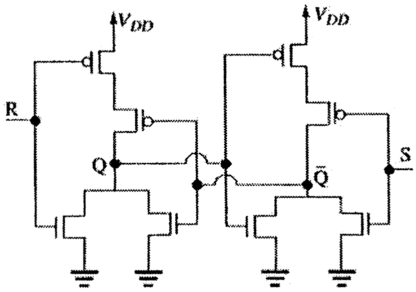
In terms of the basic matched inverter:

$(W/L)_{PA} = (W/L)_{PB} = 2(W/L)_P$

$(W/L)_{NA} = (W/L)_{NB} = (W/L)_N$

where $(W/L)_P$ and $(W/L)_N$ characterize the inverter.

16.1



16.2

$$\mu_n \frac{1}{2}\left(\frac{W}{L}\right)_5\left[(V_{DD}-V_{tp})\frac{V_{DD}}{2}-\frac{1}{2}\left(\frac{V_{DD}}{2}\right)^2\right]$$

$$= \mu_p\left(\frac{\mu_n}{\mu_p}\right)\left(\frac{W}{L}\right)_n\left[(V_{DD}-V_{tp})\left(\frac{V_{DD}}{2}\right)-\frac{1}{2}\left(\frac{V_{DD}}{2}\right)^2\right]$$

Assuming $V_{tn} = V_{pt}$ we have:

$$\mu_n\frac{1}{2}\left(\frac{W}{L}\right)_5 = \mu_p\left(\frac{\mu_n}{\mu_p}\right)\left(\frac{W}{L}\right)_n \Rightarrow \frac{1}{2}\left(\frac{W}{L}\right)_5 = \left(\frac{W}{L}\right)_n$$

$$\Rightarrow \left(\frac{W}{L}\right)_5 = 2\left(\frac{W}{L}\right)_n$$

If the flip-flop is fabricated in a 0.13-μm process, we have:

$$\left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_3 = 1 \Rightarrow W_1 = W_3 = 1 \times L_{min}$$

$$= 0.13\ \mu\text{m}$$

$$\left(\frac{W}{L}\right)_2 = \left(\frac{W}{L}\right)_4 = \left(\frac{\mu_n}{\mu_p}\right)\left(\frac{W}{L}\right)_n = 4 \times 1 = 4$$

$$\Rightarrow W_2 = W_4 = 4 \times 0.13 = 052\ \mu\text{m}$$

$$\left(\frac{W}{L}\right)_5 = \left(\frac{W}{L}\right)_6 = \left(\frac{W}{L}\right)_7 = \left(\frac{W}{L}\right)_8 = 2\left(\frac{W}{L}\right)_n$$

$$= 2 \Rightarrow W_5 = W_6 = W_7 = W_8 = 2\ \mu\text{m}$$

16.3

output $v_Q = \frac{V_{DD}}{2}$, and assuming a single equivalent transistor for Q_5 and Q_6 where $\left(\frac{W}{L}\right)_{eq} = \frac{1}{2}\left(\frac{W}{L}\right)_5 = \frac{1}{2}\left(\frac{W}{L}\right)_6$

Use eq. 13.100.

For equivalent n transistor

$$\Rightarrow V_{GS} - V_t = 1.8 - 0.5 = 1.3 > V_{DS_{satn}}$$

$$= 0.6\text{V}$$

For $P_{transistor}$

$$\Rightarrow |V_{GS}| - |V_t| = 1.8 - 0.5 = 1.3 > V_{DS_{satp}}$$

$$= 1\text{V}$$

Both operating in velocity saturation :

$$\mu_n C_{ox}\left(\frac{W}{L}\right)_{eq} V_{DS_{satn}}\left(V_{GS} - V_{tn} - \frac{1}{2}V_{DS_{satn}}\right)$$

$$\times (1 + \lambda_n V_{DS}) = \mu_p C_{ox}\left(\frac{W}{L}\right)_2 |V_{DS_{satp}}|$$

$$\times \left(|V_{GS}| - |V_{tp}| - \frac{1}{2}|V_{DS_{satp}}|\right)(1 + |\lambda_p|V_{DS})$$

$$300 \times 10^{-6} \times \left(\frac{W}{L}\right)_{eq} \times 0.6\left(1.8 - 0.5 - \frac{1}{2} \times 0.6\right)$$

$$\times \left(1 + .1 \times \frac{1.8}{2}\right) = 75 \times 10^{-6}\left(\frac{1.08}{0.18}\right)$$

$$\times 1\left(1.8 - 0.5 - \frac{1}{2} \times 1\right)\left(1 + .1 \times \left(\frac{1.8}{2}\right)\right)$$

$$\left(\frac{W}{L}\right)_{eq} = 2$$

$$\left(\frac{W}{L}\right)_{eq} = \frac{1}{2}\left(\frac{W}{L}\right)_5 = 2$$

$$\therefore \left(\frac{W}{L}\right)_5 = 4,$$

$$\left(\frac{W}{L}\right)_6 = \left(\frac{W}{L}\right)_5 = 4 = \frac{0.72\ \mu\text{m}}{0.18\ \mu\text{m}}$$

This value is greater
thus requiring 33% more width area of both n transistors as a minimum.

