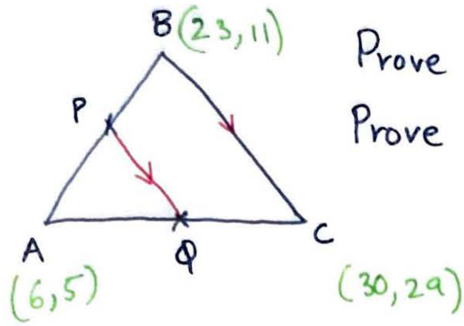
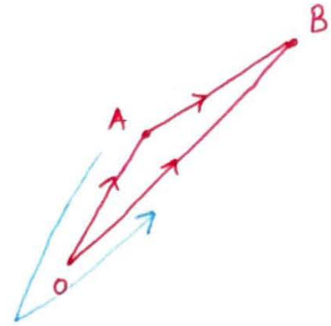
Exersice 2.1.2



Prove $PQ \parallel BC$

Prove $BC = 2PQ$



Equation of vector $\vec{PQ}$?

$\vec{OA} = (6,5) \qquad \vec{OB} = (23,11)$

$\vec{AB} = \vec{AO} + \vec{OB} = (-6,-5) + (23,11)$

$\vec{AB} = (17,6)$

$\frac{1}{2}\vec{AB} = \vec{AP} = (8.5, 3)$

$\vec{OA} = (6,5) \qquad \vec{OC} = (30,29)$

$\vec{AC} = \vec{AO} + \vec{OC} = (-6,-5) + (30,29)$

$\vec{AC} = (24,24)$

$\frac{1}{2}\vec{AC} = \vec{AQ} = (12,12)$

$\vec{PQ} = \vec{PA} + \vec{AQ} = (-8.5,-3) + (12,12)$

$\boxed{\vec{PQ} = (3.5, 9)}$

We can see that $2\vec{PQ} = \vec{BC}$

Equation of vector $\vec{BC}$?

$\vec{BC} = \vec{BA} + \vec{AC} = (-17,-6) + (24,24)$

$\boxed{\vec{BC} = (7,18)}$

When two vectors are parallel, their unit vector is same.

unit vector of $\vec{PQ} = \left(\frac{3.5}{3.5}, \frac{9}{3.5}\right) = \left(1, \frac{18}{7}\right)$

unit vector of $\vec{BC} = \left(\frac{7}{7}, \frac{18}{7}\right) = \left(1, \frac{18}{7}\right)$

Same

Exersice 2.1.4

Angle between vectors ?

$\vec{OA} = (1, 2, -1, 3)$
$\vec{OB} = (2, 1, 2, 1)$

$$a.b = ||a|| \, ||b|| \cos\theta$$

$$\vec{OA} \cdot \vec{OB} = (1\times2) + (2\times1) + (-1\times2) + (3\times1) = 2+2-2+3$$

$$\vec{OA} \cdot \vec{OB} = 5$$

$$||\vec{OA}|| = \sqrt{(1)^2 + (2)^2 + (-1)^2 + (3)^2} = \sqrt{1+4+1+9} = \sqrt{15}$$

$$||\vec{OB}|| = \sqrt{(2)^2 + (1)^2 + (2)^2 + (1)^2} = \sqrt{4+1+4+1} = \sqrt{10}$$

$$\frac{\vec{OA} \cdot \vec{OB}}{||\vec{OA}|| \, ||\vec{OB}||} = \cos\theta$$

$$\boxed{\theta = \cos^{-1}\left(\frac{5}{\sqrt{15}\sqrt{10}}\right) = 65.9^\circ}$$

Exersice 2.1.8

Region $3x - 4y > 12$?