16.4

$$V_m = \frac{r(V_{DD} - |V_{tp}|) + V_{tn}}{r + 1} \text{ where}$$

$$r = \sqrt{\frac{\mu_p W_p}{\mu_n W_n}}$$

$W_p = W_n = 0.27\ \mu\text{m}$ and $\mu_n = 4\mu_p$

$$\therefore r = \sqrt{\frac{1}{4}} = 0.5$$

$$V_m = \frac{0.5(1.8 - 0.5) + 1}{0.5 + 1} = 1.1\text{V}$$

(threshold voltage)

Assuming

$\left(\frac{W}{L}\right)_5 = \left(\frac{W}{L}\right)_6 = \left(\frac{W}{L}\right)_7 = \left(\frac{W}{L}\right)_8$ and Q_5, Q_6 have an equivalent single transistor

$\left(\frac{W}{L}\right)_{eq} = \frac{1}{2}\left(\frac{W}{L}\right)_5 = \frac{1}{2}\left(\frac{W}{L}\right)_6$, the equivalent n transistor and Q_2 are in triode region with the same current flowing through them.

$$300\mu_n \times \left(\frac{W}{L}\right)_5\left[(1.8-0.5)\frac{1.8}{2}-\frac{1}{2}\left(\frac{1.8}{2}\right)^2\right]$$

$$= 75\times10^{-6}\times\left(\frac{0.27}{0.18}\right)\left[(1.8-0.5)\frac{1.8}{2}-\frac{1}{2}\left(\frac{1.8}{2}\right)^2\right]$$

$$\left(\frac{W}{L}\right)_5 = 0.375 \Rightarrow \left(\frac{W}{L}\right)_5 = 1$$

(cannot have less than minimum)

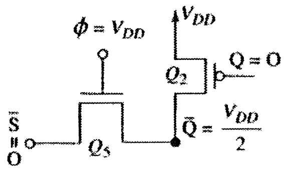
$$\left(\frac{W}{L}\right)_5 = \left(\frac{W}{L}\right)_6 = \left(\frac{W}{L}\right)_7 = \left(\frac{W}{L}\right)_8 = \frac{0.18\ \mu m}{0.18\ \mu m}$$

16.5

Q_2 is conducting and Q_5 is conducting and operating in triode region:

$$\mu_n C_{ox}\left(\frac{W}{L}\right)_5\left[(V_{DD}-V_{tn})\left(\frac{V_{DD}}{2}\right)-\frac{1}{2}\left(\frac{V_{DD}}{2}\right)^2\right]$$

$$= \mu_p C_{ox}\left(\frac{W}{L}\right)_2\left[(V_{DD}-|V_{tp}|)\frac{V_{DD}}{2}-\frac{1}{2}\left(\frac{V_{DD}}{2}\right)^2\right]$$



$$\mu_n\left(\frac{W}{L}\right)_5 = \mu_p\left(\frac{W}{L}\right)_2$$

$$\therefore \left(\frac{W}{L}\right)_5 = \frac{\mu_p}{\mu_n}\left(\frac{W}{L}\right)_2$$

16.6

Note that the devices are matched, with

a) $K_n = K_p = 20(12/6) = 40\ \mu A/V^2$, and $|V_t| = 1$ V.

For $V_I = 2.5$ V

$V_o = 2.5$ V

For $v_I = 0$ V, 5 V : one device is on, one off;

$v_o = 5$ V, 0 V

For $v_I = 1$ V, 4 V : one on, one off;

$v_o = 5$ V, 0 V

For $v_I = 1.5$ V, 3.5 V : one in saturation, one in triode mode.

$$i_D = \frac{1}{2}(40)(1.5-1)^2 = 40[(5-1.5-1)v_o - v_o^2/2]$$

Thus $0.125 = 2.5v_o - v_o^2/2$

or $v_o^2 - 5v_o + 0.25 = 0$

and $V_o[--5 \pm \sqrt{5^2-4(0.25)}]/2$

$= (5 \pm 4.8484)/2 = 0.05$V

Thus $v_o = 0.05$V or 4.95V

For $v_I = 2.0$ V, 3.0 V :

$1/2(2-1)^2 = (5-2-1)v_o - v_o^2/2$

or $(2-1)^2 = 2\times 2v_o - v_o^2$

and $v_o^2 - 4v_o + 1 = 0$

Whence $v_o = (--4 \pm \sqrt{4^2-41.1})/2$

$= (4 \pm 3.464)/2 = 0.27$ V

Thus $v_o = 0.27$ V or 4.73 V

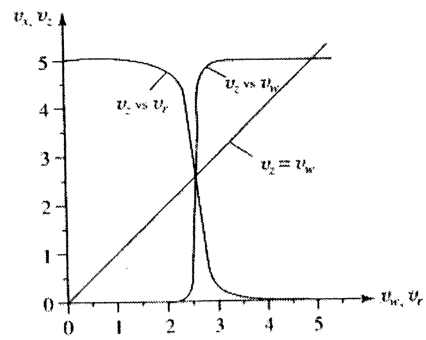
For $v_I = 2.25$ V or 2.75V :

$(2.25-1)^2 = 2(5-2.25-1)v_o - v_o^2$

$1.5625 = 3.5 - v_o - v_o^2$

$v_o^2 - 3.5v_o + 1.5625 = 0$

b)



Whence

$v_o = (- -3.5 \pm \sqrt{3.5^2 - 4(1.5625)})/2$

$= (3.5 \pm 2.45)/2 = 0.525$

Thus $v_o = 0.525$ V or 4.475 V

For $v_I = 2.5$ V, $v_o = 3.5$ V by symmetry

Now, having plotted v_Z versus v_y (or v_x versus v_w)

use the graph to find v_z versus v_w.

Work backwards : first v_z, then $v_y = v_x$ than v_w.

For $v_z = 2.5$ V, $v_y = v_x = v_w = 2.5$ V

For $v_z = 4.4$ V, $v_y = 2.25$ V;

For $v_z = 4.5$ V $v_y = 1.50$ V; for

$v_x = 1.50$ V $v_w = 2.65$ V

(c) $v_z = v_w$ line at:

point A: (0, 0)

point B: (2.5, 2.5)

point C: (5, 5)

At point B, the current flow in each inverter is:

$i_D = \frac{1}{2}(40)(2.5 - 1)^3 = 45$ µA/V

where for each transistor, $r_o = 100/(45 \times 10^{-6})$

$= 2.32$ MΩ

and $g_m = 2\left(\frac{1}{2}\right)40(2.5 - 1) = 60$ µA/V

Thus for each inverter operating at (2.5, 2.5), the voltage gain is $-(g_m + g_m)(r_o \| r_o)$

$= -g_m V_o = -60 \times 10^{-6} \times 2.22 \times 10^6 = 133$ V/V

Thus an estimate of the slope of the v_z versus v_w curve at B is (13 s) = 17.7×10^3 V/V

Correspondingly a lower bound on the width of the transition region is $(5 - 0)/(17.7 \times 10^3)$, or 0.28 mV, that is ±0.14 mV around 2.5 V.

16.7

The approximate transfer characteristic of each inverter passes through points : (0, 5), (2.0, 4.6), (2.42, 0.4), (5, 0).

For the linear centre segment between (2.0, 2.46), (2.42, 0.4) an equation is $v_O = a - bv_I$

Here : $4.60 = a - 2.00b$, and

$0.40 = a - 2.42b$

Subtract : $4.20 = 0.42b \rightarrow b = \underline{10}$

Now, $4.60 = a - 2(10) \rightarrow a = 4.6 + 20 = \underline{24.6}$

Check : $0.4 = 24.6 - 2.42(10)$ ✓

Thus the middle part of the characteristic is

$v_O = 24.6 - 10v_I$

For each device, $v_O = v_I = v$, when

$v = 24.6 - 10v$, or $11v = 24.6$, or $v = \underline{2.236\text{V}}$

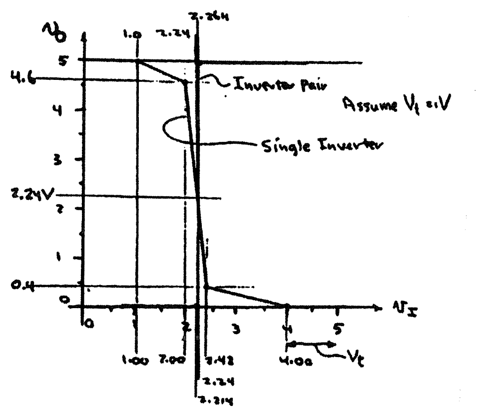
where the gain is $\Delta v_O / \Delta v_I = -b = \underline{-10\text{V/V}}$

Thus point B on the open-loop characteristic is $v_W = v_Z = \underline{2.236\text{V}}$, where the loop gain can be approximated to be at least $(-10)^2 = \underline{100\text{V/V}}$

The open-loop characteristic reaches $v_O = 5$V, where $v_I = 2.236 + (5 - 2.236)/100 = \underline{2.264\text{V}}$

and it reaches 0V where

$v_I = 2.236 - 2.236/100 = \underline{2.214\text{V}}$



16.8

(a) Q_5, Q_6 are conducting for $D = 1$ or 0

$\bar{Q} = \bar{D}$ and $Q = D$

If $D = 1$ then $\bar{Q} = 0$ and $Q = 1$

Q_1 conducts

If $D = 0$ then $\bar{Q} = 1$ and $Q = 0$

Q_4 conducts

(b) If $D = 1$ then $\bar{Q} = 0$ and $Q = 1$

when ϕ goes low, (Q_1, Q_2) conduct (Q_3 also conducts)

The value at the gate of Gz stays high (through Q_1, Q_2) $\bar{Q} = 0$ and $Q = 1$ (value is "latched")

(c) If $D = 0$ then $\bar{Q} = 1$ and $Q = 0$

when ϕ goes low, (Q_3, Q_4) conduct so gate value at G_2 is low (through Q_3, Q_4) to keep $\bar{Q} = 1$ and $Q = 0$

(d) No. The operation connects either V_{DD} or ground directly to gate of G_z which maintains values at $\bar{Q}$ and Q.

16.9

A 1Mb array requires n address bits where

$2^n = 10^6$, or $n \log_{10} 2 = 6$, $n = 6/\log_{10} 2 = 19.93$

Thus 20 bits are needed to address every cell.

For 16-bit words, $2^4 = 16$ and 4 bits are not needed.

Thus $20 - 4 =$ 16 bits of address are sufficient.

Check: $m = \log_{10}(10^6/16)/\log_{10}(2) = 4.796/0.301 = 15.93$

Use 16 ✓

Note: A "1Mb array" actually holds $2^{20} = 1024^2 = 1048{,}576$ cells.