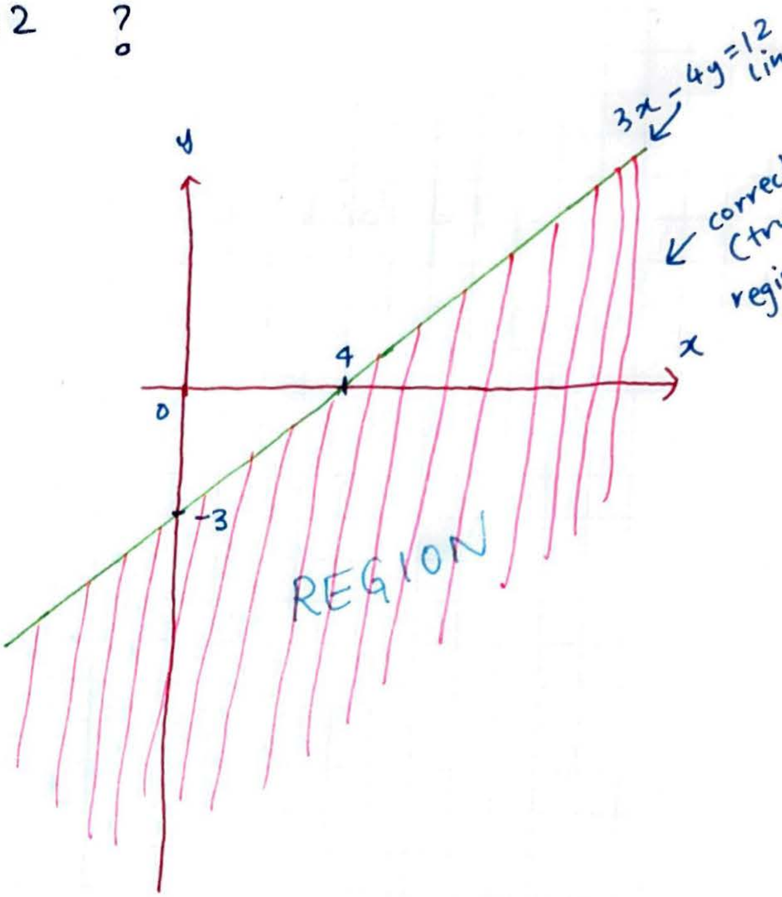
x-intercept ?

$3x - 4(y) = 12$

$3x - 4(0) = 12$

$3x = 12$

$x = \frac{12}{3} = 4$



y-intercept ?

$3(x) - 4y = 12$

$3(0) - 4y = 12$

$-4y = 12$

$y = \frac{12}{-4} = -3$

Exersice 2.1.9

$P = (2, 3) \qquad Q = (8, 11)$

ⓐ Perpendicular bisector of PQ ?

ⓑ Inequality of region containing point Q ?

ⓐ midpoint of PQ ?

$$M = \left(\frac{2+8}{2}, \frac{3+11}{2}\right) = (5, 7)$$

Gradient of line PQ ?

$$m_{PQ} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{11-3}{8-2} = \frac{4}{3}$$

Gradient of $\perp$ line to PQ ?

$$m_{\perp} \times m_{PQ} = -1$$

$$m_{\perp} = -\frac{3}{4}$$

Equation of perpendicular bisector of PQ ?

$$y - y_1 = m(x - x_1)$$

$$y - 7 = -\frac{3}{4}(x - 5)$$

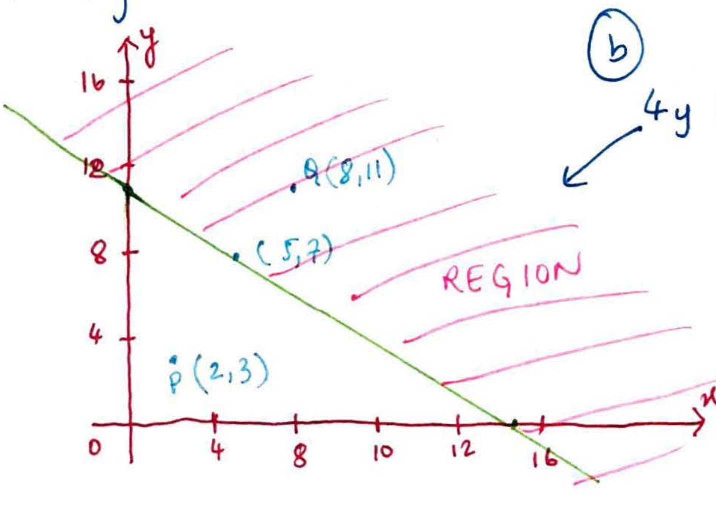
$$y - 7 = -\frac{3}{4}x + \frac{15}{4}$$