16.10

The cell area is $10^9 \times 0.38 \times 10^{-6} \times 0.76 \times 10^{-6}$

$= 0.289 \times 10^{-3} \text{m}^2$

The chip area is $19 \times 10^{-3} \times 38 \times 10^{-3}$

$= 0.722 \times 10^{-3} \text{m}^2$

Thus the peripheral circuits and interconnect occupy $(0.722 - 0.289)10^{-3} = 0.433 \text{ mm}^2$

or $\left(\frac{433}{722}\right) \times 100 = 60\%$ of the clip area.

16.11

$$\left(\frac{(W/L)_5}{(W/L)_1}\right) \le \frac{1}{\left(1 - \frac{0.5}{1.8 - 0.5}\right)^2} - 1 = 1.64$$

minimum area when $\left(\frac{W}{L}\right)_5 = 1$ so

$W_5 = 0.18\ \mu\text{m}$ Q_5 is saturated and Q_1 is in triode. Currents are equal:

$$\frac{1}{2}\mu_n C_{ox}(1)(1.8 - 0.5 - 0.2)^2 = \mu_n C_{ox}\left(\frac{W_1}{0.18\mu}\right)$$

$$\left[(1.8 - 0.5)0.2 - \frac{1}{2}(0.2)^2\right]$$

Solving for $W_1 = 0.45\ \mu\text{m}$

Check condition above :

$$\left(\frac{1}{\frac{0.45\ \mu}{0.18\ \mu}}\right) = 0.4 < 1.64$$

16.12

$$\frac{\left(\frac{W}{L}\right)_a}{\left(\frac{W}{L}\right)_n} \le \frac{1}{\left(1 - \frac{V_{tn}}{V_{DD} - V_{tn}}\right)^2} - 1$$

$$= \frac{1}{\left(1 - \frac{0.5}{2.5 - 0.5}\right)^2} - 1 = 0.78$$

$$\left(\frac{W}{L}\right)_a \le 0.78 \times 1.5 \text{ or } \left(\frac{W}{L}\right)_a \le 1.17$$

16.13

(a) 0.25 μm: $V_{DD} = 2.5$ V and $V_t = 0.5$ V

$$A{:}\left(\frac{(V_{\bar{Q}} = V_t)}{V_{DD} - V_{tn}}\right) = \frac{0.5}{2.5 - 0.5} = \frac{0.5}{2} = 0.25$$

$$\Rightarrow \frac{\left(\frac{W}{L}\right)_5}{\left(\frac{W}{L}\right)_1} \approx 0.8$$

$$\Rightarrow \frac{(W/L)_5}{(W/L)_1} \approx 0.8$$

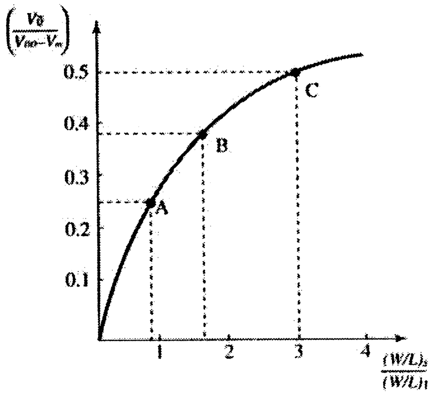
(b) 0.18 μm: $V_{DD} = 1.8$ V and $V_t = 0.5$ V

$$\text{B:}\left(\frac{0.5}{1.8 - 0.5}\right) = 0.385 \Rightarrow \frac{\left(\frac{W}{L}\right)_5}{\left(\frac{W}{L}\right)_1} \approx 1.7$$

$$\Rightarrow \frac{(W/L)_5}{(W/L)_1} = 1.64$$

(c) 0.13 μm: $V_{DD} = 1.2$ V and $V_t = 0.4$ V

$$C: \left(\frac{0.4}{1.2 - 0.4}\right) = 0.5$$



$$\frac{\left(\frac{W}{L}\right)_5}{\left(\frac{W}{L}\right)_1} = 3$$

$$\left(\frac{W}{L}\right)_5 / \left(\frac{W}{L}\right)_1 = 3$$

16.14

When $V_{\bar{Q}} \le V_t = 0.5$ V:

Q_5: $V_{GS} - V_t = 1.8 - 0.5 = 1.3 > 0.6$ V and $V_{DS} = 1.8 - 0.5 > V_{DS_{sat}} = 0.6$

Q_1: $V_{GS} - V_t = 1.8 - 0.5 = 1.3 > 0.6$ V and $V_{DS} = 0.5 - 0 < V_{DS_{sat}} = 0.6$

(not in velocity saturation)

Only Q_5 is in velocity saturation.

using Eq. 13.100:

$$i_D = \mu_n C_{ox}\left(\frac{W}{L}\right)_5 V_{DS_{sat}}\left(V_{GS} - V_t - \frac{1}{2}V_{DS_{sat}}\right) \times (1 + \lambda V_{DS})$$

Neglecting $\lambda (\lambda = 0)$

$$\mu_n C_{ox}\left(\frac{W}{L}\right)_5 0.6\left(1.8 - 0.5 - 0.5 - \frac{1}{2}(0.6)\right)$$

$$= \mu_n C_{ox}\left(\frac{W}{L}\right)_1 0.6\left[(1.8 - 0.5)0.5 - \frac{1}{2}(0.5)^2\right]$$

$$\frac{\left(\frac{W}{L}\right)_5}{\left(\frac{W}{L}\right)_1} = 1.75$$

without velocity saturation: (Eq. 15.4)

$$\frac{\left(\frac{W}{L}\right)_5}{\left(\frac{W}{L}\right)_1} \le \frac{1}{\left(1 - \frac{0.5}{1.8 - 0.5}\right)^2} - 1 = 1.64$$

16.15

With body effect considerations:

$$V_t = V_{t0} + r[\sqrt{2\phi_f + V_{SB}} - \sqrt{2\phi_f}]$$

$$V_t = 0.5 + 0.3[\sqrt{0.8 + 0.5} - \sqrt{0.8}] = 0.574 \text{ V}$$

$$\left(\frac{\left(\frac{W}{L}\right)_5}{\left(\frac{W}{L}\right)_1}\right) \le \left[\frac{1}{\left(1 - \frac{0.574}{1.8 - 0.574}\right)^2} - 1\right] = 2.54$$

without body effect: $V_{tn} = V_{t0}$

$$\left(\frac{\left(\frac{W}{L}\right)_5}{\left(\frac{W}{L}\right)_1}\right) \le \left[\frac{1}{\left(1 - \frac{0.5}{1.8 - 0.5}\right)^2} - 1\right] = 1.64$$

16.16

$\left(\frac{W}{L}\right)_5$ for body effect can have a large maximum ratio.

$$\left(\frac{\left(\frac{W}{L}\right)_5}{1}\right) \le \left[\frac{1}{\left(1 - \frac{0.4}{1.2 - 0.4}\right)^2} - 1\right] = 3$$

For V_Q kept below V_{tn}, Eq. 15.10 becomes

$$\left(1 - \frac{V_{tn}}{V_{DD} - V_{tn}}\right)^2 = 1 - \left(\frac{\mu_p}{\mu_n}\right)\frac{\left(\frac{W}{L}\right)_4}{\left(\frac{W}{L}\right)_6}$$

$$\left(\frac{W}{L}\right)_4 = \frac{\left(1 - \frac{V_{tn}}{V_{DD} - V_{tn}}\right)^2}{(\mu_p/\mu_n)}$$

Assuming $\frac{\mu_n}{\mu_p} = 4$

$$\left(\frac{W}{L}\right)_4 = 3$$

$$\Rightarrow \Delta t = \frac{C_{\bar{B}}\,\Delta V}{I_5} \text{ where } I_5$$

is obtained from

$$I_5 = \frac{1}{2} \times 430 \times 10^{-6} \times 1 \times (1.2 - 0.4 - 0.4)^2$$

$$= 34.4\ \mu\text{A}$$

$$\Delta t = \frac{C_{\bar{B}}\,\Delta V}{I_5} = \frac{2 \times 10^{-12} \times 0.2}{34.4 \times 10^{-6}} = 11.6 \text{ ns.}$$

(c) $\left(\frac{W}{L}\right) = 3$

(Eq. 15.1):

$$I_5 = \frac{1}{2} \times 430 \times 10^{-6} \times 3(1.2 - 0.4 - 0.4)^2$$

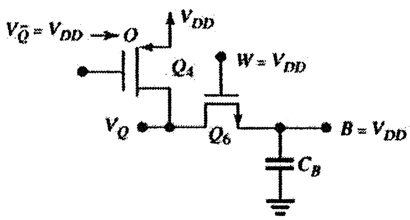
$$= 103\ \mu\text{A}$$

$$\Delta t = \frac{C_{\bar{B}}\,\Delta V}{I_5} = \frac{(2 \times 10^{-12} \times 0.2)}{103 \times 10^{-6}} = 3.9 \text{ ns.}$$

16.17

Storing a 0: $V_Q = 0$, $V_{\bar{Q}} = V_{DD}$

To write a 1 → B line raised to V_{DD}, $\bar{B}$ line lowered, and word line raised to V_{DD}. V_Q changes to V_{DD} and $V_{\bar{Q}} = 0$. Relevant transistors:



Q_4 in saturation and Q_6 in triode, which is the same as the text for writing a 0.

16.18

$$\left(\frac{W}{L}\right)_p \le \left(\frac{W}{L}\right)_a \times 4 \times \left[1 - \left(1 - \frac{0.5}{2.5 - 0.5}\right)^2\right]$$

$$\left(\frac{W}{L}\right)_p \le 1.75\left(\frac{W}{L}\right)_a$$

16.19

For a 1Mb square array there are 1024 rows and 1024 columns.

Thus the bit-line capacitance is $10^{-15}(1024 \cdot 1 + 12)$ or 1.036 pF

When storing a '1', the voltage on C_s is $(V_{DD} - V_t)$ or $(5 - 1.5) = 3.5V$. With precharge to $V_{DD}/2 = 2.5V$, the change in voltage on $C_s = 3.5 - 2.5 = 1.0V$,

For $C_s = 25fF$, the bit-line voltage resulting is $25/(25 + 1036) \times 1 = 23.6\,mV$

When storing a '0', the voltage on C_s is 0V and the change is $2.5 - 0 = 2.5V$ with a resulting bit-line signal of $25/(25 + 1036) \times 2.5 = 58.9\,mV$

16.20

Let $\left(\frac{W}{L}\right)_5 = \frac{0.13\mu m}{0.13\mu m}$

Let $V_{\bar{Q}} = V_{tn} = 0.4V$.