$$y = -\frac{3}{4}x + \frac{43}{4}$$

$\to$ y-intercept $= \frac{43}{4} \approx 10.75$

x-intercept $= \frac{43}{3} \approx 14.33$

$$4y = -3x + 43$$



ⓑ

$$4y + 3x > 43$$

Exersice 2.1.13

Region $x_1 + 3x_2 + 2x_3 < 6$

x_1 intercept

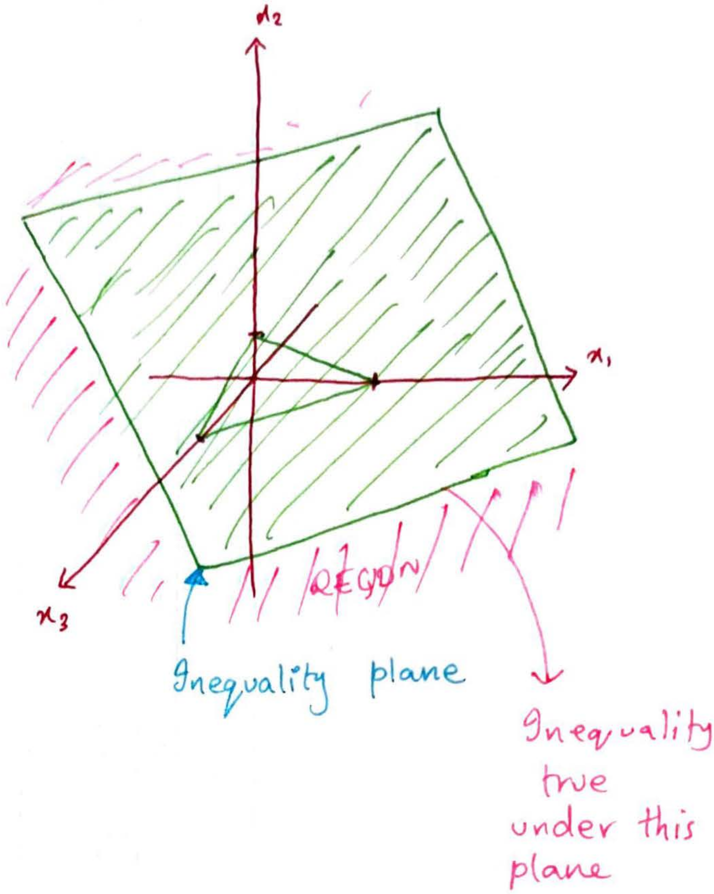
$x_1 < 6$

x_2 intercept

$x_2 < 2$

x_3 intercept

$x_3 < 3$



Exersice 2.1.14

$P = (2, 1, 1)$

plug in to plane equation

$x_1 - x_2 - 2x_3 = \text{test}$

$2 - 1 - 2(1) = \text{test}$

$\text{test} = -1$

$\text{test} < 4$

∴ ~~Eq~~ Inequality of ~~plane~~ region seperated by plane & containing point $(2, 1, 1)$:-

$x_1 - x_2 - 2x_3 < 4$

Exersice 2.2.1

$$y = T(x_1, x_2, x_3)$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 2x_1 - x_2 + 3x_3 \\ x_1 + 2x_2 + x_3 \\ x_1 + x_2 + x_3 \end{bmatrix}$$

$$y_1 = 2x_1 - x_2 + 3x_3$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 2 & -1 & 3 \\ 1 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\frac{dy_1}{dx_1} = -1$$

$$\begin{bmatrix} \frac{dy_1}{dx_2} \\ \frac{dy_2}{dx_1} \\ \frac{dy_3}{dx_1} \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\begin{bmatrix} \frac{dy_1}{dx_2} \\ \frac{dy_2}{dx_1} \\ \frac{dy_3}{dx_1} \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$$

Exersice 2.2.2

$$y = T(x)$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} w_{11} & w_{12} & w_{13} & w_{14} \\ w_{21} & w_{22} & w_{23} & w_{24} \\ w_{31} & w_{32} & w_{33} & w_{34} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

$$\frac{\partial y_3}{\partial x_2} = w_{32} \qquad \frac{\partial y_3}{\partial x_3} = w_{33}$$

$$\frac{\partial y_3}{\partial w_{24}} = 0 \qquad \frac{\partial y_2}{\partial w_{23}} = x_3$$

Small Change in w_{24} does not affect y_3