$I_5 = I_1$

$$\frac{1}{2}\mu_n C_{OX} \times 1 \times (1.2 - 0.4 - 0.4)^2$$

$$= \mu_n C_{OX} \times \left(\frac{W}{L}\right)_1 \times \left[(1.2 - 0.4)0.4 - \frac{1}{2}(0.4)^2\right]$$

Solving for $\left(\frac{W}{L}\right)_1 = .33$ so choose

$$\left(\frac{W}{L}\right)_1 = \frac{0.13\mu m}{0.13\mu m} = 1$$

Checking

$$1 \le \left[\frac{1}{\left(1 - \frac{0.4}{1.2 - .4}\right)^2} - 1\right] = 3$$

Let $\left(\frac{W}{L}\right)_4 = \frac{0.13\mu m}{0.13\mu m}$

Let $V_Q = V_{tn} = 0.4V$.

$\therefore I_4 = I_6$

$$\frac{1}{2}\mu_p C_{OX} \times 1 \times (1.2 - 0.4)^2 = \mu_p C_{OX} \times \left(\frac{W}{L}\right)_6$$

$$\times \left[(1.2 - 0.4)0.4 - \frac{1}{2}(0.4)^2\right]$$

$$\left(\frac{W}{L}\right)_6 = 1.33 \therefore \left(\frac{W}{L}\right)_6 = 2 = \frac{0.26\ \mu m}{0.13\ \mu m},$$

$$\left(\frac{W}{L}\right)_2 = \left(\frac{W}{L}\right)_3 = \frac{0.13\ \mu m}{0.13\ \mu m}$$

Checking

$$\frac{\left(\frac{W}{L}\right)_p}{\left(\frac{W}{L}\right)_a} \le \left(\frac{\mu_n}{\mu_p}\right)\left[1\left(1 - \frac{V_{tn}}{V_{DD} - V_{tn}}\right)^2\right]$$

(Assume $\left(\frac{\mu_n}{\mu_p} \cong 4\right)$

$$\frac{1}{2} \le 4\left[1 - \left(1 - \frac{0.4}{1.2 - 0.4}\right)^2\right] = 3$$

16.21

If the memory array has n columns, it has 2n rows and $2n^2$ cells

Refresh time is

$2n\,(30)\,10^{-9}\text{s} = (1.00 - 0.98)\,\text{s} \times 10^{-3}\text{s}$

Whence

$n = 0.02 \times 8 \times 10^{-3} / 40 \times 10^{-9} = 4000$

The corresponding memory capacity is

$2n^2 = 2(4000)^2$ or 32 M bits

16.22

For leakage current I, the voltage change on C in time T is $V = IT/C$

Correspondingly, $1 = I \times 10 \times 10^{-3}/20 \times 10^{-15}$, and the maximum leakage is $I = 20 \times 10^{-15}/10 \times 10^{-3} = \underline{2pA}$

16.23

For leakage current I, the voltage change on c in time T is $V = \dfrac{IT}{c}$

Hence,

$$0.2\text{V} = \frac{I \times 10^{-3} \times 10}{20 \times 10^{-15}} \Rightarrow I = 0.4 \times 10^{-12}$$

$= 0.4$ pA is the maximum leakage current.

16.24

For the bit-line output to reach $0.9\,V_{DD} = 2.7$ V from $V_{DD}/2 = 1.5$ V in 2ns for an initial bit-line signal at $0.1/2 = 0.05$ V:
signal of $0.1/2 = 0.05$ V:

$2.7 = 1.5 + 0.05e^{2/\hat{j}}$

whence $2/\hat{j} = \ln[(2.7 - 1.5)/0.05] = 3.178$

and $\hat{j} = 2/3.178 = 0.629$ ns

Thus $C/G_m = 0.629 \times 10^{-9}$s, and $G_m = 1 \times 10^{-12}/(0.629 \times 10)$
$= 1.589$ mA/V

For matched inverters $g_{mn} = g_{mp} = G_m/2 = 1.589/2$
$= 0.795$ mA/V

Now $g_m = k'(W/L)(v_{GS} - V_t)$

and $0.795 \times 10^{-3} = 100 \times 10^{-6}\,(W/L)_n[3.0/2 - 0.8)$

Thus $(W/L)_n = 0.795 \times 10^{-3}/(100 \times 10^{-6})/0.7$
$= 11.36$

Now, for devices assumed to have length L μm (or, alternatively, for each micron of device length) $W_n = 11.36$ μm and $W_p = 3(11.36) = 34.1$ μm

Now, for a differential input signal of 0.2V (and 0.1V on each bit-line), the response time is t, where $2.7 = 1.5 + 0.1\,e^{t/0.629}$ whence $t = 0.629 \ln(2.7 - 1.5)/0.1 = 1.56$ ns

16.25

Note that for the inverters

$k_n = k'_n\,(W/L)_n = 100(6/1.5) = 400\ \mu\text{A/V}^2$

$k_p = k'_p\,(W/L)_p = (100/2.5)(15/1.5) = 400\ \mu\text{A/V}^2$

Thus we see that the inverters are matched,

Generally, $i_o = 1/2\,k_n\,(v_{GS} - V_t)^2$ and $g_m = 2i_o/$
$3V_{ov} = kn(v_{gs} - v_t)$

Now, at $v_{GS} = v_o = V_{DD}/2 = 3.3/2 = 1.65$ V.

$g_{mn} = 400\,(1.65 - 0.8) = 340\ \mu\text{A/V}$

Thus $G_m = g_{mn} + g_{mp} = (340) = 680\ \mu\text{A/V}$

For a bit-line capacitance of 0.8 pF $\hat{j}$-C/G_m

or $\hat{j} = 0.8 \times 10-^{12}/680 \times 10^{-6} = 1.176$ ns

Now, for 0.9 V_{DD} reached in 2ns, for a signal Δv,
$0.9(3.3) = 1.65 + \Delta v e^{2/1.76}$

or $\Delta v = (2.97 - 1.65)/5.478 = 0.241$ V

Thus the initial voltage between B lines must be $2(0.241) = 0.482$ V

If an additional 1ns is allowed : $t = 2+1 = 3$ ns and $\Delta v = (2.97 - 1.65)/e^{3/1.176} = 0.103$ V allowing a signal to be used of $2(0.103) = 0.206$ V

Now, with the original bit-line signal of 0.241 V, and a delay of 3 ns:

$2.97 = 1.65 + 0.241e^{3/\hat{j}}$

and $e^{3/\hat{j}} = (2.97 - 1.65)/0.241 = 5.477$

$3/\hat{j} = \ln(5.477) = 1.7006$

whence $\hat{j} = 3/1.7006 = 1.764$ ns

Thus $C = G_m\,\hat{j} = 680 \times 10^{-6} \times 1.764 \times 10^{-9} =$
1.20 pF

This is an increase (from 0.8 pF) of $\left(\dfrac{1.2 - 0.8}{0.8}\right)$

$100 \times 50\%$

For the longer line, the initial delay to establish a suitable signal becomes 150% of 5ns = 7.5ns

16.26

(a) For an initial difference between bit lines of ΔV, each bit-line signal is $\Delta V/2$.

for the rising line: $v_B = \frac{V_{DD}}{2} + \frac{\Delta V}{2} e^{t/(C_B/G_m)}$

$$e^{t/(C_B/G_m)} = \frac{2}{\Delta V} \cdot (0.9 - 0.5)V_{DD} = \frac{0.8V_{DD}}{\Delta V}$$

Taking the natural log of both sides

$$\ln e^{t/(C_B/G_m)} = \ln\left(\frac{0.8V_{DD}}{\Delta V}\right)$$

$$t_d = \left(\frac{C_B}{G_m}\right)\ln\left(\frac{0.8V_{DD}}{\Delta V}\right) \text{ as stated}$$

(b) For reduction of one half the original, G_m has to be doubled. $G_m \propto (W/L)$

G_m is doubted by <u>doubling</u> the width of all transistors

(c) $V_{DD} = 1.8$ V, $\Delta V = 0.2$ V

original design:

$$t_d = \left(\frac{C_B}{G_m}\right)\ln\left(\frac{0.8(1.8)}{0.2}\right) = 0.9\left(\frac{C_B}{G_m}\right)$$

Reducing ΔV by 4: $\Delta V = \frac{0.2}{4} = 0.05$ V

$$t_d = \left(\frac{C_B}{G_{m2}}\right)\ln\left(\frac{0.8(1.2)}{0.05}\right) = 1.5\left(\frac{C_B}{G_{m2}}\right)$$

for these to be equal:

$$\frac{0.9}{G_m} = \frac{1.5}{G_{m2}} \text{ and } G_{m2} = \frac{1.5}{0.9}G_m = 1.7\,G_m$$

Thus the transistors must be made 70% wider (or increased by a factor of 1.7x)

16.27

For the DRAM arrangement, the signal is applied to only one side; Thus in comparison to the SRAM treatment, the applied signal is only half as large.

Now, the specification must be met for either a '0' a '1' stored. The worst case is a differential signal of 40 mV (corresponding to a single-side signal of 70 mV)

Thus $2.0 = 20\times10^{-3}e^{5/\hat{j}}$, and $5 = \hat{j}\ \ln(2/20\times10^{-3})$, or $5 = \hat{j}\ \ln(100) \times 4.605\ \hat{j}$, whence $\hat{j} = 1.086$ ns (next)

For a 1pF bit-line capacitance, $G_m\ C/\hat{j}$ or $G_m = 1 \times 10^{-12}/1.086 \times 10^{-9} = 0.921$ mA/V, with 0.921 = 0.46 mA/V from each transistor.

Now, for the n-channel device, $g_m = k'_n(W/L)_n(v_{GS} - V_t)$ or $0.46 \times 10^{-3} = 100 \times 10^{-6}(W/L)_n(2.5-1)$

Thus $(W/L)_n = (0.46/0.1)/1.5 = 3.07$

For matched inverters, $(W/L)_p = 2.5(3.07) = 7.68$

When a '1' is read, the response time will be $t = \hat{j}\ \ln(2/20 \times 10^{-3}) = 1.086 \ln 100 = 5$ns

(note: this is as designed!)

When a '0' is read, $t = 1.086 \times 10^{-9} \ln(2/(100/2) \times 10^{-3}) = 4.01$ns

16.28

$$\Delta t = \frac{CV_{DD}}{I}$$

$$I = \frac{CV_{DD}}{\Delta t} = \frac{60 \times 10^{-15} \times 1.2}{0.3 \times 10^{-9}} = 240\mu A$$

$$p = V_{DD} I = 1.2 \times 240\mu = 288\mu W$$

16.29

Here $2^n = 512$, $n\log_{10}2 = \log_{10} 512$, n = 2.709/ 0.301 = 9.00
Thus the number of bits is 9
The decoder has 512 output line, one of which is active (high). The NOR away requires true and complement input lines for each bit: 2 × 9 = 18
Each row uses 9 NMOS for a total of 9 × 512 = 4608 NMOS and 512 PMOS, for a total of 5120 transistors.

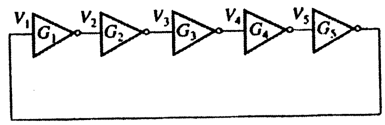
16.30

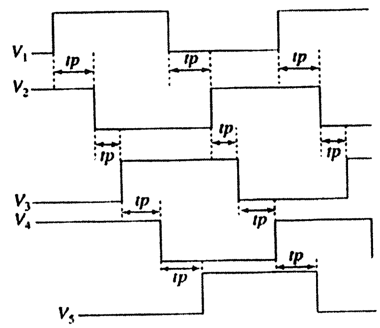
For a 256 K bit square away, there are $(256 \times 1024)^{1/2}$ = 512 rows and columns
Number of column-address bits is $\log_2 512 = 9$
Two multiplexors are needed, since both true and complement bit lines are required. For each multiplexor, there are 512 output lines.
For each (half) multiplexor, 512 NMOS needed for a total of 1024 NMOS pass gates.
For the 512 output NOR decoder itself, 512 × 9 = 4608 NMOS and 512 PMOS are needed.
The address-bit inverters need 9 NMOS and 9 PMOS Overall, the need is for 1024 + 4608 + 9 = 5641 NMOS and 512 + 9 = 521 PMOS, for a total of 6162 transistors.

16.31

From the solution . above, a square 256 K-bit array has 512 rows and 512 columns for which 9 row and 9 column address bits are needed
Check: $2^{9+9} = 2^{18} = 262144$
For the tree 9 levels of pass gates are needed.
The total number of pass gates is N = 2+4+8+16+32+64+128+256+512
See that N = 2 + 2 (N − 512), or N = 2 + 2N − 1024, whence N = 1022
Thus a tree column decoder for 9 bits needs 1022 pass transistors
For true and complement bit lines, a total of 2(1022) = 2044 pass transistors are needed. Compare this with the number required beyond the input inverters namely 6162 − 18 = 6144

16.32





$$t_P = \frac{1}{2}(t_{PLH} + t_{PHL}) = \frac{1}{2}(6n + 4n) = 5 \text{ ns.}$$

$$f = \frac{1}{10t_P} = 20 \text{ MHz}$$

16.33

$N = 11$

$$f = 20\ \mu\text{Hz} = \frac{1}{2Nt_p} = \frac{1}{2(11)t_p}$$

$\therefore t_p = 2.3$ nsec

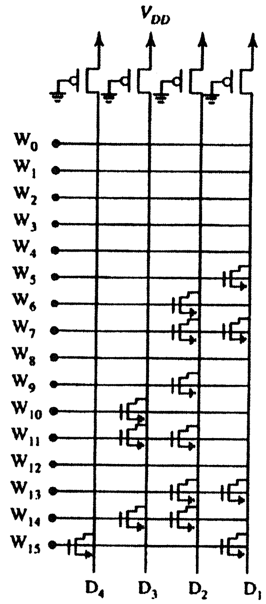
16.34

Note that the output is high if no word is selected. Thus, logically, high must correspond to logic 0 (and no transistor, as noted in the text).
Correspondingly, the words stored in are 0100, 0000, 1000, 1001, 0101, 0001, 0110, and 0010.

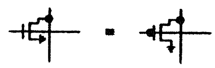
16.35

Need $z = x + y$

X	Y	Z
00	00	0000
00	01	0000
00	10	0000
00	11	0000
01	00	0000
01	01	0001
01	10	0010
01	11	0011
10	00	0000
10	01	0010
10	10	0100
10	11	0110
11	00	0000
11	01	0011
11	10	0110
11	11	1001



Note that a total of 14NMOS and 4PMOS are used.



16.36

(a) For the PMOS, with $V_B = 2.5$ V

$L_D = (90/3)10^{-6}(12/1.2)[(5-1)2.5 - 2.5^2/2]$

$= 30 \times 10^{-6}(10)[4(2.5) - 2.5^2/2]$

$= 2.0625$ mA

Thus the average charging current is 2.06 mA
Time for precharge $t = CV/I$
whence

$t = 1 \times 10^{-12}(5-0)/(2.06 \times 10^{-3}) = 2.42$ ns

(b) For the word-line rise,

$T = RC = 5 \times 10^3 \times 2 \times 10^{-12} = 10$ ns

Here, $v_w = 5(1 - e^{-t/10})$

Thus the rise time (10% to 90%) is essentially the time t to 90%, where

$0.9(5) = 5(1 - e^{-t/10})$

$e^{-t/10} = 0.1$

and $t = -10 \ln(0.1) = 23$ ns

At the end of one time constant, $T =] = 10$ ns

and $v_w = 5(1 - e^{-10/10}) = 3.16$ V

For discharge,

$i_{D_{av}} = 1/2\, k_n' \,(W/L)_n(v_{gs} - V_t)^2$

$= 1/2(90)(3/1.2)(3.16 - 1)^2 = 525\ \mu A$

Thus, the bit-line voltage will lower by 1V in

about $\Delta t = C\Delta V / i_{D_{av}} = 1 \times 10^{-12}$, (× ...

...) $\times 1/(525 \times 10^{-6}) = 1.90$ ns